① Cylindrical $\iiint f(x,y,z)\,dx\,dy\,dz$

$$\iiint f(r,\theta,z)\,\boxed{r}\,dr\,d\theta\,dz$$

volume magnification

$x = r\cos\theta$	$r = \sqrt{x^2+y^2}$
$y = r\sin\theta$	$\theta = \arctan y/x$
$z = z'$	$z' = z$

Area factor for polar

jacobian determinant

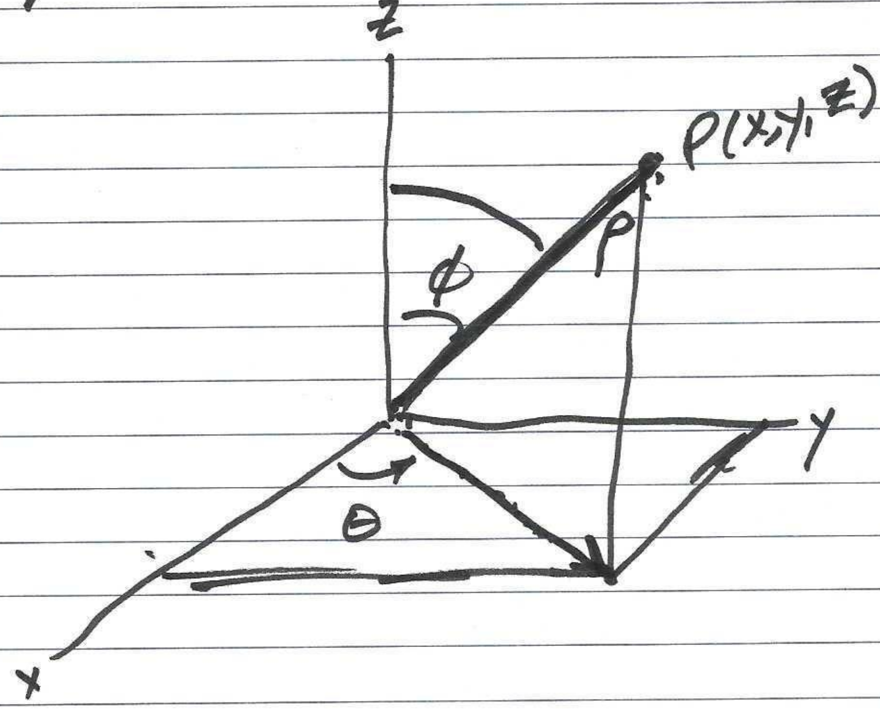
$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} x_r & y_r \\ x_\theta & y_\theta \end{vmatrix} = x_r y_\theta - y_r x_\theta$$

$$= r$$

②

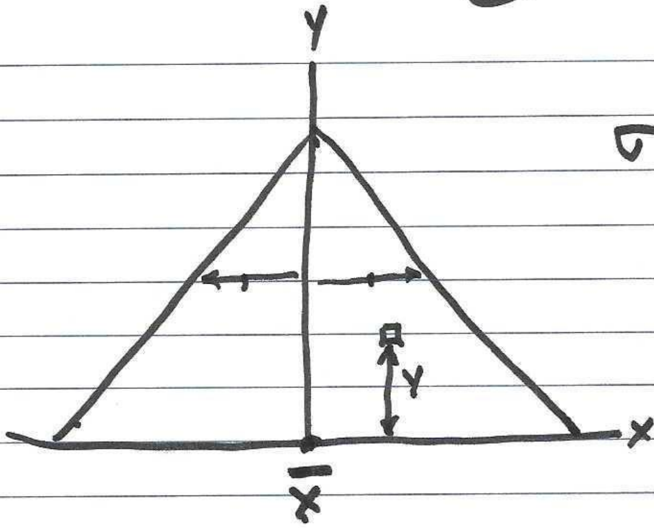
$$\frac{\partial(x,y,z)}{\partial(r,\theta,z')} = r$$

