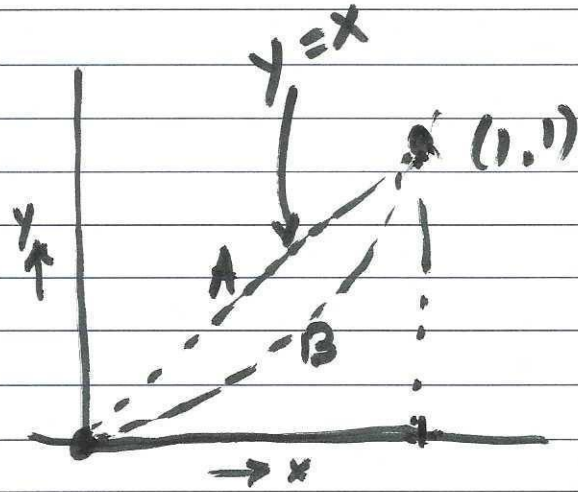
Line Integrals



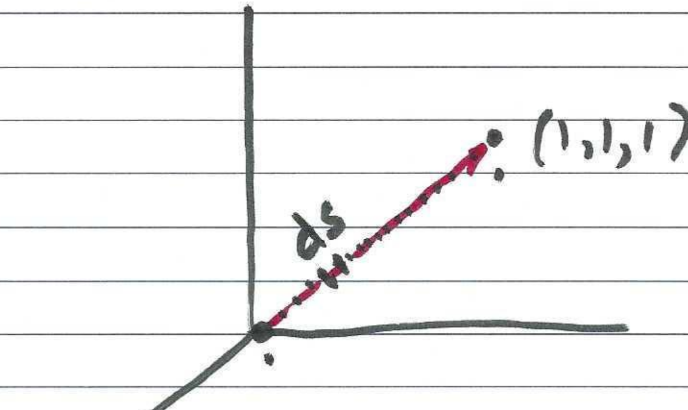
$\int_{(0,0)}^{(1,1)} f(x,y)\,\underline{ds}$ is a <u>line</u> integral

$$f(x,y) = x^2 y$$

1) $$\int_{(0,0)}^{(1,1)} x^2 y\,dx = \int_{(0,0)}^{(1,1)} x^2 \cdot x\,dx = \left.\frac{x^4}{4}\right|_0^1 = \frac{1}{4}$$

2) $$\int_{(0,0)}^{(1,1)} x^2 \cdot x^2\,dx = \left.\frac{x^5}{5}\right|_0^1 = \frac{1}{5}$$

$$f(x,y,z) = x - 3y^2 + z$$



$$\frac{ds}{dt}\,dt = ds$$

$$\mathbf{r}(t) = t\,\hat{i} + t\,\hat{j} + t\,\hat{k} \qquad 0 \le t \le 1$$

$$\frac{ds}{dt} = |v(t)| \qquad \int_C f(x,y,z)\,ds = \int_{t=a}^{t=b} f(x(t), y(t), z(t))\,|v(t)|\,dt$$

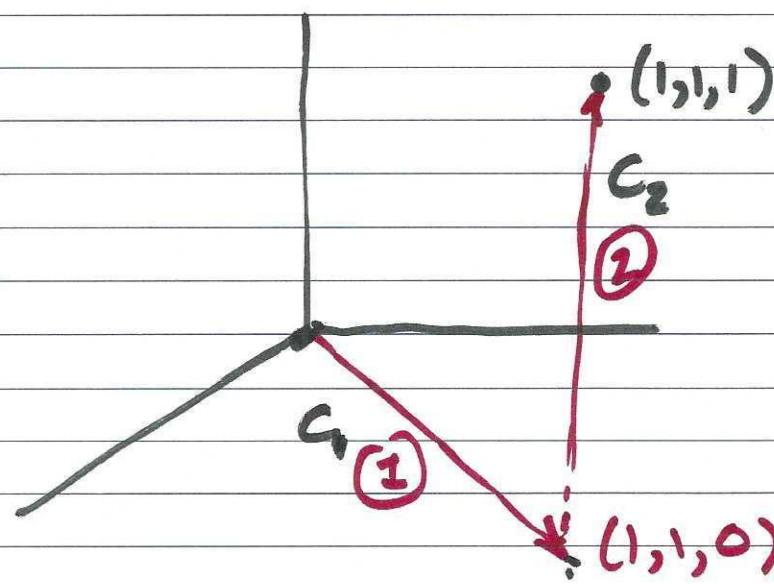
$$|v(t)| = |\mathbf{r}'(t)| \qquad \frac{d\mathbf{r}(t)}{dt} = \hat{i} + \hat{j} + \hat{k}$$

$$\left|\frac{d\mathbf{r}(t)}{dt}\right| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

