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From 5.2

Given $f(x)$, if $f'(x_0) > 0$ f is increasing

$f'(x_0) < 0$ f is decreasing

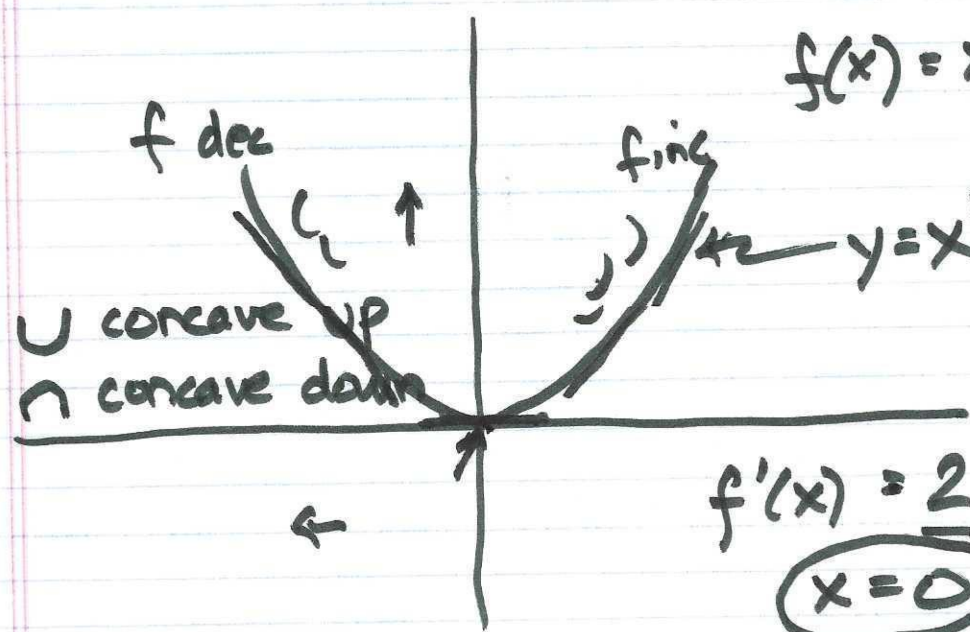
$f'(x_0) = 0$ f is stationary

5.3: Higher derivatives:

$f(x) = xe^x$

$f'(x) = (1)\cdot e^x + xe^x = e^x(x+1)$

$f''(x) = e^x(x+1) + e^x\cdot(1) = e^x(x+2)$



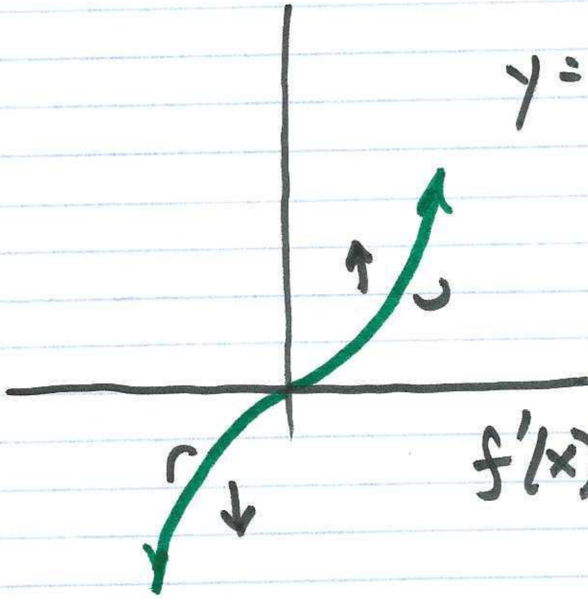
$f(x) = x^2$

$f'(x) = 2x = 0 \Rightarrow$

$x = 0$

$f''(x) = (2x)' = 2 > 0$

(2)



