11/7

What function differentiated is $2x$?

How about x^2? $(x^2)' = 2x$

Also x^2+1, x^2+2, x^2+100, $x^2+\pi$

<u>all</u> have derivative $2x$.

Special symbol

integrator

$$F(x) = \int f(x)\,dx$$

integral sign

integrand

$$\int 2x\,dx = x^2 + C$$

General $F(x) = \int f(x)\,dx$

Specific $x^2 + C = \int 2x\,dx$ C, any constant

2

Integration Rules

constant

① $\int k\,dx = kx + C$ constant

② $\int x\,dx = \frac{x^2}{2} + C$ linear function

③ $\int x^2\,dx = \frac{x^3}{3} + C$ square

④ $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ general $(n \neq -1)$

⑤ $\int \frac{dx}{x} = \ln x + C$ logs

⑥ $\int A f(x)\,dx = A \int f(x)\,dx$ constant mult rule

⑦ $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

⑧ $\int (a_n x^n \pm a_{n-1} x^{n-1} \pm \cdots a_1 x + a_0)\,dx =$

$= \frac{a_n x^{n+1}}{n+1} \pm \frac{a_{n-1} x^n}{n} \pm \cdots \frac{a_1 x^2}{2} + a_0 x + C$

9 $\int e^x dx = e^x + C$

10 $\int e^{kx} dx$, (k constant) $= \dfrac{e^{kx}}{k} + C$

11 $\int a^x dx = \dfrac{a^x}{\ln a} + C$

another base (>0 ; $\neq 1$)

Prob's:

11 $\int (2z+3)\,dx = 2zx + 3x + C$

$\int (2x+3)\,dx = x^2 + 3x + C$

14 $\int (5x^2 - 6x + 3)\,dx = \dfrac{5x^3}{3} - 6 \cdot \dfrac{x^2}{2} + 3x + C$

$\int 5x^2 - 6x + 3\,dx = \boxed{\frac{5}{3}x^3 - 3x^2 + 3x + C}$

★ Now Rule: $\int x^r dx \; [r \in \mathbb{R} \, (\neq -1)] = \dfrac{x^{r+1}}{r+1} + C$

Ex. $\int \sqrt{x}\, dx = \int x^{1/2} dx = \dfrac{x^{3/2}}{(3/2)} + C$

$= \dfrac{2}{3} x^{3/2} + C$

#18 $\int (t^{1/4} + \pi^{1/4}) dt = \dfrac{t^{5/4}}{(5/4)} + \pi^{1/4} \cdot t$

#24 $\int (56 t^{5/2} + 18 t^{7/2}) dt = 2$

$\dfrac{56 t^{7/2}}{(7/2)} + \dfrac{18 t^{9/2}}{(9/2)} + C$

$\downarrow \qquad \downarrow$

$16 t^{7/2} + 4 t^{9/2} + C$

(5)

#28 $\int \left(\sqrt{u} + \frac{1}{u^2}\right) du = \int \left(u^{1/2} + u^{-2}\right) du =$

$$= \frac{u^{3/2}}{(3/2)} + \frac{u^{-1}}{(-1)} + C$$

$$\boxed{= \frac{2}{3} u^{3/2} - \frac{1}{u} + C}$$

#38 $\int \frac{2y^{1/2} - 3y^2}{6y} dy \stackrel{?}{=} \int \left(\frac{1}{3} y^{-1/2} - \frac{y}{2}\right) dy =$

$$= \frac{1}{3} \frac{y^{\frac{1}{2}}}{(1/2)} - \frac{1}{2} \cdot \frac{y^2}{2} + C$$

$$= \boxed{\frac{2}{3}\sqrt{y} - \frac{y^2}{4} + C}$$

#44 $\displaystyle\int \frac{1-2\sqrt[3]{z}}{\sqrt[3]{z}}\,dz \stackrel{?}{=} \int\left(z^{-1/3}-2\right)dz$