$$\frac{\partial(x,y,z)}{\partial(\rho,\theta,\phi)} = \rho\sin\phi$$



Fact: $$\frac{\partial(r,\theta)}{\partial(x,y)} = \frac{1}{\frac{\partial(x,y)}{\partial(r,\theta)}}$$

$$\frac{dy}{dx} = \frac{1}{dx/dy}$$

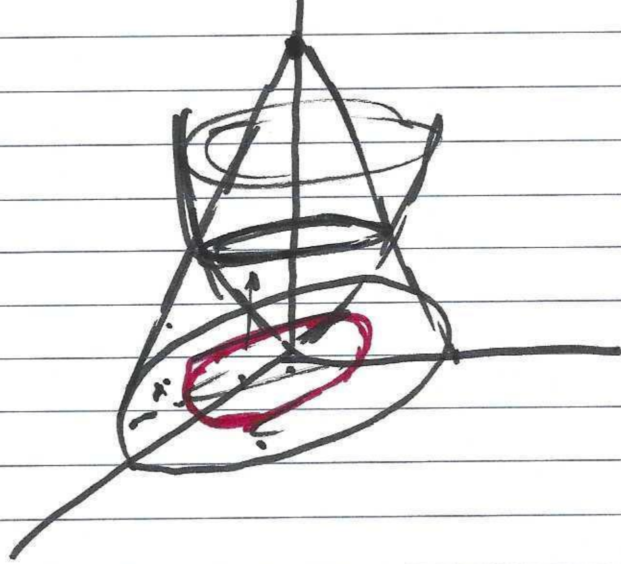


$\sigma = k|x|$

M_x

M_y

$$\frac{\iint \sigma \, y \, dx\,dy}{\iint dx\,dy} = \bar{y}$$

