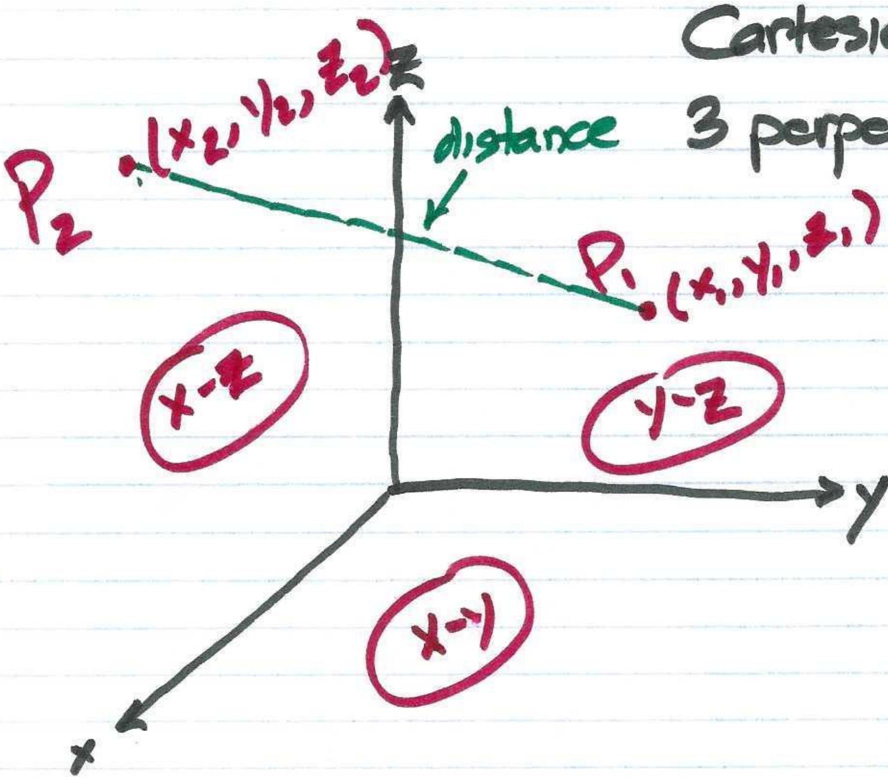
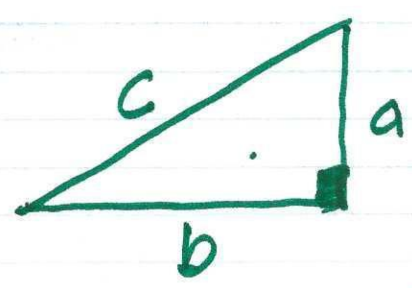
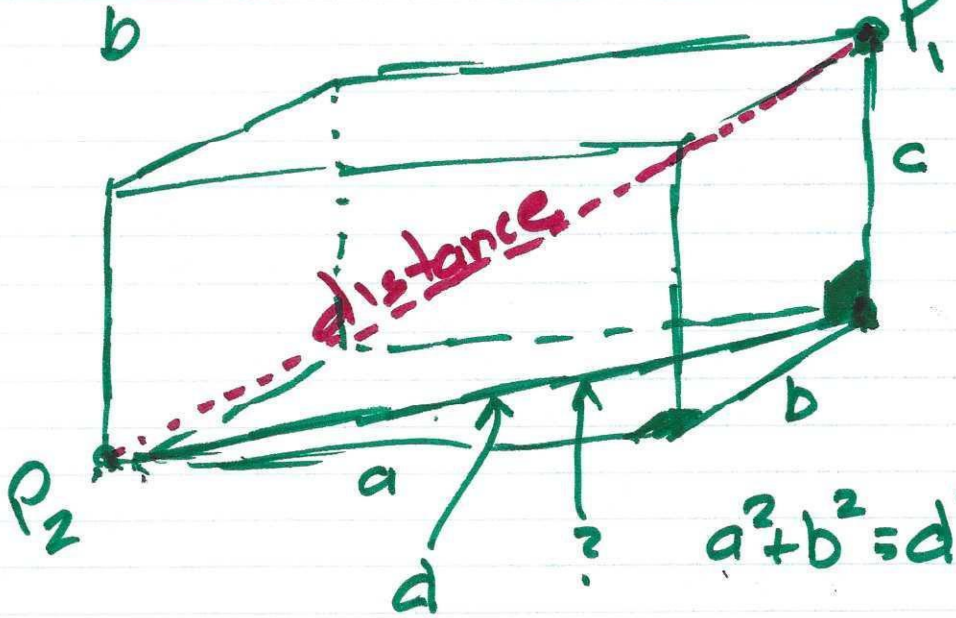
Vector Spaces

