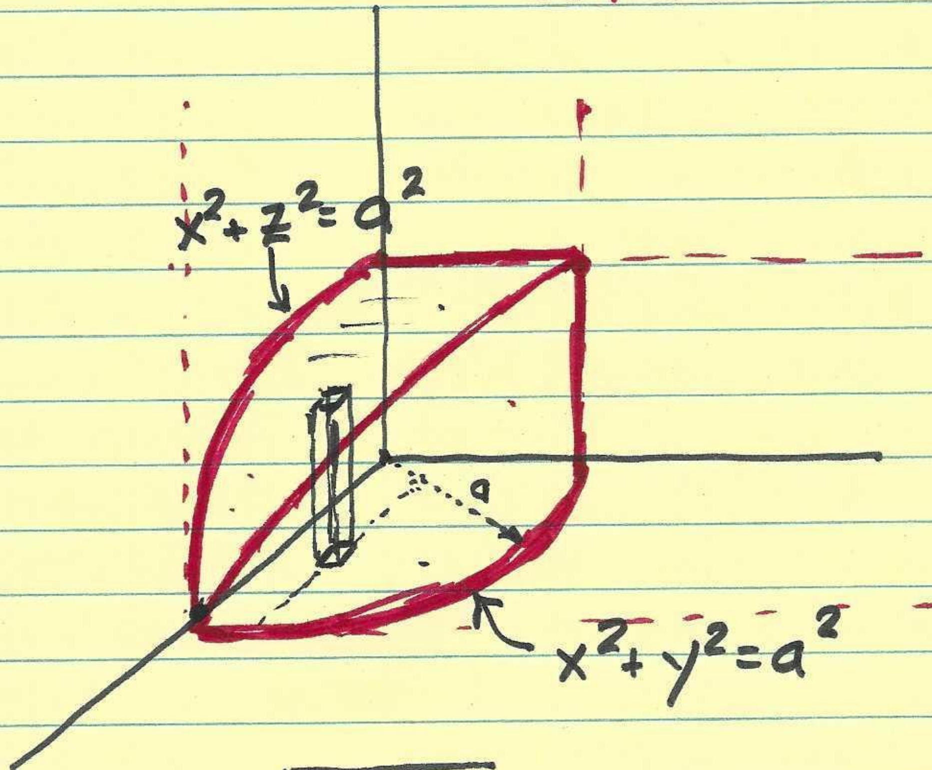
11/12

Intersection of two cylinders
of radius "a"



So... $z = \sqrt{a^2-x^2}$

$y = \sqrt{a^2-x^2}$

$$\text{Volume} = 8 \int_0^a \int_0^{\sqrt{a^2-x^2}} \int_0^{\sqrt{a^2-x^2}} dz\, dy\, dx$$

2

$$V = 8\int_0^a \int_0^{\sqrt{a^2-x^2}} \Big[z\Big]_0^{\sqrt{a^2-x^2}} dy\,dx$$

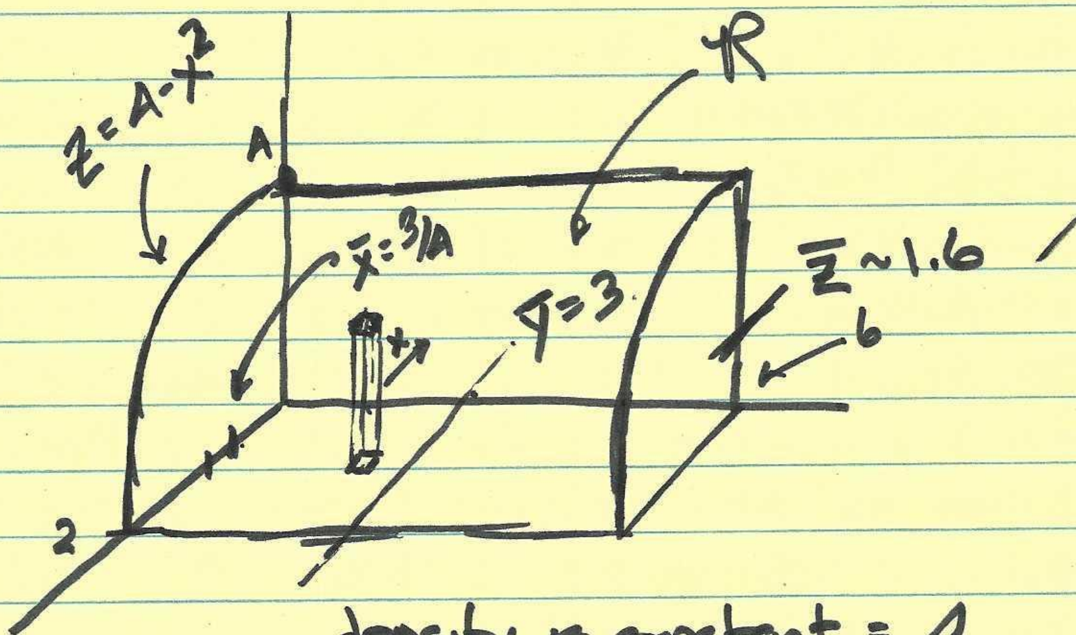
$$= 8\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2}\,dy\,dx$$

