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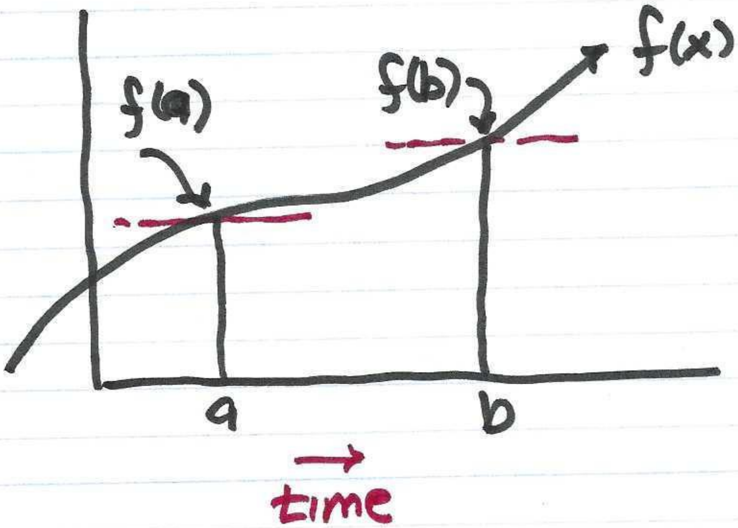
Ch 3.3

Rates of Change



$f(b) - f(a)$ = net change a to b

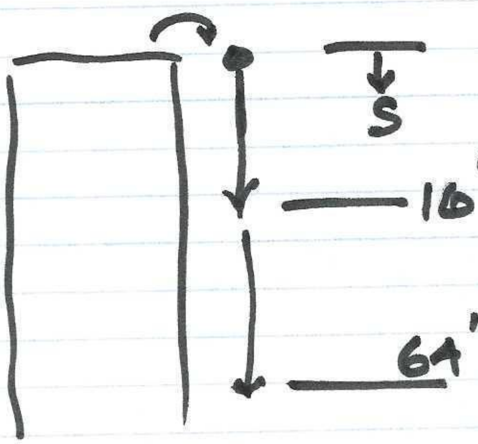
$b - a$ = " " in time

$$\bar{f}_{a,b} = \frac{f(b) - f(a)}{b - a}$$

Average change $\dfrac{f(b)-f(a)}{b-a}$

Instantaneous change

$$\lim_{\Delta t \to 0}\left(\frac{f(a+\Delta t)-f(a)}{\Delta t}\right)$$



distance is $s(t)$

$$s = 16t^2$$

Average Speed $t=0 \to t=1$

16 ft/sec

$t=1 \to t=2$

48 ft/sec

$$\lim_{\Delta t \to 0}\left(\frac{s(1+\Delta t)-s(1)}{\Delta t}\right)$$

③

$$s(1+\Delta t) = 16(1+\Delta t)^2$$
$$s(1) = 16$$
$$\Delta t = \text{increment of time}$$

$$\lim_{\Delta t \to 0}\left(\frac{\cancel{16}\left(1 + 2\Delta t + (\Delta t)^2\right) - \cancel{16}}{\Delta t}\right)$$

$$= \lim_{\Delta t \to 0}\left(16 \cdot 2 + \Delta t\right) \to 2 \cdot 16 = \underline{32 \text{ fps}}$$

$$t = t_0$$

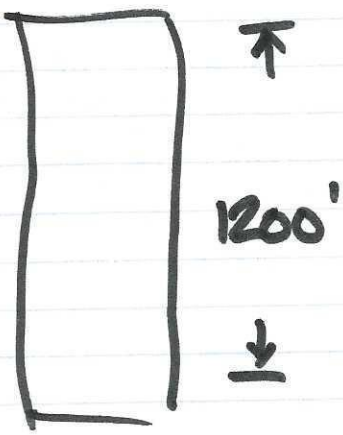
$$s(t_0+\Delta t) = 16(t_0+\Delta t)^2$$
$$s(t_0)$$
$$\Delta t = \text{increment of time}$$

$$\lim_{\Delta t \to 0}\left(\frac{16(t_0+\Delta t)^2 - 16t_0^2}{\Delta t}\right)$$

$$\lim_{\Delta t \to 0} \left(\frac{16(\cancel{t_0^2} + 2t_0\Delta t + (\Delta t)^2 - \cancel{16t_0^2})}{\Delta t} \right)$$

$$= \lim_{\Delta t \to 0} 16(2t_0 + \Delta t) = (32t_0 + 16\underbrace{\Delta t}_{0})$$

