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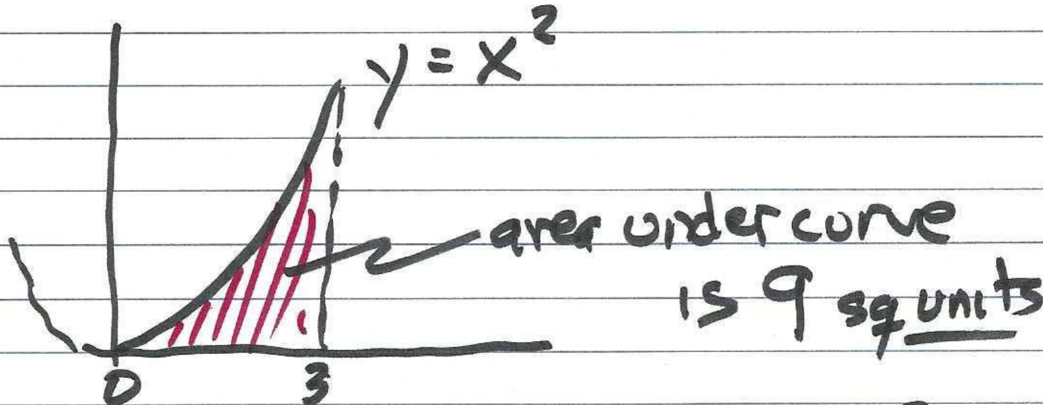
## 7.3 Areas

The definite integral

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$




$$\int_0^3 x^2\,dx = F[x] = \left[\frac{x^3}{3}\right]_0^3 = \frac{3^3}{3} = 9$$

$$\left(\frac{x^3}{3}\right)' = \frac{3x^2}{3} = x^2 !$$

If $f(x)$ is a rate of change,

then $\displaystyle\int_a^b f(x)\,dx$ is the total net change

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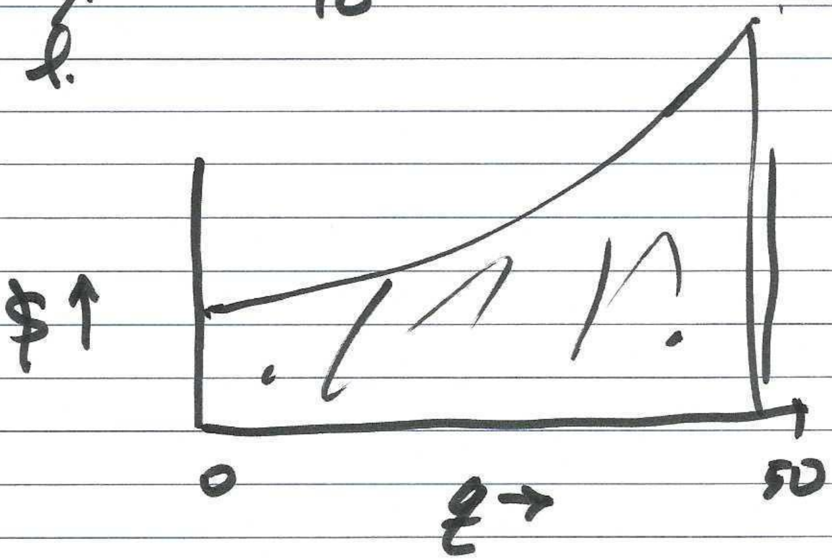
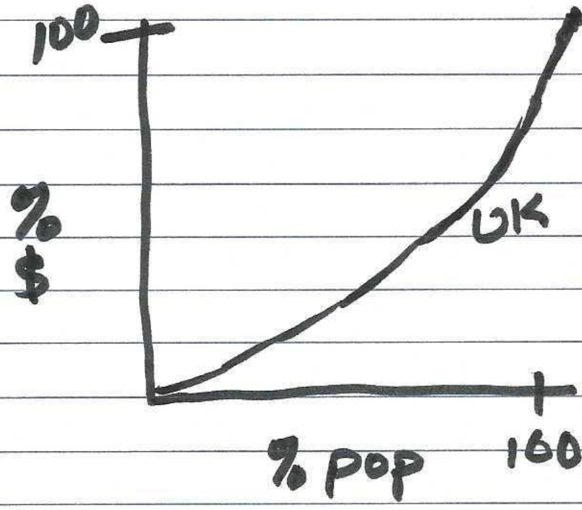
$$D(q) = \frac{q^2}{10} - 10q + 260 \qquad \Big|\; 0 \le q \le 50$$

\$ ℓ.

