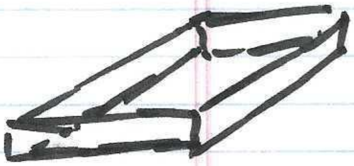
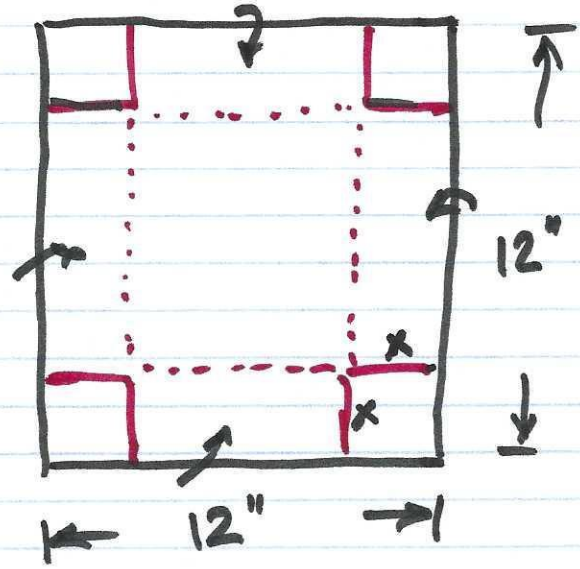
Extrema (6.1)





area of one clipped corner is x^2

what is height of candy box ... x

side of box is $12-2x$

Vol of box:

$$V(x) = x(12-2x)^2$$

$$V'(x) = 0$$

$$V(x) = x(144-48x+4x^2)$$

$$V(x) = 4x^3-48x^2+144x$$

$$V'(x) = 12x^2-96x+144 = 0$$

$$V'(x) = x^2-8x+12 = 0$$

$(x-2)(x-6)=0$

$\Rightarrow$ critical pts are $x=2$ & $x=6$

Note $x=0$ is boundary pt., but $V(0)=0$
so we discard.

max

min

Calc: $V''(x) = 24x - 96$

$V''(2) = -48 \Rightarrow$ max

$\underline{V''(6)} = +48 \Rightarrow$ min is going to be 0

$x=2$ gives a max

$V(2) = 2(12-4)^2 = 2 \cdot 64 = 128$ cu. in. ✓

Script: ① Given $f(x)$

② Calc. $f'(x)$ & set equal to zero

③ From solutions to step ② identify candidate points (places where $f'(x)$ DNE or boundary)

④ Calc $f''(x)$ @ places where $f'(x)=0$

⑤ If c is a point from ③, eval. $f''(c)$ & conclude if $f''(c)>0$ $f(c)$ is a min
if $f''(c)<0$ $f(c)$ is a max | if $f''(c)=0$ no info

#19
p.344

$C(x) = x^3 + 37x + 250$

Check for min on $x \in [1, 10]$

$C'(x) = \underline{3x^2+37} = 0$

$3x^2 = -37$

+ no solution

★ $C(1) = 288$

$C(10) = 1620$

$C(20) = 8990$

#57

$f(x) = \dfrac{x^2+36}{2x}$ (% Se in soil)

$(1 \le x \le 12)$

$f'(x) = \dfrac{(2x)(2x) - (2)(x^2+36)}{4x^2} = 0$

$= 4x^2 - 2x2 - 72 = 0$

$= 2x^2 - 72 = 0$

$x^2 = 36$ so $x = \pm 6$

$x=6$ is only candidate

$x=0$ is candidate from $f'(x)$ DNE

Check $f(1)$ & $f(12)$

④

$f''(x) = ?$

$$f'(x) = \frac{4x^2 - 2x^2 - 72}{4x^2} = \frac{2x^2 - 72}{4x^2}$$

$$= \frac{x^2 - 36}{2x^2}$$

$$f''(x) = \frac{(2x^2)(2x) - (4x)(x^2 - 36)}{4x^4}$$

$$= \frac{\cancel{4x^3} - \cancel{4x^3} + 1\cancel{44x}^{\,36}}{\cancel{4}x^{\cancel{4}\,3}}$$

$$f''(x) = \frac{36}{x^3} \Rightarrow f''(6) = \frac{36}{216} = \frac{1}{6} > 0$$