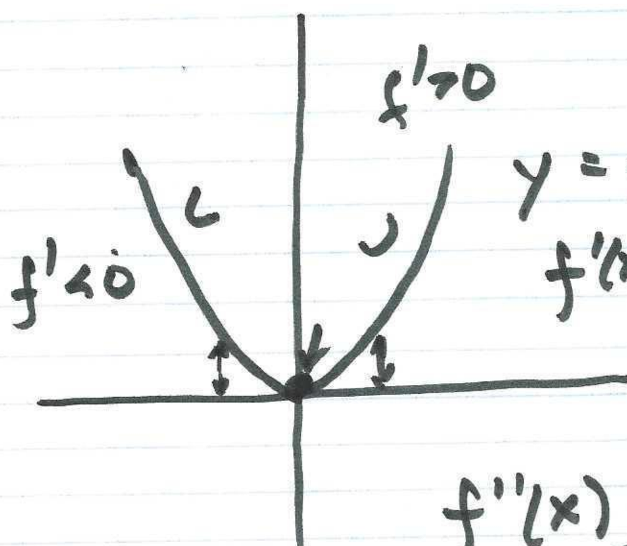
$y = f(x) = x^3$

$f'(x) = 3x^2 = 0$

$\underline{x = 0}$

A point where concavity of graph changes from + to − or − to + is called an "inflection point".

$f''(x) = 6x$ $\quad 6x$ @ $x = 0$ is 0 ?