$$= 8\int_0^a \Big[\sqrt{a^2-x^2}\Big]\sqrt{a^2-x^2}\,dx$$

$$= 8\int_0^a (a^2-x^2)\,dx = 8\left[a^2x-\frac{x^3}{3}\right]_0^a$$

$$= 8\left(\frac{2}{3}a^3\right) = \boxed{\frac{16}{3}a^3}$$

3



density is constant $= \rho$

$$\text{Mass} = \rho \cdot \text{Vol} = \rho \iiint_R dx\,dy\,dz$$

$$= \rho \int_{x=0}^{2} \int_{y=0}^{6} \int_{z=0}^{4-x^2} dz\,dy\,dx$$

$$= \rho \int_0^2 \int_0^6 \Big[z \Big]_0^{4-x^2} dy\,dx$$

④

$$= \rho \int_0^2 \int_0^6 (4-x^2)\,dy\,dx$$

$$= \rho \int_0^2 \Big[4y - x^2y\Big]_{y=0}^{6} dx$$

$$= \rho \int_0^2 (24 - 6x^2)\,dx$$

$$= \rho \Big[24x - 2x^3\Big]_0^2 = \rho(48-16) = \boxed{32\rho}$$

Find $(\bar{x}, \bar{y}, \bar{z})$

$\bar{y} = 3$ by symmetry

$$\bar{x} = ? \quad M_{yz} = \int_0^2 \int_0^6 \int_0^{4-x^2} x\,dz\,dy\,dx$$

5

$$M_{yz} = \int_0^2 \int_0^6 [x(4-x^2)]\,dy\,d\cancel{x}$$

$$= \int_0^2 \int_0^6 (4x - x^3)\,dy\,d\cancel{x}$$

$$= \int_0^2 \left[(4xy - x^3y)\right]_{y=0}^{6} dx$$

$$= \int_0^2 (24x - 6x^3)\,dx$$

$$M_{yz} = \left[\frac{24x^2}{2} - \frac{6x^4}{4}\right]_0^2$$

$$= 48 - 24 = \boxed{24}$$

So... $\bar{x} = \frac{24}{32} = \frac{3}{4}$

?

