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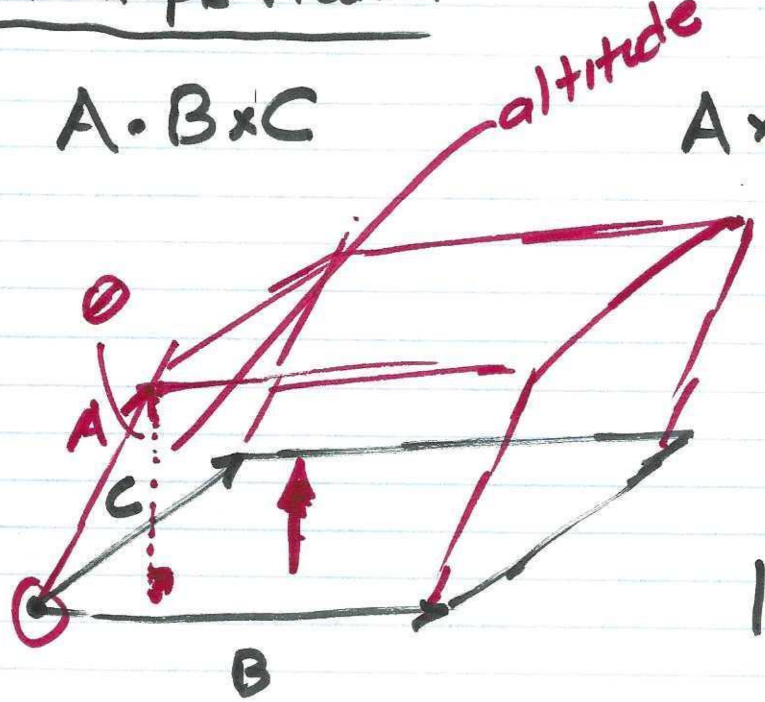
Scalar Triple Product

$A \cdot B \times C$ $\quad$ $A \times (B \times C)$

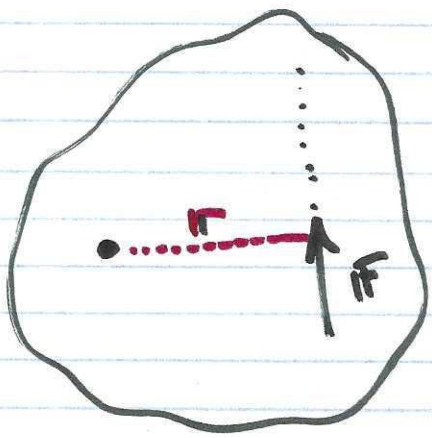


$|B \times C| = \text{Area}$

$A \cdot B \times C = |A| \cos\theta \, |B \times C|$

$A \cdot B \times C$ = volume of parallelepiped

$$A \cdot B \times C = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix} = \text{volume}$$