$y = f(x) = x^4$

$f'(x) = 4x^3 = 0$

$\Rightarrow \underline{x = 0}$

$f''(x) = 12x$

$12 \cdot 0 = 0$

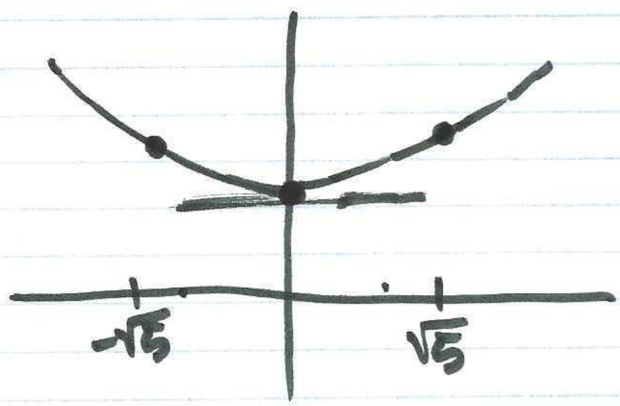
#15 p. 314

$f(x) = \sqrt{x^2+4} = (x^2+4)^{1/2}$

$f'(x) = \dfrac{1}{2\sqrt{x^2+4}} = 0$ no solution

domain = $\mathbb{R}$

no points where $f'(x)$ DNE



+ here means standard orientation

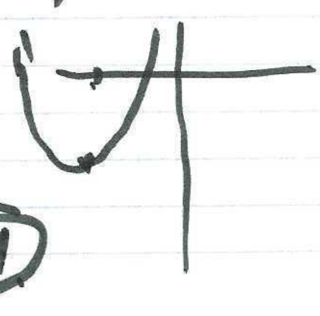
#41 $f(x) = x^2 + 10x - 9$

$f'(x) = 2x + 10 = 0 \Rightarrow x = -5$

$f''(x) = 2 > 0 \Rightarrow$ concave up

Conclude: $f(x)$ has local (global) min

@ $x = -5$



$f(-5) = (-5)^2 + 10(-5) - 9$

$25 - 50 - 9 = -34$

44 $f(x) = -x^3 - 12x^2 - 45x + 2$

$f'(x) = -3x^2 - 24x - 45 = 0$

$\div -3$ $\quad = x^2 + 8x + 15 = 0$

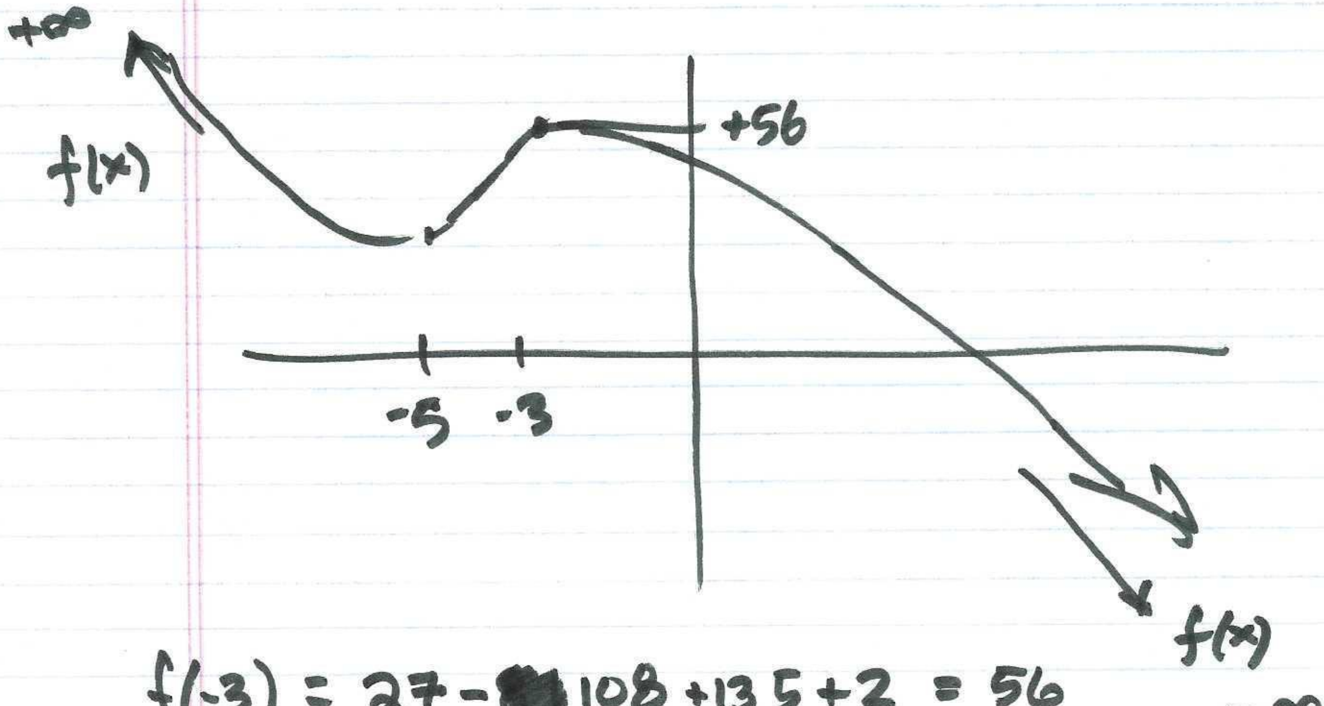
$(x+5)(x+3) = 0$

crit #'s are $-5, -3$

$f''(x) = -6x - 24$

$f''(-3) = -6$ concave ↓ max

$f''(-5) = +6$ concave ↑ min



$f(-3) = 27 - 108 + 135 + 2 = 56$

5

x is factor from 0 to 1 for BW

$$R(x) = C\,\underbrace{x(1-e^{-kx})}$$

C ← depends on site

$$R'(x) = C\left[(1)(1-e^{-kx}) + (x)(ke^{-kx})\right]$$
$$= C\left[1 - e^{-kx} + xke^{-kx}\right]$$
$$\rightarrow \quad = C\left[1 + e^{-kx}(kx-1)\right]$$

When is $R'(x) > 0$ i.e. $R(x)$ increasing

If $kx > 1$ i.e. $x > \frac{1}{k}$ revenue is up

$$R''(x) = C\left[e^{-kx}(kx-1)\right]'$$
$$= C\left[\underline{(-ke^{-kx})(kx-1)} + \underline{(e^{-kx})(k)}\right]$$

When is this positive

$$k\cancel{e^{-kx}} > k\cancel{e^{-kx}}(kx-1)$$
$$\cancel{k} > \cancel{k}(kx-1) \Rightarrow \underline{1 > (kx-1)}$$