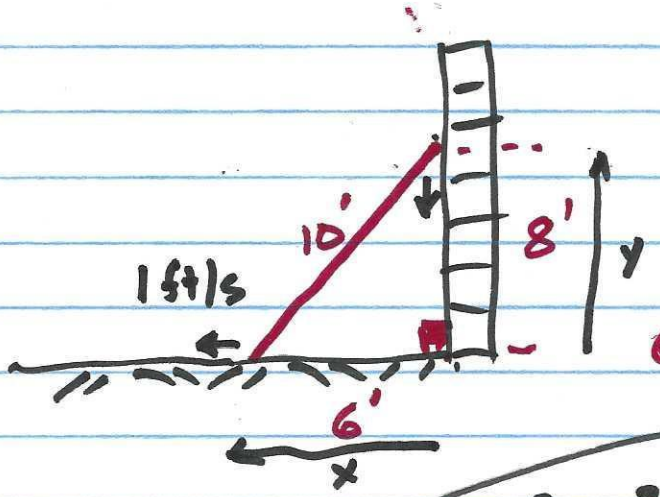
10/29

Exam #4 on Thurs. 10/31

6.5 Related Rates



$6^2 + 8^2 = 36 + 64 = 100 = 10^2$

$x^2 + y^2 = 10^2$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = -\frac{x}{y}\frac{dx}{dt}$$

$x = 6$
$y = 8$
$\frac{dx}{dt} = 1$

$$\frac{dy}{dt} = -\frac{6}{8}(1) = -\frac{3}{4}\ \text{ft/s}$$

To solve related rate problems:

(1) Express geometric or other relation between two variables

(2) Differentiate across the board everything with respect to time

(3) You should have 2 rates from step 2. One is known, one is unknown

(4) Solve for unknown rate in terms of the known rate and values of the other variables as needed

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Peanut demand function is:

$$P(q) = 50 - \frac{q}{200}$$

Find rate of change of revenue w/r/t time when daily production is 200 units/day

Revenue = price × quantity

(3)

q 9

$$R(q) = q\left(50 - \frac{q}{200}\right)$$

Recall: q is changing at rate of 50 cases per day.

$$R(q) = 50q - \frac{q^2}{200}$$

$$\frac{dR}{dt} = \frac{dR}{dq}\cdot\frac{dq}{dt}$$

$$\frac{dR}{dt} = 50\frac{dq}{dt} - \frac{q}{100}\frac{dq}{dt}$$

(200 → $\frac{q}{100}$; $\frac{dq}{dt}$ = +50; $\frac{dq}{dt}$ = +50)

$$\frac{dR}{dt} = (50)(50) - \frac{200}{100}\cdot 50$$

$$= 2500 - 100 = 2400 \text{ \$/day}$$

(a)

$$V = k(R^2 - r^2)$$

$k = 375$

Suppose blood vessel has radius $R = 0.08$mm

Cold weather causes vessel to contract

@ $\frac{dR}{dt} = -0.01$ mm/min.

How fast is blood velocity changing.

Want to know $\frac{dV}{dt}$ given $\frac{dR}{dt}$.

$$\frac{dV}{dt} = 2kR \cdot \frac{dR}{dt} + 0$$

$$\frac{dV}{dt} = 2(375)(0.08)\cdot(-0.01) = -0.6 \frac{mm}{min^2}$$

#13 $C(x) = 0.2x^2 + 10{,}000$

How fast is $C(x)$ changing when x is changing @ 12 units/mo. & production level is @ 80 units?

$$\frac{dC}{dt} = 0.4x\frac{dx}{dt} + \not{0}$$

$$= (0.4)(80)(+12) = (.4)(960) = \_\_\_$$

#21 $b = 0.22\,m^{0.87}$

m is mass (gm)
b = brain mass (gm)