polar

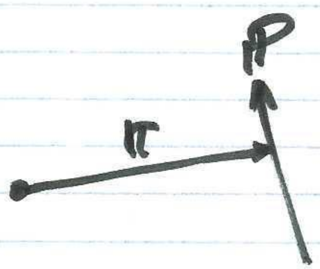
$$\mathbf{r} \times \mathbf{F} = \tau$$

vector

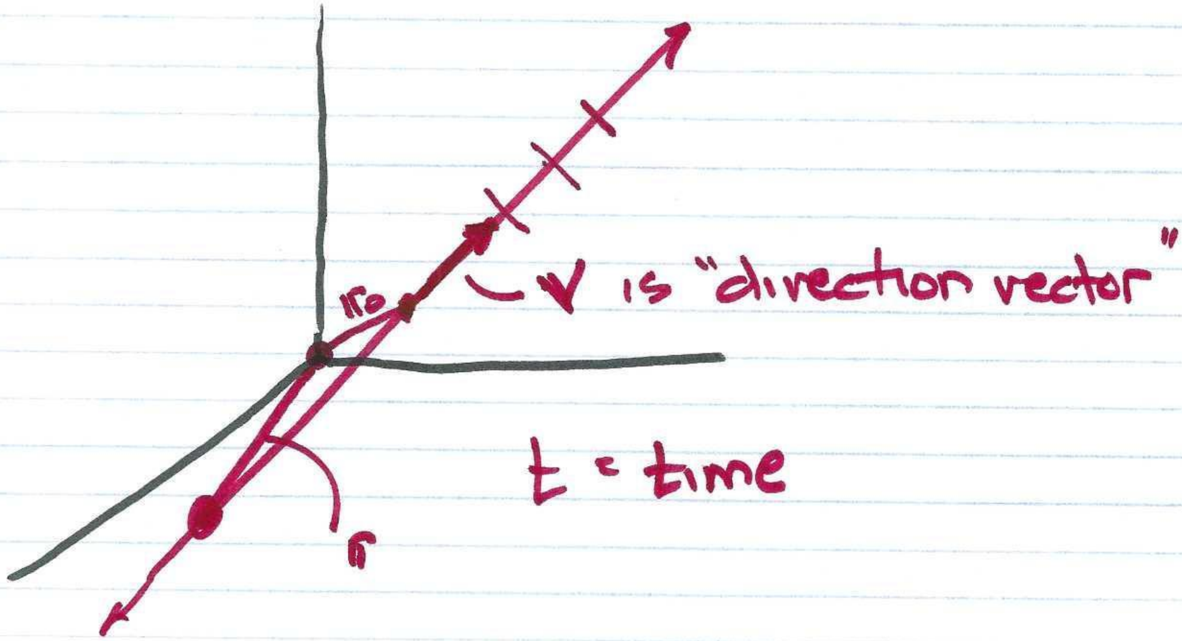
axial

$\theta \quad \dot{\theta} \quad \ddot{\theta}$

$$\tau = M\ddot{\theta}$$



$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$



$$\mathbf{r}(t) = \mathbf{r}_0 + t\,\mathbf{v}$$ standard eqn for line in space

$$\mathbf{r}(t) = \langle x(t), y(t), z(t) \rangle$$

$$x(t) = x_0 + t v_x$$

$$y(t) = y_0 + t v_y$$

$$z(t) = z_0 + t v_z$$

co-ordinate eqns

$\mathbf{v}$ is velocity, $|\mathbf{v}|$ is speed

Ex: Given line segment that connects

$$P_1 = (-3, 2, -3) \quad P_2 = (1, -1, 4)$$

$$\mathbf{v} = \overrightarrow{P_1P_2} = \langle 4, -3, 7 \rangle$$

$\mathbf{r}_0 = \langle -3, 2, -3 \rangle$ takes you from origin to line

$$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v} \qquad t \in (-\infty, \infty)$$

$$\mathbf{r}(t) = \langle -3, 2, -3 \rangle + t\langle 4, -3, 7 \rangle$$

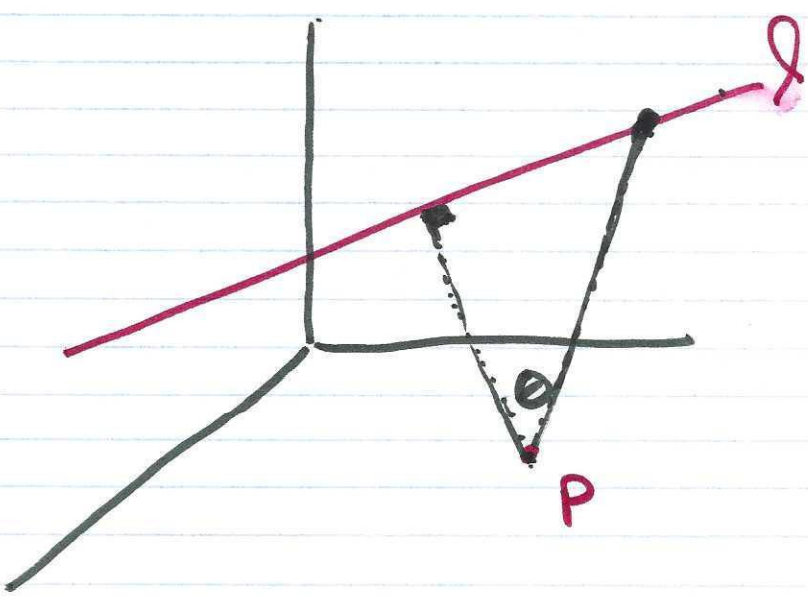
$$\left.\begin{aligned} x(t) &= -3 + 4t \\ y(t) &= 2 - 3t \\ z(t) &= -3 + 7t \end{aligned}\right\} \text{also called "parametric" eqns.}$$