$$= \frac{z^{2/3}}{(2/3)} - 2z + C = \frac{3}{2}z^{2/3} - 2z + C$$

Applications: <u>Fixed + Variable = Total Cost</u>

#49. <u>MC</u> $= C'(x) = 4x - 5$ $\quad FC = \$8$

$x =$ <u>quantity</u>

$$C(x) = \int C'(x)\,dx = \int (4x-5)\,dx =$$

$$C(x) = 4\left(\frac{x^2}{2}\right) - 5x + C_1$$

$$= 2x^2 - 5x + C_1$$

$C(0) =$ fixed $= 8$

$$C(0) = 2\cdot 0 - 5\cdot 0 + C_1 = 8 \Rightarrow C_1 = 8$$

#49 cont'd $C(x) = 2x^2 - 5x + 8$

#53 $C'(x) = x^{2/3} + 2$ (8 units cost \$58)

$$C(x) = \int (x^{2/3} + 2)\,dx = \frac{x^{5/3}}{(5/3)} + 2x + C_1$$

$$C(x) = \frac{3}{5}x^{5/3} + 2x + C_1$$

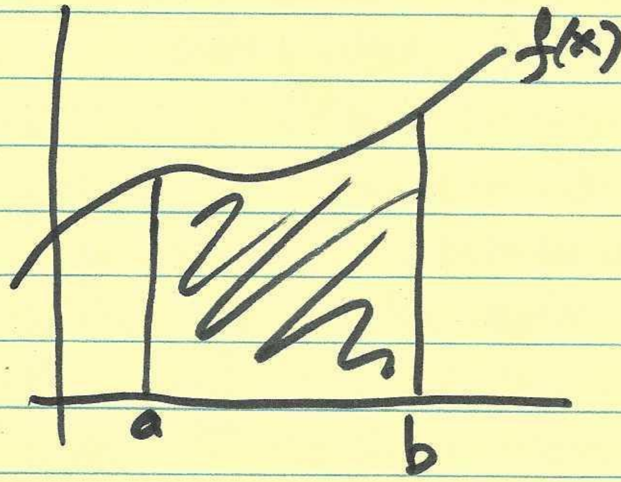
$$C(8) = \frac{3}{5}(8)^{5/3} + 2 \cdot 8 + C_1 = 58$$

$$= \frac{3}{5}(32) + 16 + C_1 = 58$$

$$19.2 + 16 + C_1 = 58$$

$$C_1 = 58 - 35.2 = 22.8$$

8



$$\int f(x)\,dx = F(x) + C$$

eval @ a & b &

take difference

$$P'(x) = \sqrt{x} + \frac{1}{2}$$

x in 000 burgers

P(x) in \$

If no burgers are sold, $P(x) = -\$1000$

$$P(x) = \int \left(\sqrt{x} + \frac{1}{2}\right) dx = \int \left(x^{1/2} + \frac{1}{2}\right) dx =$$

$$= \frac{x^{3/2}}{(3/2)} + \frac{x}{2} + C_1$$

$P(x) = \frac{2}{3}x^{3/2} + \frac{x}{2} + C_1$

$P(0) = -\$1{,}000 = C_1$

$P(x) = \frac{2}{3}x^{3/2} + \frac{x}{2} - 1000$

If you sell 64 burgers, what is profit

$P(64) = \frac{2}{3}64^{3/2} + \frac{64}{2} - 1000$

$= \left(\frac{2}{3}\right)(512) + 32 - 1000$

#63 $P'(x) = x(50x^2 + 30x)$

x in 00 lbs
\$ in \$

$P(0) = -40$

$P(x) = \int(50x^3 + 30x^2)\,dx$

$= \frac{50x^4}{4} + \frac{30x^3}{3} + C_1$

10

$P(x) = 12.5x^4 + 10x^3 + C_1$

$P(0) = 0 + 0 + C_1 = -40$

$P(x) = 12.5x^4 + 10x^3 - 40$

$P(200) = (12.5)(2^4) + 10 \cdot 2^3 - 40$ $\qquad \boxed{\begin{array}{l}(2 \times 10^2)^4 \\ 16 \times 10^8\end{array}}$

$P(200) = (12.5)(16) + 80 - 40$