Cartesian system
3 perpendicular axes





$a^2 + b^2 = c^2$

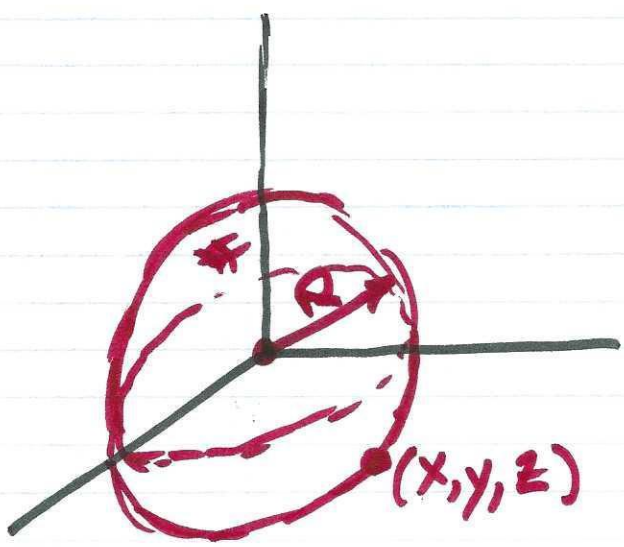


$a^2 + b^2 = d^2$

$$(\text{cross diag dist})^2 = \underline{a^2+b^2}+c^2$$

so .. the distance formula for points in space is...

$$\text{dist} = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$



$$\sqrt{x^2+y^2+z^2} = R$$

Move center of sphere to (a,b,c)

$$(x-a)^2+(y-b)^2+(z-c)^2=R^2$$

Eqn of sphere of radius R centered @ (a,b,c).

#51 Center @ $(-2,0,2)$ radius $=2\sqrt{2}$
p.718

$$(x+2)^2 + y^2 + (z-2)^2 = 8$$

#55 $(x^2+4x+4)+(y^2)+(z^2-4z+4)=0+4+4$

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sidebar

$$x^2+ax+b \to x^2+\left(\frac{a}{2}\right)^2-\left(\frac{a}{2}\right)^2+b+ax$$

$$\downarrow$$

$$\left(x+\frac{a}{2}\right)^2+\left(b-\left(\frac{a}{2}\right)^2\right)$$

#58 $3x^2+3y^2+3z^2+2y-2z=9$

$\div 3$: $x^2+y^2+z^2+\frac{2}{3}y-\frac{2}{3}z=3$

$x^2+\left(y^2+\frac{2}{3}y+\frac{1}{9}\right)+\left(z^2-\frac{2}{3}z+\frac{1}{9}\right)=3+\frac{2}{9}$