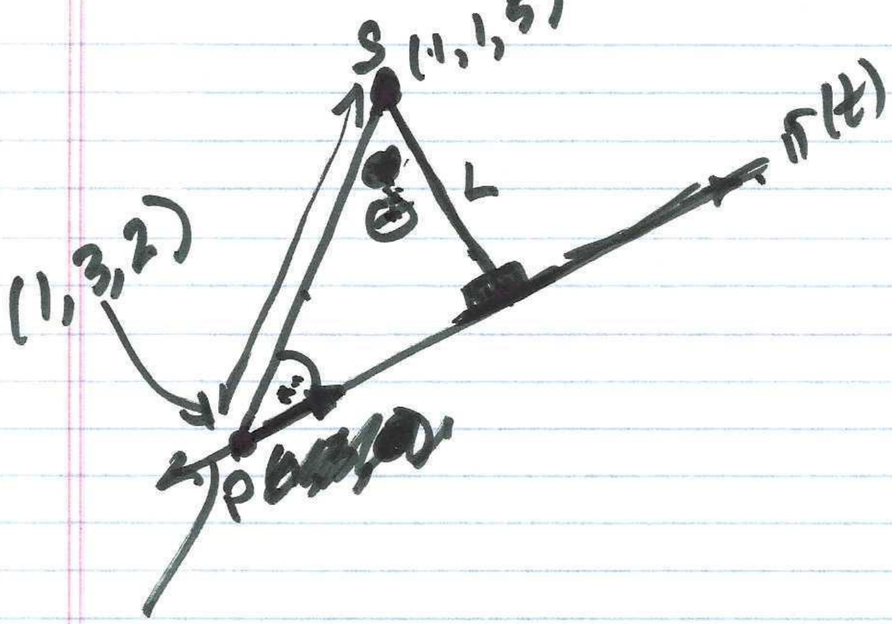
Ex: Helicopter starts @ $(0,0,0)$ & flies in direction of $(1,1,1)$ @ 60 fps. Where is it @ $t=10$?

$$\mathbf{r}_0 = \langle 0,0,0 \rangle \qquad \mathbf{v} = \langle 1,1,1 \rangle \cdot 60$$

$$\mathbf{r}(t) = \langle 0,0,0 \rangle + t\langle 1,1,1 \rangle \cdot 60$$

$$\mathbf{r}(10) = \langle 600, 600, 600 \rangle$$





$x(t) = 1 + t$

$y(t) = 3 - t$

$z(t) = 2t + 2$

$$\mathbf{r}(t) = \langle 1,3,2\rangle + t\langle 1,-1,2\rangle$$

$$\text{dist} = \frac{\vec{PS} \times \langle 1,-1,2\rangle}{|\langle 1,-1,2\rangle|}$$

this is the unit vector ∥ to line

$$\vec{PS} \times \langle 1,-1,2\rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & -2 & 3 \\ 1 & -1 & 2 \end{vmatrix}$$

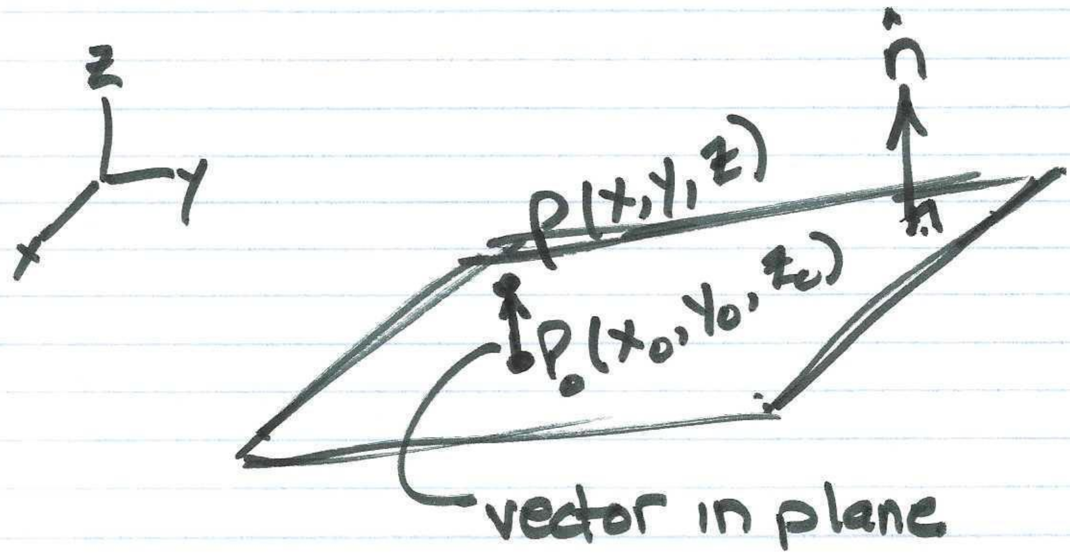
$$\hat{i}(-1) - \hat{j}(-3) + \hat{k}(2) = \langle -1, 3, 2\rangle$$