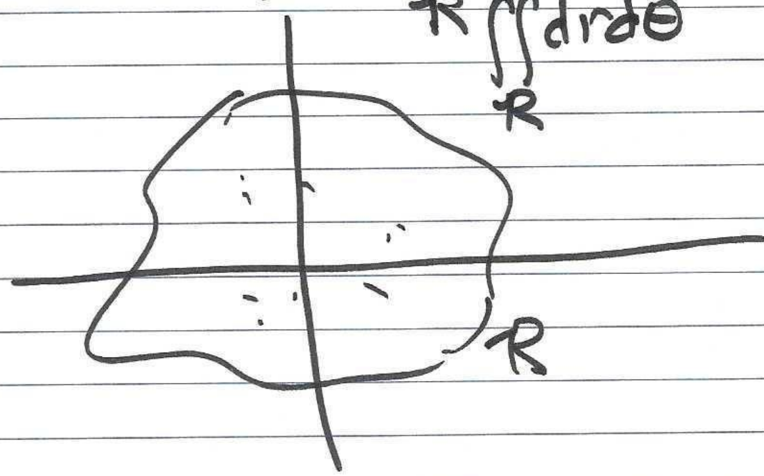


$$z = x^2 + y^2$$

$f(r,\theta)$ on R

$$\bar{f} = \frac{\iint_R f(r,\theta)\,dr\,d\theta}{\iint_R dr\,d\theta}$$

$\bar{f}_R$



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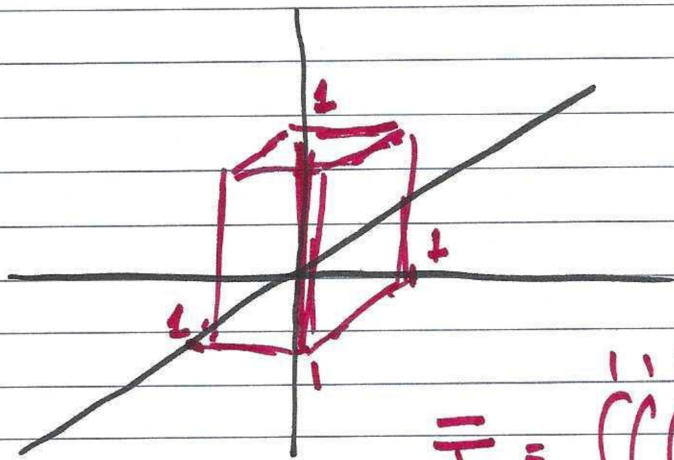
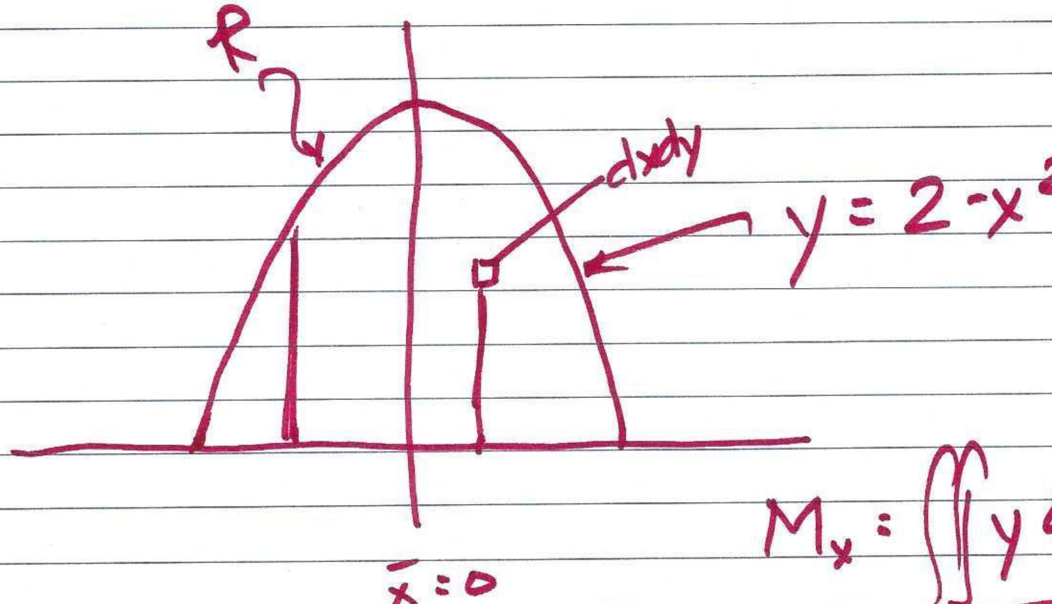
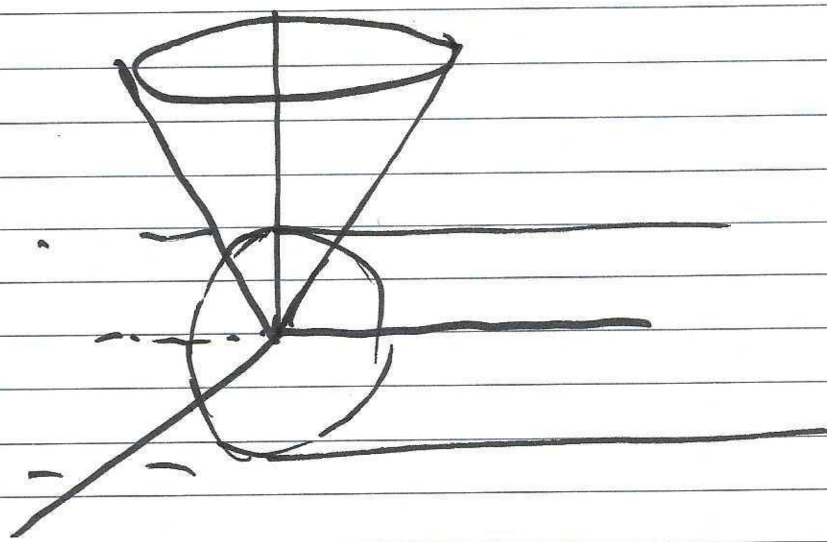