$x^2+\left(y+\frac{1}{3}\right)^2+\left(z-\frac{1}{3}\right)^2=\frac{29}{9}$

center @ $\left(0,-\frac{1}{3},+\frac{1}{3}\right)$ $R=\frac{1}{3}\sqrt{29}$

#60 $(x-1)^2+(y-2)^2+(z+1)^2=103+2x+4y-2z$

$x^2-2x+1+y^2-4y+4+z^2+2z+1=103+2x+4y-2z$

$(x^2-4x+1)+(y^2-8y+4)+(z^2+4z+1)=103$
$+3$
$+12$
$+3$

$(x^2-4x+4)+(y^2-8y+16)+(z^2+4z+4)$

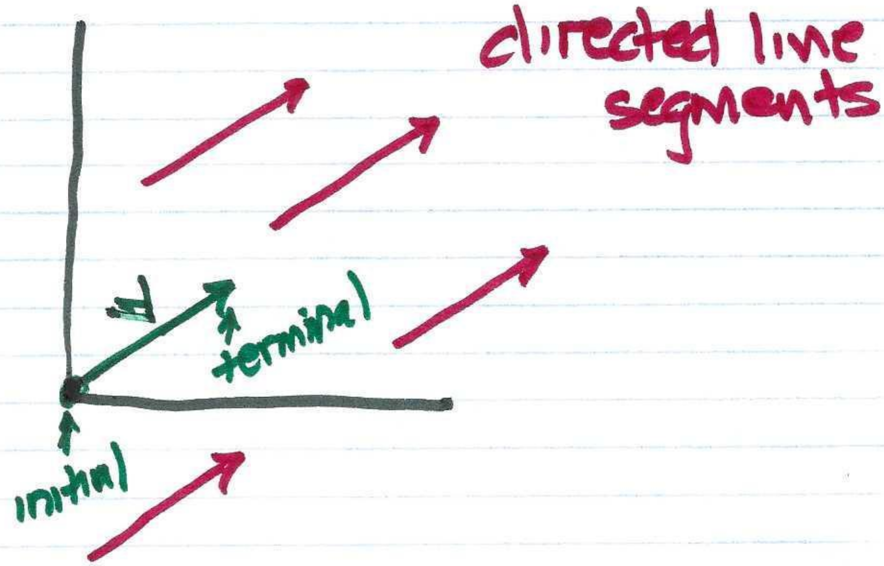
$(x-2)^2+(y-4)^2+(z+2)^2=\underline{121}$

center @ $(2,4,-2)$ $R=11$

Vectors

A vector is a math object that has length and direction:

$\vec{v}$ $\bar{v}$

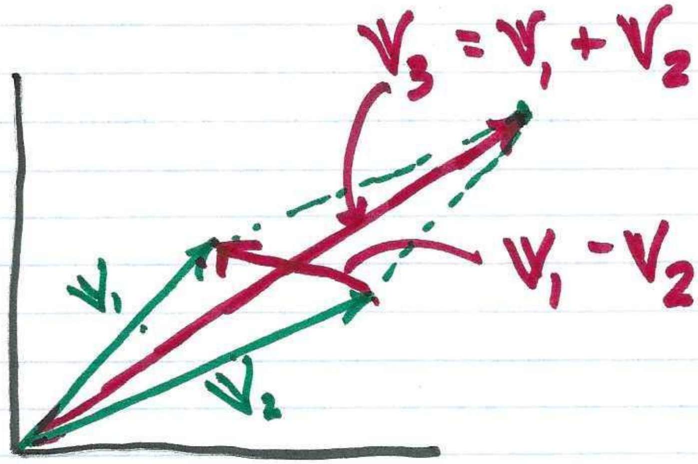


What is directed line segment between $P_1\ (x_1, y_1, z_1)$ to $P_2\,(x_2, y_2, z_2)$

$$\langle x_2 - x_1,\ y_2 - y_1,\ z_2 - z_1 \rangle$$

length is same as the "norm"

$$v = \underbrace{\langle v_1, v_2, v_3 \rangle}_{\text{components}} \qquad \underbrace{(v_1, v_2, v_3)}_{\text{co-ordinates of a point}}$$



$$\mathbb{V}_1 - \mathbb{V}_2 + \mathbb{V}_2 = \mathbb{V}_1$$

Sum/Difference

$$\mathbb{V}_1 = \langle v_1, v_2, v_3 \rangle$$

$$\mathbb{V}_2 = \langle w_1, w_2, w_3 \rangle$$

$$\mathbb{V}_1 + \mathbb{V}_2 = \langle v_1 + w_1, v_2 + w_2, v_3 + w_3 \rangle$$

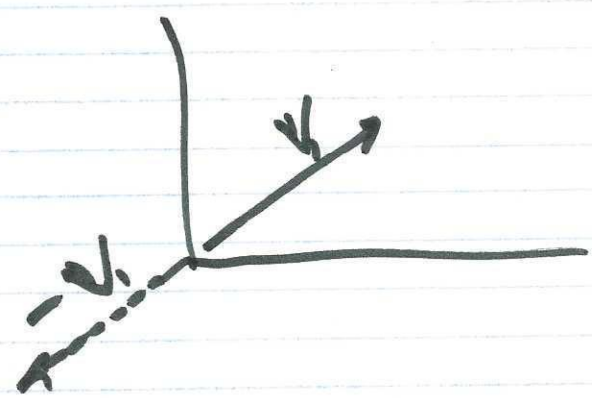
scalar

$\oplus$

Given $\mathbf{v}_1$, $\mathbf{v}_2$, & $\mathbf{v}_3$ the following hold:

1. $\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{v}_2 + \mathbf{v}_1$ commutativity
2. $\mathbf{v}_1 + (\mathbf{v}_2 + \mathbf{v}_3) = (\mathbf{v}_1 + \mathbf{v}_2) + \mathbf{v}_3$ associativity
3. There is a zero vector $\langle 0,0,0 \rangle$
4. $\mathbf{v}_1 + \mathbf{0} = \mathbf{v}_1$
5. For every $\mathbf{v}_1$ there is a $-\mathbf{v}_1$ such that $\mathbf{v}_1 + (-\mathbf{v}_1) = \mathbf{0}$

we usually just write $\mathbf{v}_1 - \mathbf{v}_1 = \mathbf{0}$



8

Can multiply vectors by scalars

$\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ by scalar $r \in \mathbb{R}$

$r\mathbf{v} = \langle rv_1, rv_2, rv_3 \rangle$

6 $r(\mathbf{v}_1 + \mathbf{v}_2) = r\mathbf{v}_1 + r\mathbf{v}_2$

7 $r_1(r_2 \mathbf{v}) = (r_1 r_2)\mathbf{v}$

p.722 gives this summary

8 $1 \cdot \mathbf{v} = \mathbf{v}$

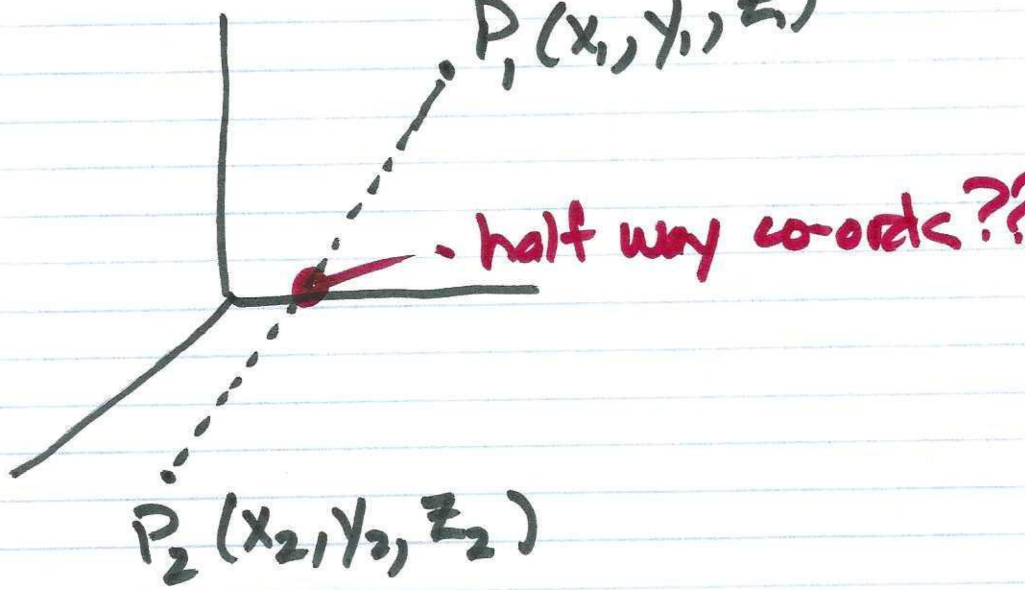
9 $(r_1 + r_2)\mathbf{v} = r_1\mathbf{v} + r_2\mathbf{v}$

To unitize vector $\mathbf{v}$, form $\dfrac{\mathbf{v}}{|\mathbf{v}|} = \hat{u}_{\mathbf{v}}$

