10/1

Product / Quotient Derivatives

1) Given $f(x) \cdot g(x)$

The derivative of product is

$$f(x)g'(x) + f'(x)g(x)$$

Some examples:

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$$y(x) = (\overset{f}{3x^2+2})(\overset{g}{2x-1}), \text{ find } y'(x)$$

$$y'(x) = (3x^2+2)(2) + (6x)(2x-1)$$

#6

$$y(x) = (\overset{f}{5x^2-1})(\overset{g}{4x+3})$$

$$y'(x) = (5x^2-1)(4) + (10x)(4)$$

#9

$$k(t) = (t^2-1)^2 = t^4-2t^2+1$$

$$k'(t) = 4t^3-4t \checkmark$$

$$k(t) = (t^2-1)(t^2-1)$$

Same

$$k'(t) = (t^2-1)(2t) + (2t)(t^2-1)$$

$$(2t^3-2t) + (2t^3-2t)$$

$$4t^3-4t \checkmark$$

$\sqrt{x} = x^{1/2}$
so $(\sqrt{x})' = \frac{1}{2}x^{1/2 - 1}$

#11 $y(x) = (x+1)(\sqrt{x}+2)$

$y'(x) = (x+1)\left(\frac{1}{2\sqrt{x}}\right) + (1)(\sqrt{x}+2)$

#13 $P(y) = (y^{-1}+y^{-2})(2y^{-3}-5y^{-4})$

$P'(y) = (y^{-1}+y^{-2})(-6y^{-4}+20y^{-5}) +$
$(-y^{-2}-2y^{-3})(2y^{-3}-5y^{-4})$

Q?: Given $f(x) = g(x)h(x)k(x)$

$f'(x) = g(x)h(x)k'(x) + g(x)h'(x)k(x) +$
$g'(x)h(x)k(x)$

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② Given $\frac{f(x)}{g(x)}$ The derivative of the quotient

is $\frac{g(x)f'(x) - g'(x)f(x)}{[g(x)]^2}$

3

$$\left(\frac{f}{g}\right)' = \frac{gf' - fg'}{g^2}$$

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$$f(x) = \frac{6x+1}{3x+10}$$

$$f'(x) = \frac{(3x+10)(6) - (6x+1)(3)}{(3x+10)^2}$$

#18

$$y(t) = \frac{9-7t}{1-t}$$

$$y'(t) = \frac{(1-t)(^-7) + (+1)(9-7t)}{(1-t)^2}$$

#20

$$y(x) = \frac{x^2-4x}{x+3}$$

$$y'(x) = \frac{(x+3)(2x-4) - (x^2-4x)(1)}{(x+3)^2}$$

#26

$$r(t) = \frac{\sqrt{t}}{2t+3}$$

$$r'(t) = \frac{(2t+3)\left(\frac{1}{2\sqrt{t}}\right) - (\sqrt{t})(2)}{(2t+3)^2}$$

#31 $$f(x) = \frac{(3x^2+1)(2x-1)}{5x+4}$$

$$f'(x) = \frac{(5x+4)\left[(3x^2+1)(2)+(6x)(2x-1)\right]}{\text{cont'd}} \rightarrow$$

$$\frac{-(3x^2+1)(2x-1)(5)}{(5x+4)^2}$$

Given overall cost function $C(x)$ where C in \$

and x is units of production.

$$\frac{C(x)}{x} = \bar{C}(x) \rightarrow \text{average cost}$$

Define $\bar{C}'(x) \rightarrow$ marginal avg. cost

$$\bar{R}(x) = \frac{R(x)}{x} \longrightarrow \text{average revenue}$$

$$\bar{R}'(x) \rightarrow \text{marginal avg. revenue}$$

#54 p. 240 (Marginal Revenue)

demand function $p = D(q)$ — demand; price; quantity

Show that marginal revenue is:

$$R'(q) = D(q) + qD'(q)$$

same

$$R(q) = p \cdot q = \underline{D(q) \cdot q}$$

$$R'(q) = D(q)(1) + D'(q) \cdot q$$

#55 (Marginal Avg Cost) $\bar{C}'(x)$

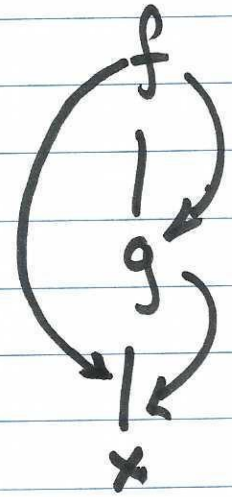
We know $\bar{C}(x) = \frac{C(x)}{x}$

Need to differentiate $\frac{C(x)}{x}$

$$\bar{C}'(x) = \left(\frac{C(x)}{x}\right)' = \frac{xC'(x) - (1)C(x)}{x^2}$$

(4.3) Chain Rule

Given $f(g(x))$ Want is derivative of f w/r/t x



$$[f(g(x))]' = \underline{f'(g(x))} \cdot \underline{g'(x)}$$

p. 250 good examples

#12 $f(x) = -8x + 9$; $g(x) = \frac{x}{5} + 4$

$$f(g(x)) = -8\left(\frac{x}{5} + 4\right) + 9 \leftarrow \star$$

Clearer: $f(g) = -8g + 9$; $g(x) = \underline{\frac{x}{5} + 4}$

$$[f(g(x))]' = (-8) \cdot \left(\frac{1}{5}\right) = -\frac{8}{5}$$

Useful formula: (General Power Rule)

Suppose $f(g) = g^n$ $\quad g^3$

$g(x)$ -- some formula $\quad x^2+2$

$$f(g(x)) = (x^2+2)^3$$

$$= (x^2+2)^2(x^2+2)$$

$$= (x^4+4x^2+4)(x^2+2)$$

$$= x^6+4x^4+4x^2+2x^4+8x^2+8$$

$$= x^6+6x^4+12x^2+8$$

$$\blacksquare^3 \qquad 3\,\blacksquare^2 \cdot \blacksquare'$$

derivative $= \boxed{6x^5+24x^3+24x}$

$$[f(g(x))]' = 3(x^2+2)^2 \cdot 2x$$

$$= 6x(x^4+4x^2+4)$$

$$= \boxed{6x^5+24x^3+24x}$$

same!

#27 $y(x) = (8x^4 - 5x^2 + 1)^4$

$y'(x) = 4(8x^4 - 5x^2 + 1)^3 \cdot (32x^3 - 10x)$

#33 $g(t) = -3\sqrt{7t^3 - 1}$

$-3(7t^3 - 1)^{1/2}$

$g'(t) = -3\left(\frac{1}{2}\right)(7t^3 - 1)^{-\frac{1}{2}} \cdot (21t^2)$

$= -\frac{3}{2} \cdot \frac{1}{\sqrt{7t^3 - 1}} \cdot 21t^2$

$= -\frac{63}{2} \frac{t^2}{\sqrt{7t^3 - 1}}$

#38 $y = (x^3 + 2)(x^2 - 1)^4$

$y'(x) = (x^3 + 2) \cdot \left[(x^2 - 1)^4\right]' + (3x^2)(x^2 - 1)^4$

$4(x^2 - 1)^3 \cdot (2x)$