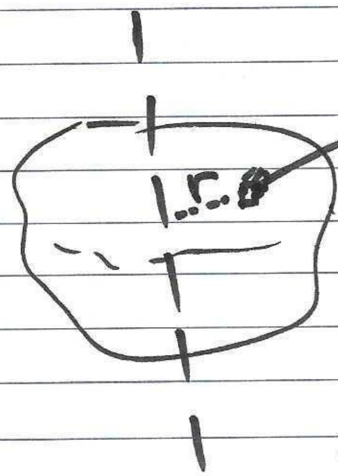
$$T = 1 - e^x e^y e^z$$

$$\bar{T} = \frac{\int_0^1\int_0^1\int_0^1 (1 - e^x e^y e^z)\,dx\,dy\,dz}{\int_0^1\int_0^1\int_0^1 dx\,dy\,dz}$$



$$\bar{y} = \frac{M_x = \iint_R y\,dx\,dy}{\iint dx\,dy}$$



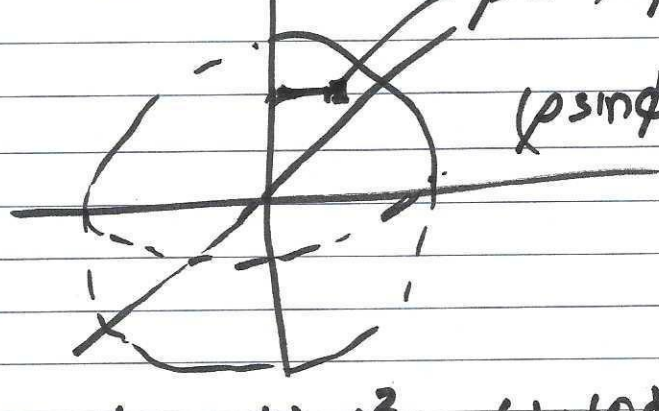


$\sigma(x,y,z)\,dx\,dy\,dz = dm$

$dI = r^2\,dm$

$r^2 \qquad dm$

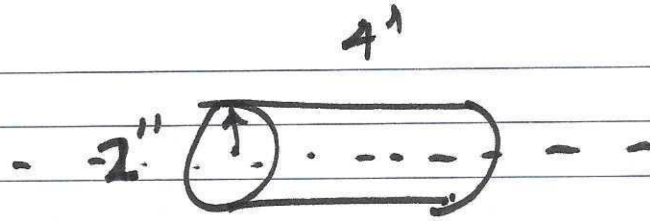
$$\iiint (\rho \sin\phi)^2$$



$\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$

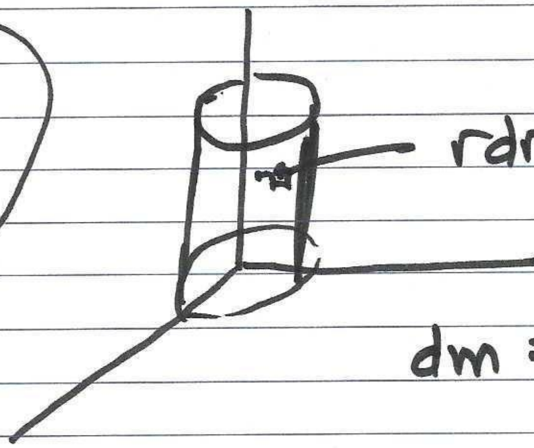
$(\rho \sin\phi)^2$

$dm = \sigma(\rho,\theta,\phi)\cdot\rho^2 \sin\phi \, d\rho \, d\theta \, d\phi$