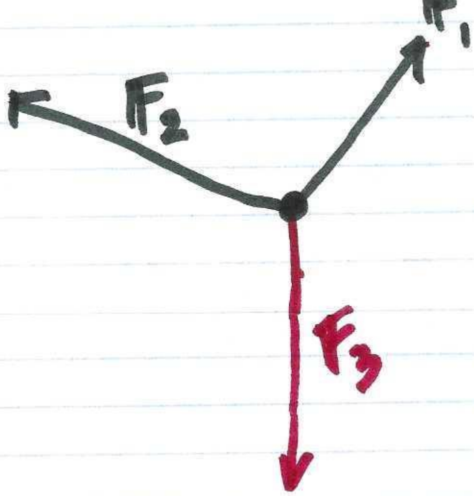
$|\cdot|$ is the length function (or norm function)

$\hat{u} \rightarrow$ direction

10



$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

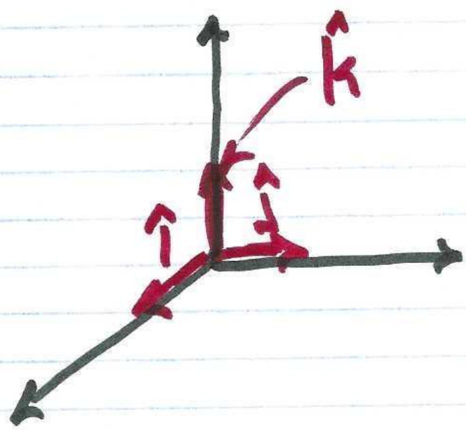


$\underline{F_3 + F_1 + F_2 = 0}$.

$F_1 = \langle 2, 2 \rangle$

$F_2 = \langle 3, 1 \rangle$

⑨



$$\mathbf{v} = \langle v_1, v_2, v_3 \rangle = v_1\hat{i} + v_2\hat{j} + v_3\hat{k}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ x & y & z \end{array}$$

$$5\langle 1, 2, 3\rangle = \langle 5, 10, 15\rangle$$

same

$$5(\hat{i} + 2\hat{j} + 3\hat{k}) = 5\hat{i} + 10\hat{j} + 15\hat{k}$$

If $\mathbf{v}$ represents velocity, what is the associated speed

speed is |velocity|

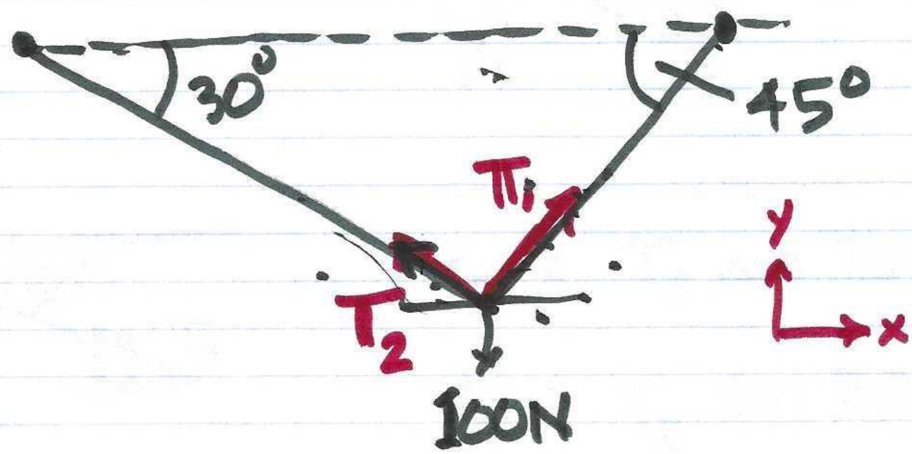
$$|\mathbf{v}| \cdot \hat{u}_v$$

$\mathbb{F}_3 = \langle x, y \rangle$

$\langle x, y \rangle + \langle 2, 2 \rangle + \langle 3, 1 \rangle = \langle 0, 0 \rangle$