$$M_{xy} = \int_0^2 \int_0^6 \int_0^{4-x^2} z \, dz\,dy\,dx$$

$$= \int_0^2 \int_0^6 \left[\frac{z^2}{2} \right]_0^{4-x^2} dy\,dx$$

$$= \int_0^2 \int_0^6 \frac{1}{2}\left(16 - 8x^2 + x^4\right) dy\,dx$$

$$= \frac{1}{2} \int_0^2 \left(16y - 8x^2y + x^4y\right)\Big|_0^6 dx$$

$$= \frac{1}{2} \int_0^2 \left(96 - 48x^2 + 6x^4\right) dx$$

$$M_{xy} = \frac{1}{2}\left(96x - 16x^3 + \frac{6x^5}{5}\right)\Big|_0^2$$

7

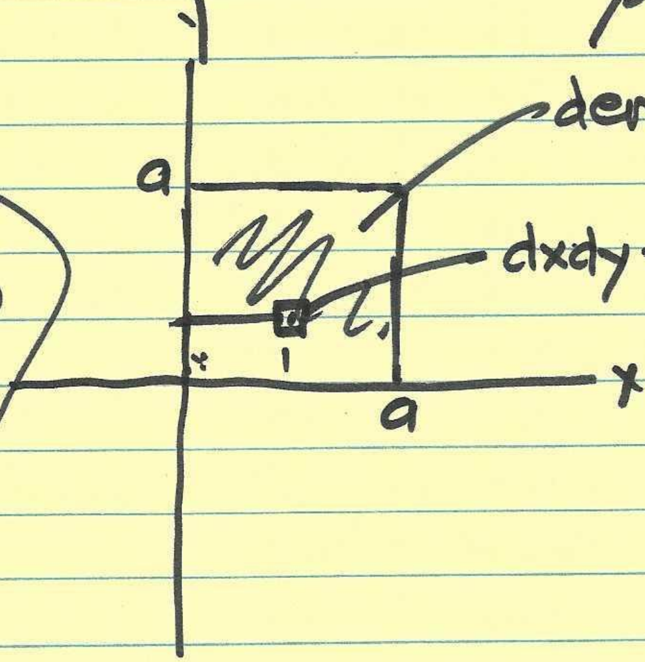
$$M_{xy} = \frac{1}{2}\left(192 - 128 + \frac{6}{5}(32)\right)$$

$$= \underline{51.2}$$

$$\bar{z} = \frac{51.2}{32} = \boxed{1.6}$$

Moments about an axis

σ = areal density
ρ = vol "



density ρ

$dx\,dy \cdot \sigma = dm$

Gen'l

$$\boxed{dI = r^2 dm}$$

$$\underline{F} = m\underline{a}$$

$$\tau = I\ddot{\theta}$$

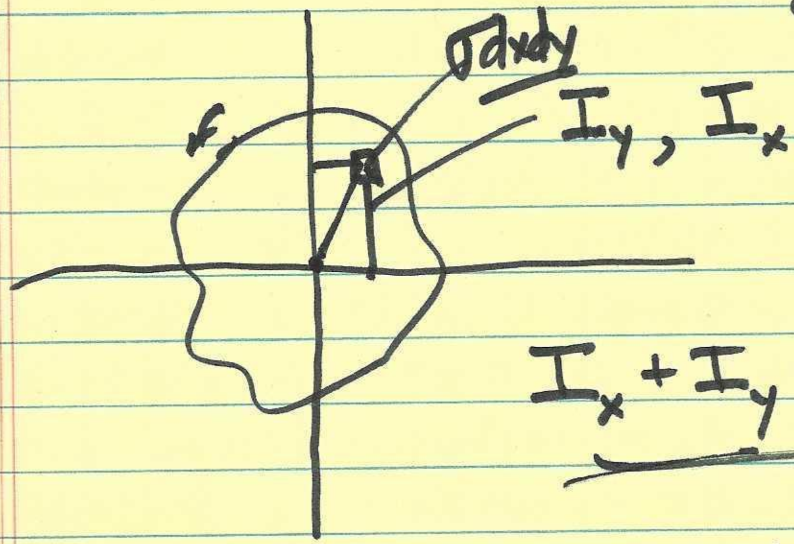
torque   moment of inertia

$$dI = x^2 \sigma\, dx\, dy$$

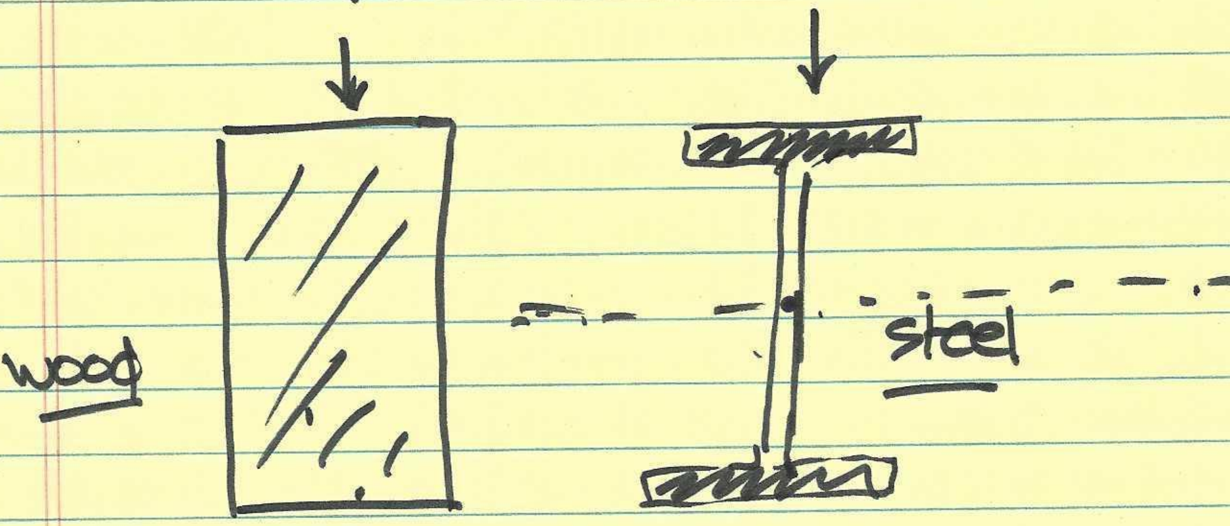
$$I = \sigma \int_0^a \int_0^a x^2 dx\, dy$$