σ = density

$dI = r^2 dm$



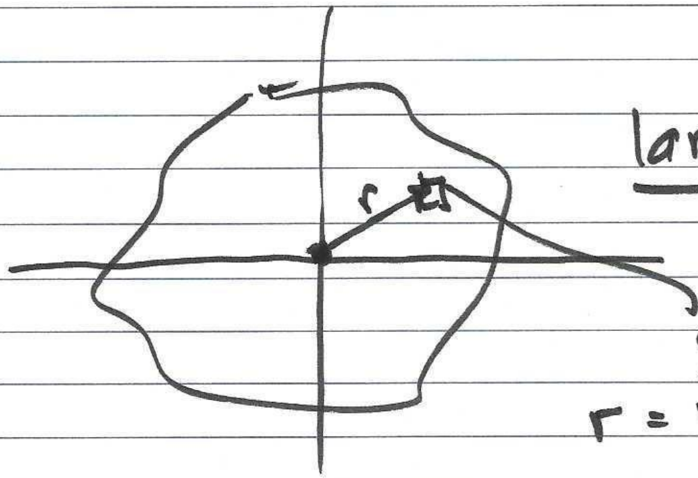
$r\,dr\,d\theta\,dz = dV$

$dm = \sigma r\,dr\,d\theta\,dz$

$$\iiint r^2 (\quad) =$$

$$I = \sigma \int_0^{2\pi}\int_0^{1}\int_0^{4} r^3\,dr\,d\theta\,dz$$

laming

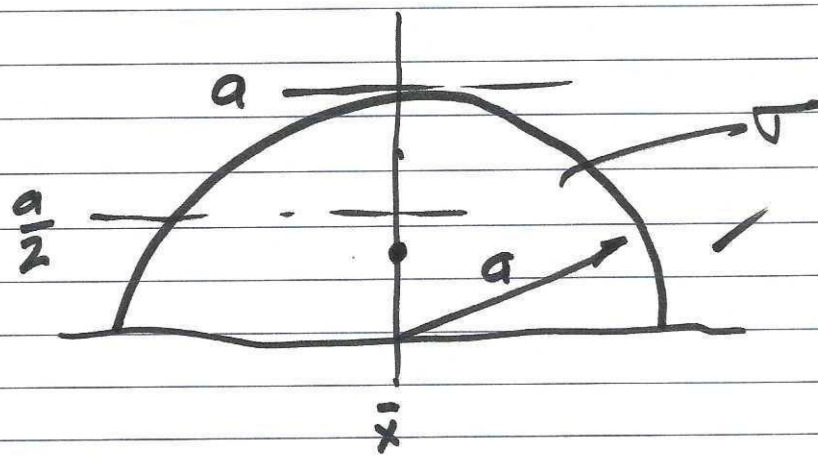


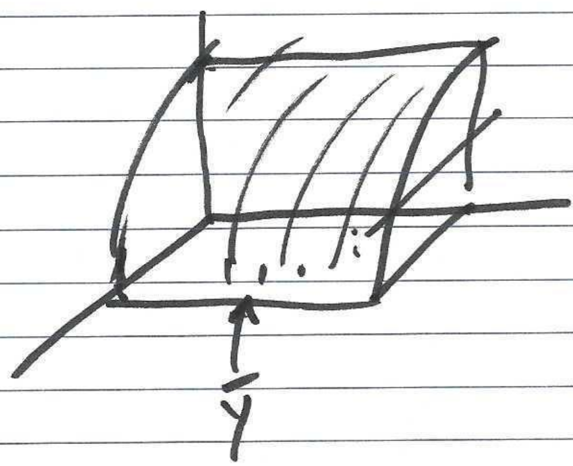
$$r = \sqrt{x^2+y^2}$$

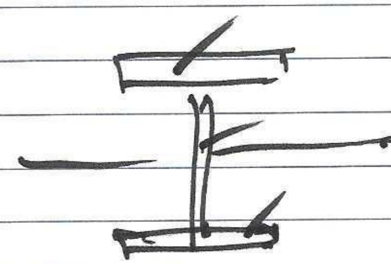
$$dI = r^2\,dm = \underline{\sigma(x^2+y^2)\,dxdy}$$

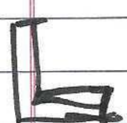
$$\sigma x^2\,dxdy + \sigma(\cancel{x}y^2\,dxdy)$$

$$I_z = I_y + I_x$$









$A_s \bar{x}_s + A_\Delta \bar{x}_\Delta = \overline{}$

$A_s + A_\Delta$