$$|\langle -1, 3, 2\rangle| = \sqrt{1+9+4} = \sqrt{14}$$

$$|\langle 1, -1, 2\rangle| = \sqrt{1+1+4} = \sqrt{6}$$

$$\text{dist} = \frac{\sqrt{14}}{\sqrt{6}} = \sqrt{7/3}$$

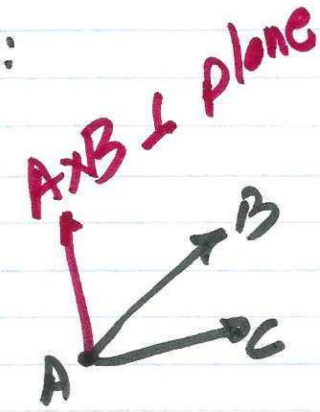
Eqn. of Plane in Space



$$\overrightarrow{P_0P} \cdot \hat{n} = 0$$

⑧

Given 3 pts in plane:



$A = (0,0,0)$

$B = (1,2,3)$

$C = (-1,-1,2)$

$\vec{AB} = \langle 1, 2, 3 \rangle$

$\vec{AC} = \langle -1, -1, 2 \rangle$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & -1 & 2 \end{vmatrix}$$

$$= \hat{i}(7) - \hat{j}(5) + \hat{k}(1)$$

$$= \langle 7, -5, 1 \rangle$$

$$\hat{n} = \frac{\langle 7, -5, 1 \rangle}{\sqrt{75}}$$

$$\sqrt{49+25+1} = \sqrt{75} = 5\sqrt{3}$$

$$\langle x, y, z \rangle \cdot \hat{n} = 0$$

$$\frac{1}{5\sqrt{3}}\left(7x - 5y + z\right) = 0$$

$$\boxed{\frac{7}{5\sqrt{3}}x - \frac{1}{\sqrt{3}}y + \frac{1}{5\sqrt{3}}z = 0}$$

$$\alpha x + \beta y + \gamma z = D$$

$$ax + by + cz = d$$

normal vector to plane (called direction vector of plane) is

$$\langle a, b, c \rangle$$

9-13

Given direction vector $\langle 1, 2, 3 \rangle$ for a plane thru $(1, 1, 1)$; find eqn of plane