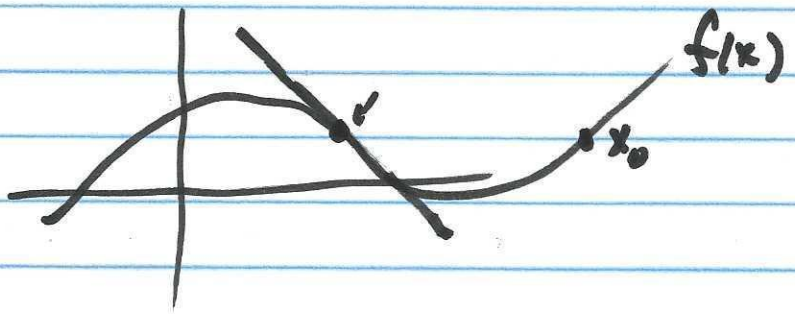
Suppose we have a 25 g fetus gaining mass at rate of 0.25 g/day

$$\frac{db}{dt} = (0.22)(0.87)\,m^{-0.13}\frac{dm}{dt}$$

$\frac{db}{dt}$ ↑ ; m ↑ 25g ; $\frac{dm}{dt}$ ↖ 0.25g/day

Review:

Increasing / Decreasing Functions

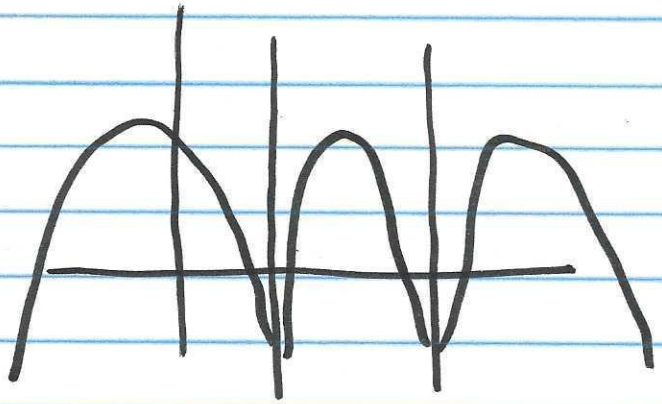


Evaluate $f'(x)$ @ x_0 then if

$f'(x_0) > 0$ inc

< 0 dec

Relative Extrema:



Critical value of $f(x)$ is an "x" where

(i) $f'(x) = 0$

(ii) $f'(x)$ DNE

What are endpts of domain?

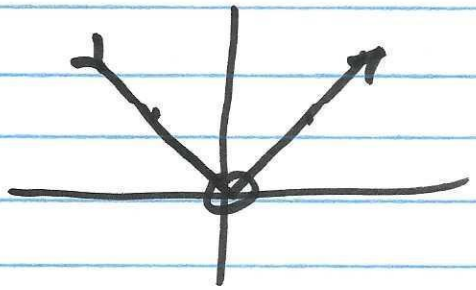
f might be defined on less than $\mathbb{R}$

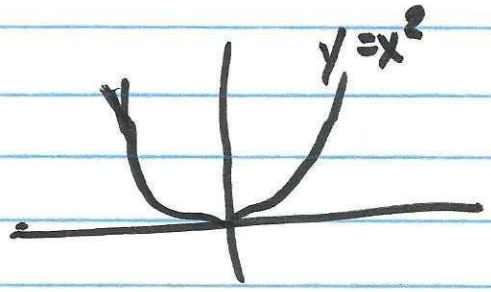
<u>boundaries of domain</u>

Where do ~~ext~~ max/min's occur:

@ either critical values or domain boundaries

Ex Given $f(x) = |x|$, find all extrema





Ex $f(x) = x^2$ $f'(x) = 2x = 0$ @ $x = 0$

Concavity

decides max/min status after $f'(x) = 0$

Look @ $f''(x)$ and $f''(x) > 0$ concave up/min

$f''(x) < 0$ " down/max

$f''(x) = 0$ no conclusion

What is an inflection point (inflexion)

Restaurant sells 800 bottles of wine / yr

$1 / bottl storage per year

$5 / bott to replace