$$= 32t_0$$



$$16t^2 = 1200'$$

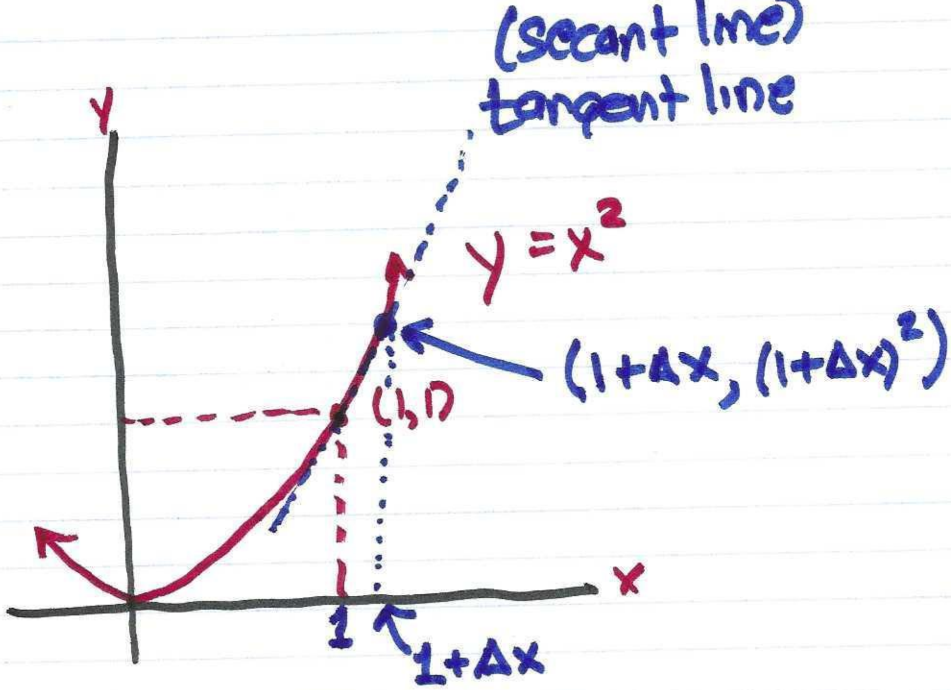
$$t^2 = \frac{1200}{16}$$

$$t = \sqrt{75} \cong 8.6$$

$$32 \cdot 8.6 \text{ fps} = \underline{275 \text{ fps}}$$

$$60 \text{ mph} = 88 \text{ fps}$$

187.5 mph



slope of blue (secant line) $= \dfrac{(1+\Delta x)^2 - 1}{(1+\Delta x) - 1}$

slope of tangent (when $\Delta x \to 0$)

$$\lim_{\Delta x \to 0} \left(\frac{(1+\Delta x)^2 - 1}{\Delta x} \right)$$

$$\lim_{\Delta x \to 0} \left(\frac{\cancel{1} + 2\Delta x + (\Delta x)^2 \cancel{- 1}}{\Delta x} \right)$$

$$= \lim_{\Delta x \to 0} \left(2 + \Delta x \right) = 2$$

What we are calculating by taking limits of difference quotients is the "derivative" of the function.

$$f(x) \rightsquigarrow f'(x)$$

In summary:

$$f'(x) := \lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$$

$$f(x) = \text{constant} = c$$

constant $\lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right) = \lim_{\Delta x \to 0} \left(\frac{c - c}{\Delta x} \right)$

x $\lim_{\Delta x \to 0} \left(\frac{(x+\Delta x) - x}{\Delta x} \right) = 1$

$$c \cdot f(x) = \lim_{\Delta x \to 0} \left(\frac{c\, f(x+\Delta x) - c\, f(x)}{\Delta x} \right)$$

$$= c \cdot \underbrace{\lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)}_{f'(x)}$$

Table of Derivatives:

$f(x)$	$f'(x)$
c, constant	0
x	1
x^2	$2x$
x^3	$3x^2$
$\vdots$	
x^n	nx^{n-1}
$c\,f(x)$	$c \cdot f'(x)$
$\sqrt{x} = x^{1/2}$	$\frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2x^{1/2}} = \frac{1}{2\sqrt{x}}$
$\frac{1}{x} = x^{-1}$	$(-1)x^{-1-1} = -\frac{1}{x^2}$

Consider polynomial :

$$f = 6x^4 - 2x^3 + 5x^2 + x - 7$$

$$6(4x^3) - 2(3x^2) + 5(2x) + 1 - 0$$

$$f' = \boxed{24x^3 - 6x^2 + 10x + 1}$$

p.187 Example 6

Let $f(x) = 2x^3 + 4x$

Find : $f'(x), f'(2), f'(-3)$

$$f'(x) = 2(3x^2) + 4(1) = \underline{6x^2 + 4}$$

$$f'(2) = 6 \cdot 2^2 + 4 = 28$$

$$f'(-3) = 6(9) + 4 = 58$$