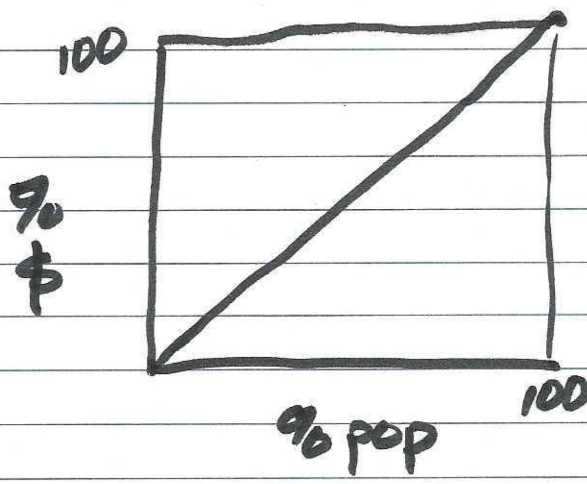



$$\int_0^{50} D(q)\,dq = \int_0^{50} \left(\frac{q^2}{10} - 10q + 260\right) dq$$

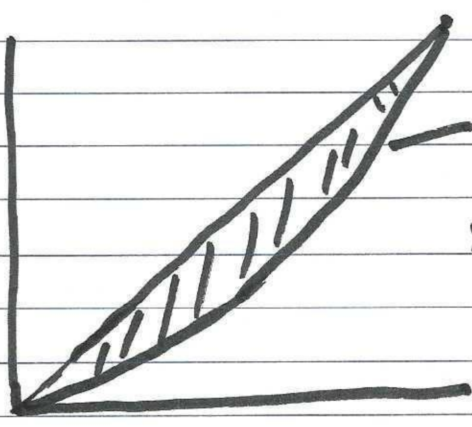
$$= \left[\frac{q^3}{30} - 5q^2 + 260q\right]_0^{50} = \frac{50^3}{30} - 5\cdot 50^2 + 260\cdot 50$$




reality

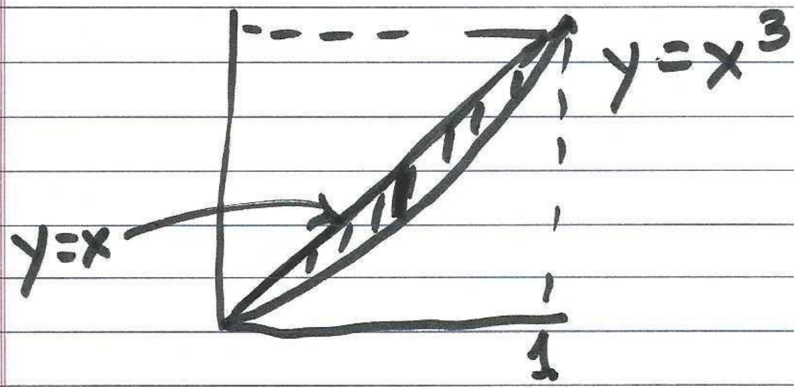



socialistic ideal



measure of inequality is $\frac{1}{2}$ this area

(4)




$$\text{Area} = \int_0^1 (x - x^3)\,dx$$

$$= \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

inequality index = $1/8 \sim \underline{.125}$

p. 436

7 $$\int_{-1}^{2}(5t-3)\,dt = \left[\frac{5t^2}{2} - 3t\right]_{-1}^{2} =$$

$$\left(\frac{5\cdot 2^2}{2} - 3\cdot 2\right) - \left(\frac{5\cdot(-1)^2}{2} - 3(-1)\right) =$$

$$(10-6) - \left(\frac{5}{2}+3\right) = \left(4 - \frac{11}{2}\right) = -\frac{3}{2}$$