so @ $x = 6$ months Se level is a min

Recall $f(x) = \dfrac{x^2+36}{2x}$

$$f(1) = \frac{37}{2} = 18.5$$

$$f(12) = \frac{180}{24} = \frac{15}{2} = 7.5$$

$$f(6) = 6.0$$

⑤

#61

$$M(s) = -\frac{1}{45}s^2 + 2s - 20$$

Check speeds between 30 ; 65 mph

$$M'(s) = -\frac{2}{45}s + 2$$

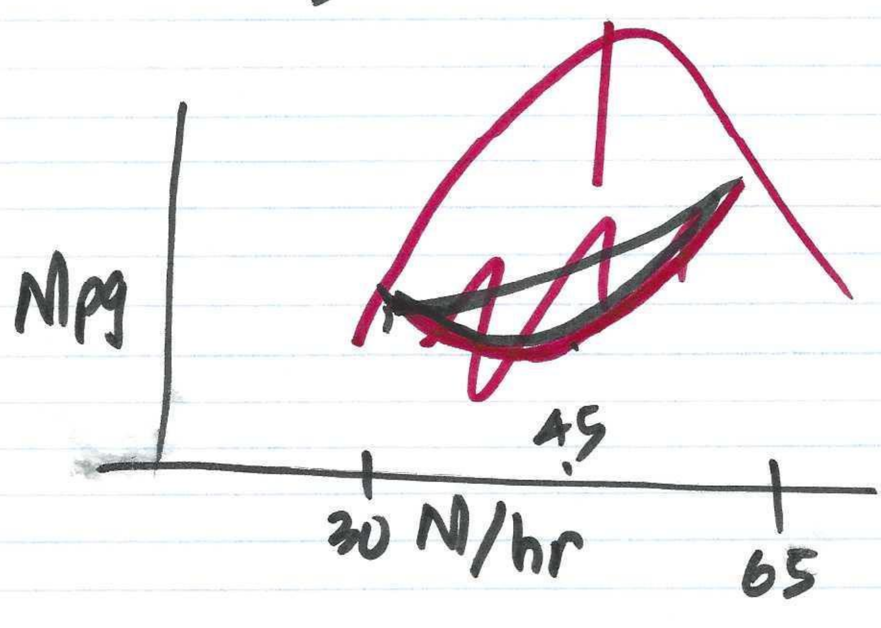
$$M'(s) = 0 : \frac{2}{45}s = 2 \Rightarrow s = 45$$

$$M''(s) = -\frac{2}{45} < 0 \text{ so max}$$

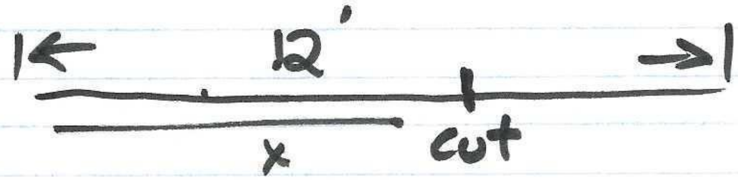
$$M(30) = -\frac{1}{45}(900) + 40 = -20 + 40 = 20$$

$$M(45) = -\frac{1}{45}(45)^2 + 90 - 20 = 25$$

$$M(65) = -\frac{1}{45}(65)^2 + 130 - 20 = \_\_\_$$



⑥



Let x be the piece formed into a circle

x into circle

12 − x into square

Circle x is circumference so $\frac{x}{2\pi}$ is radius

area is then $\pi\left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$

Square $12-x$ is perimeter of square, so

one side is $\frac{12-x}{4}$

area is then $\left(\frac{12-x}{4}\right)^2 = \frac{144-24+x^2}{16}$

$$f(x) = \frac{x^2}{4\pi} + \left(\frac{12-x}{4}\right)^2$$

$$f'(x) = \frac{2x}{4\pi} + 2\left(\frac{12-x}{4}\right)\left(-\frac{1}{4}\right)$$

$$= \frac{x}{2\pi} + \frac{x-12}{8} = 0$$

(7)

$$\frac{8+2\pi}{16\pi}$$

$$x\left(\frac{1}{2\pi}+\frac{1}{8}\right)-\frac{3}{2}=0$$

$$x\left(\frac{1}{2\pi}+\frac{1}{8}\right)=\frac{3}{2}$$

$$x=\frac{3}{2}\cdot\frac{16\pi}{8+2\pi}\rightarrow$$

$$\frac{3}{2}\cdot\frac{50.27}{14.28}=5.28'$$

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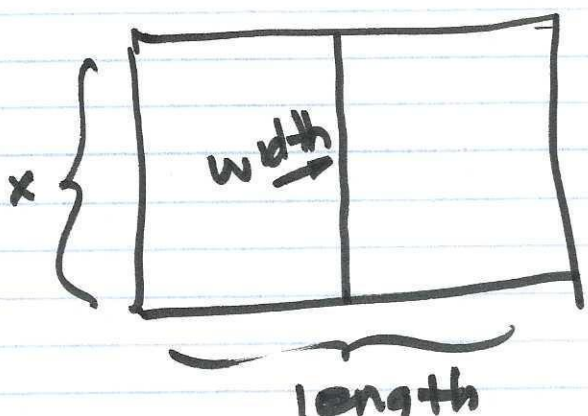
p. 353
#16



2400 m fence

width $3x$

length $\dfrac{2400-3x}{2}$

$$A(x)=(3x)\left(\frac{2400-3x}{2}\right)$$

8

$$A(x) = \frac{3}{2}\left(2400x - 3x^2\right)$$

$$A'(x) = \frac{3}{2}\left(2400 - 6x\right) = 0$$

$$2400 - 6x = 0$$

$$x = 400$$

$$A''(x) = -9 < 0 \text{ so max}$$

$$A(400) = \frac{3}{2}\left(960{,}000 - 3\cdot(160{,}000)\right)$$

$$= \frac{3}{2}(480{,}000) = \underline{720{,}000\ m^2}$$