③

$$I = \int_0^1 (t - 3t^2 + t)(\sqrt{3})\,dt \quad \leftarrow \frac{ds}{dt}$$

$$= \sqrt{3}\int_0^1 (2t - 3t^2)\,dt = \sqrt{3}\left(t^2 - t^3\right)\Big|_0^1 = \underline{0}$$



On C_1 : $\mathbf{r}_1(t) = t\,\hat{i} + t\,\hat{j} + 0\,\hat{k}$ | $|v(t)|$ $\mathbf{r}_1'(t) = \hat{i} + \hat{j}$

C_2 : $\mathbf{r}_2(t) = (1)\,\hat{i} + (1)\,\hat{j} + t\,\hat{k}$ | $\mathbf{r}_2'(t) = \hat{k}$

④ $\boxed{f(x,y,z) = x - 3y^2 + z}$

$$\int_{(0,0,0)}^{(1,1,1)} f(x,y,z)\,ds = \int_{C_1} f(x,y,z)\,ds + \int_{C_2} f(x,y,z)\,ds$$

$$\int_{C_1} (t - 3t^2 + 0)\,|v_1(t)|\,dt = \int_{t=0}^{1} (t - 3t^2)(\sqrt{2})\,dt$$

$$\int_{C_2} (1 - 3\cdot 1^2 + t)\,|v_2(t)|\,dt = \int_{0}^{1} (-2 + t)(1)\,dt$$

$$\int_{C_1} \rightarrow \sqrt{2}\left(\frac{t^2}{2} - t^3\right)\Big|_0^1 = \sqrt{2}\left[\left(\frac{1}{2} - 1\right) - 0\right] = \boxed{-\frac{\sqrt{2}}{2}}$$

$$\int_{C_2} \rightarrow \left(-2t + \frac{t^2}{2}\right)\Big|_0^1 = \left[-2 + \frac{1}{2}\right] = \boxed{-\frac{3}{2}}$$

So... $\int_{(0,0,0)}^{(1,1,1)} (x-3y^2+z)\,ds = -\frac{\sqrt{2}}{2} + \left(-\frac{3}{2}\right) = -\frac{(3+\sqrt{2})}{2}$

$\int \mathbf{f}\cdot d\mathbf{s}$

two choices for each

$\int f\,ds$ scalar/scalar

$\int \mathbf{f}\,ds$ vector/scalar

$\int (x\,\hat{i} + x^2\,\hat{j} + \sin x\,\hat{k})\,dx$

$\int f\,d\mathbf{s}$ scalar/vector

important $\boxed{\int \mathbf{f}\cdot d\mathbf{s} \text{ vector/vector}}$

$$f(x,y,z) = x\hat{i} + y^2x\hat{j} + xz\hat{k}$$

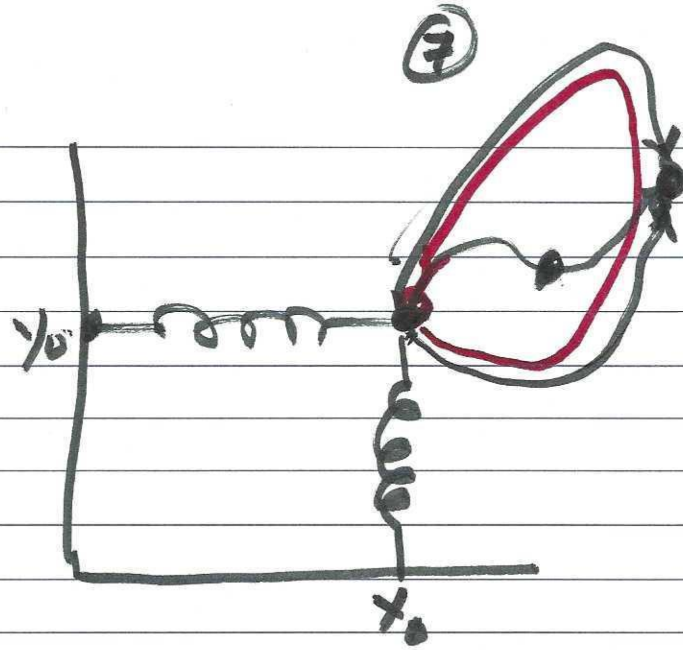
$$d\mathbf{r} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\underline{\int f(x,y,z)\cdot d\mathbf{r} = \int x\,dx + \int xy^2\,dy + \int xz\,dz}$$

$$\nabla f(x,y,z) = \hat{i}\frac{\partial f}{\partial x} + \hat{j}\frac{\partial f}{\partial y} + \hat{k}\frac{\partial f}{\partial z}$$

$$\underline{\int_C \nabla f\cdot d\mathbf{r}}$$

$$\int_C \mathbf{F}\cdot d\mathbf{r} = \underline{\text{Work}}$$



Conservative Field

$$\oint_C \mathbf{F}\cdot d\mathbf{r} = 0$$

Contour integral