$x = -5 \quad y = -3$

#47
p. 727



$T_1 = \langle x_1, y_1 \rangle \quad T = \langle x_2, y_2 \rangle$

$y_1 + y_2 = 100 \qquad y_1 + y_2 - 100 = 0$

$x_1 = -x_2 \qquad x_1 + x_2 = 0$

13

$$T_1 \frac{\sqrt{2}}{2} + T_2 \cdot \frac{1}{2} + 100 = 0$$

$$\boxed{\text{(ii)} \quad \sqrt{2}\, T_1 + T_2 = +100}$$

Preview 12.3

Scalar multiplication $r \cdot \mathbf{v}$

- Dot product (scalar product) $\mathbf{v} \cdot \mathbf{w} = r$

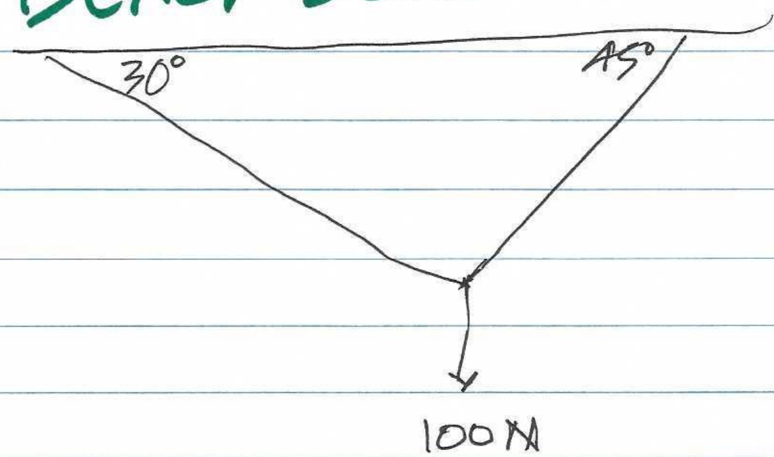
- Cross product $\mathbf{v} \times \mathbf{w} = \mathbf{u}$

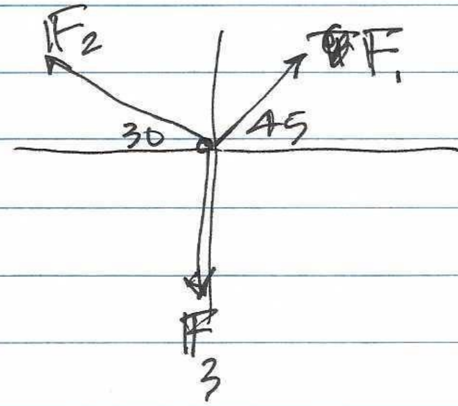
$$\mathbf{v} = \langle v_1, v_2, v_3 \rangle$$

$$\mathbf{w} = \langle w_1, w_2, w_3 \rangle$$

$$\mathbf{v} \cdot \mathbf{w} = \underline{v_1 w_1 + v_2 w_2 + v_3 w_3}$$

Better Solution:





horiz comp: $|F_1| \cos 45° = T_1 \frac{\sqrt{2}}{2}$

$- |F_2| \cos 30° = -T_2 \frac{\sqrt{3}}{2}$

$|F_3| = 0$

vert comp: $|F_1| \sin 45 = T_1 \frac{\sqrt{3}}{2}$

$|F_2| \sin 30° = T_2 \cdot \frac{1}{2}$

$|F_3| = -100$

$$(i)\ \frac{\sqrt{2}}{2}T_1 - \frac{\sqrt{3}}{2}T_2 = 0$$

$$(ii)\ \frac{\sqrt{2}}{2}T_1 + \frac{T_2}{2} = 100$$

$$(i)\ \sqrt{2}\,T_1 - \sqrt{3}\,T_2 = 0$$

$$(ii)\ \sqrt{2}\,T_1 + T_2 = 200$$

$$(ii) - (i) \Rightarrow (1+\sqrt{3})T_2 = 200$$

$$2.732\,T_2 = 200 \Rightarrow T_2 = 73.2\text{ N}$$

$$(i) \Rightarrow T_1 = \frac{\sqrt{3}\,T_2}{\sqrt{2}} \Rightarrow 1.22\,T_2 = T_1$$

$$\text{so } T_1 = 89.7\text{ N}$$

Check:

Horiz forces: ① $(89.7)(.707) = 63.43$ to right

② $(73.2)(.866) = 63.4$ to left

Vert: ① $(89.7)(.707) = 63.43$ up

② $(73.2)(.5) = 36.6$ up

$\}$ 100 up