10 $$\int_{-2}^{3}(-x^2-3x+5)\,dx = \left[-\frac{x^3}{3} - \frac{3x^2}{2} + 5x\right]_{-2}^{3} =$$

$$\left(-\frac{27}{3} - \frac{27}{2} + 15\right) - \left(\frac{8}{3} - 6 - 10\right) =$$

you can do this

#12

$$\int_3^9 \sqrt{2r-2}\,dr \quad \text{use substitution}$$

$$u = 2r-2 \Rightarrow du = 2dr$$

$$\int_{r=3}^{r=9} \sqrt{u}\,\frac{du}{2} = \frac{1}{2}\int_{r=3}^{r=9} u^{1/2}\,du$$

$$= \frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{3}u^{3/2}\right]_{r=3}^{r=9} = \left[\frac{1}{3}(2r-2)^{3/2}\right]_3^9$$

$$\left(\frac{1}{3}(64) - \frac{1}{3}(8)\right) = \frac{1}{3}\cdot 56 = \frac{56}{3}$$

7

#21 $\int_{1/2}^{1} (p^3 - e^{4p})\,dp = \left[\frac{p^4}{4} - \frac{e^{4p}}{4}\right]_{1/2}^{1} =$

$\frac{1}{4}\left[p^4 - e^{4p}\right]_{1/2}^{1} = \frac{1}{4}\left[\left(1 - e^4\right) - \left(\frac{1}{16} - e^2\right)\right]$

$= \frac{15}{64} + \frac{e^2 - e^4}{4}$

#30 $\int_{1}^{3} \frac{\sqrt{\ln x}}{x}\,dx$ let $u = \ln x$

then $du = \frac{dx}{x}$

$\int_{x=1}^{x=3} u^{1/2}\,du \rightarrow \left[\frac{2}{3}u^{3/2}\right]_{x=1}^{x=3} \rightarrow \frac{2}{3}\left[(\ln x)^{3/2}\right]_{1}^{3}$

$= \frac{2}{3}\left((\ln 3)^{3/2}\right)$

#45 $\text{Area} = \int_0^2 (4-x^2)\,dx - \int_2^3 (4-x^2)\,dx$

$$= \left[4x - \frac{x^3}{3}\right]_0^2 - \left[4x - \frac{x^3}{3}\right]_2^3$$

$$\left(8-\frac{8}{3}\right) - \left((12-9) - \left(8-\frac{8}{3}\right)\right)$$

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#59 $P'(t) = (3t+3)(t^2+2t+2)^{1/3}$ ←

$$P = \int_0^3 P'(t)\,dt$$

$$P'(t) = 3(t+1)(t^2+2t+2)^{1/3}$$

$$3\left[(t+1)^3(t^2+2t+2)\right]^{1/3}$$

$$(t^3+3t^2+3t+1)$$

(9)

$P'(t) = 3(t+1)\left((t+1)^2+1\right)^{1/3}$

Try: $u = t+1 \Rightarrow du = dt$

$3u\left(u^2+1\right)^{1/3}$

$\cancel{3\left(u^5+u^3\right)^{1/3}}$

Let $u^2+1 = v$

$2u\,du = dv$

$3\int_{t=0}^{3} v^{1/3}\frac{dv}{2} = \frac{3}{2}\int_{t=0}^{3} v^{1/3}dv$

$\left[\frac{3}{2}\cdot\frac{3}{4}v^{4/3}\right]_{t=0}^{3} \rightarrow$ back sub:

$\frac{9}{8}\left(u^2+1\right)^{4/3} = \left[\frac{9}{8}\left((t+1)^2+1\right)^{4/3}\right]_0^3$

10

$$\frac{9}{8}\left[17^{4/3} - 2^{4/3}\right] = \underline{?}$$

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$$L'(t) = \frac{80 \ln(t+1)}{t+1}$$

$$L(1 \text{ day}) = \int_0^{24} \frac{80 \ln(t+1)\, dt}{t+1}$$

$x = t+1$

$dx = dt$

$$= 80 \int_{t=0}^{24} \frac{\ln x}{x} dx$$

$d(\ln x) = \frac{dx}{x}$

Let $u = \ln x$

$du = \frac{\ln x}{x} dx$

$$= 80 \int_{t=0}^{24} du =$$

$$80 \int_{t=0}^{24} du = 80 \left[u\right]_{t=0}^{24} = 80 \left[\ln x\right]_{t=0}^{24}$$

11

$$L(1\ \text{day}) = 80\left[\ln(t+1)\right]_0^{24}$$

$$= 80(\ln 25 - \ln 1) = \boxed{80 \ln 25}$$