$$1x + 2y + 3z = D$$

$$1 + 2 + 3 = 6$$

$$x + 2y + 3z = 6$$

Given two planes

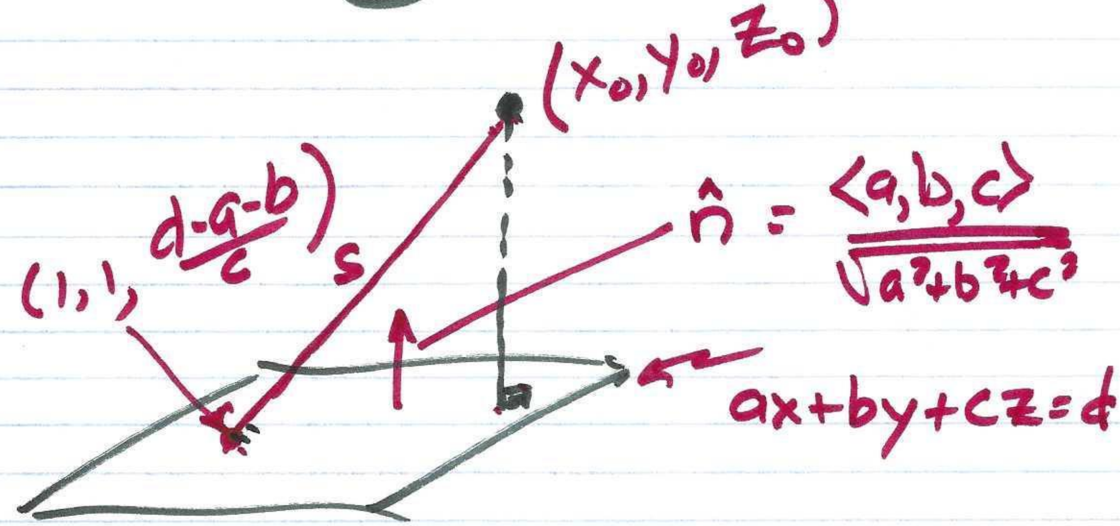
1) $x + 3y - 4z = 5 \rightarrow \langle 1, 3, -4 \rangle$

2) $2x - y + 5z = 7 \rightarrow \langle 2, -1, 5 \rangle$

$$\theta = \arccos\left(\frac{2 - 3 - 20}{\sqrt{26}\,\sqrt{30}}\right)$$

$$\theta = \arccos\left(\frac{-21}{\sqrt{780}}\right)$$

10



$x=1 \quad y=1$

$$a+b+cz=d$$
$$cz=d-a-b$$
$$z=\frac{d-a-b}{c}$$

$$S = \left\langle x_0-1,\ y_0-1,\ z_0-\frac{d-a-b}{c}\right\rangle$$

$S \cdot \hat{n}$ = dist point to plane

Given plane $2x - 3y + 5z = 10$

How far away from plane is the origin.

$$\hat{n} = \frac{\langle 2, -3, 5 \rangle}{\sqrt{38}}$$

$x = 1$ $y = 1$ $\qquad 2 - 3 + 5z = 10$

$$5z = 11$$

$$z = 11/5$$

This point is in the plane:

$$\left(1, 1, \frac{11}{5}\right)$$

So is $(0, 0, 2)$

What is distance from $(0,0,0)$ to $(0,0,2)$

This distance is 2.

Vector $\langle 0, 0, 2 \rangle$

Now dot w/ $\hat{n}$: $\langle 0,0,2 \rangle \cdot \dfrac{\langle +2, -3, 5 \rangle}{\sqrt{38}}$

Distance is $\frac{10}{\sqrt{38}} = ?$

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p.750 #17

Angle between planes:

(i) $x + y = 1$ $\langle 1, 1, 0 \rangle$

(ii) $2x + y - 2z = 2$ $\langle 2, 1, -2 \rangle$

$$\theta = \arccos\left(\frac{2+1+0}{\sqrt{2} \cdot 3}\right) = \frac{3}{3\sqrt{2}} = \frac{1}{\sqrt{2}}$$

$$\theta = \arccos\left(\frac{1}{\sqrt{2}}\right)$$

#18 (i) $5x + y - z = 10$ $\langle 5, 1, -1 \rangle$

(ii) $x - 2y + 3z = -1$ $\langle 1, -2, 3 \rangle$

$$\theta = \arccos\left(\frac{0}{\_\ \ \_}\right) = \frac{\pi}{2}$$

13

p. 749 #39

point is $(2,-3,4)$

plane is $x+2y+2z=13$

$\hat{n} = \dfrac{\langle 1,2,2\rangle}{3}$

point in plane is $(13,0,0)$

vector from plane to point is

$S = \langle 11, 3, -4\rangle$

$\text{dist} = \hat{n}\cdot S = \dfrac{\langle 1,2,2\rangle}{3}\cdot\langle 11,3,-4\rangle$

$= \frac{1}{3}\left(11+6-8\right) = 3$

#40 point is $(0,0,0)$

plane is $3x+2y+6z=6$

$\hat{n} = \dfrac{\langle 3,2,6\rangle}{7}$

(14)

$(0,0,1)$ is point in plane

$\langle 0,0,1 \rangle$ vector point to plane

dist $= \dfrac{\langle 0,0,1\rangle \cdot \langle 3,2,6\rangle}{7} = \frac{6}{7}$