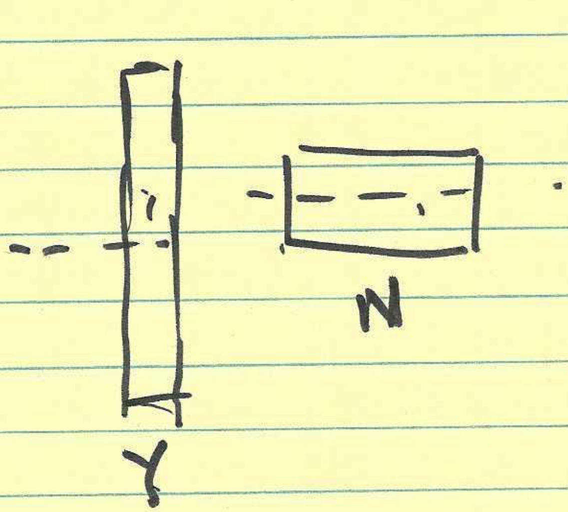
$$= \sigma \int_0^a \left[\frac{x^3}{3}\right]_0^a dy = \sigma \frac{a^3}{3} \int_0^a dy = \boxed{\sigma \frac{a^4}{4}}$$

$$dI = r^2 dm$$

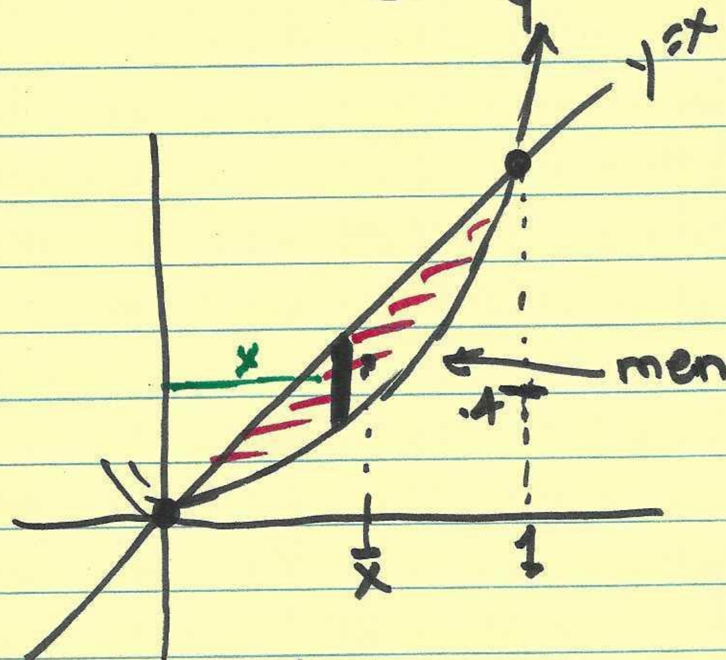


$$I_x + I_y = I_z$$





⑩



$x^2 = x$
@ 0,1

$$A = \int_0^1 \int_{x^2}^{x} dy\,dx$$

$$= \int_0^1 (x - x^2)\,dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \boxed{\frac{1}{6}}$$

$$M_y = \int_0^1 \int_{x^2}^{x} x\,dy\,dx = \int_0^1 \left[xy\right]_{y=x^2}^{x} dx$$

$$= \int_0^1 (x^2 - x^3)\,dx$$

$$M_y = \left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1 = \frac{1}{3} - \frac{1}{4} = \boxed{\frac{1}{12}}$$

$$\bar{x} = \frac{1/12}{1/6} = \frac{1}{2}$$

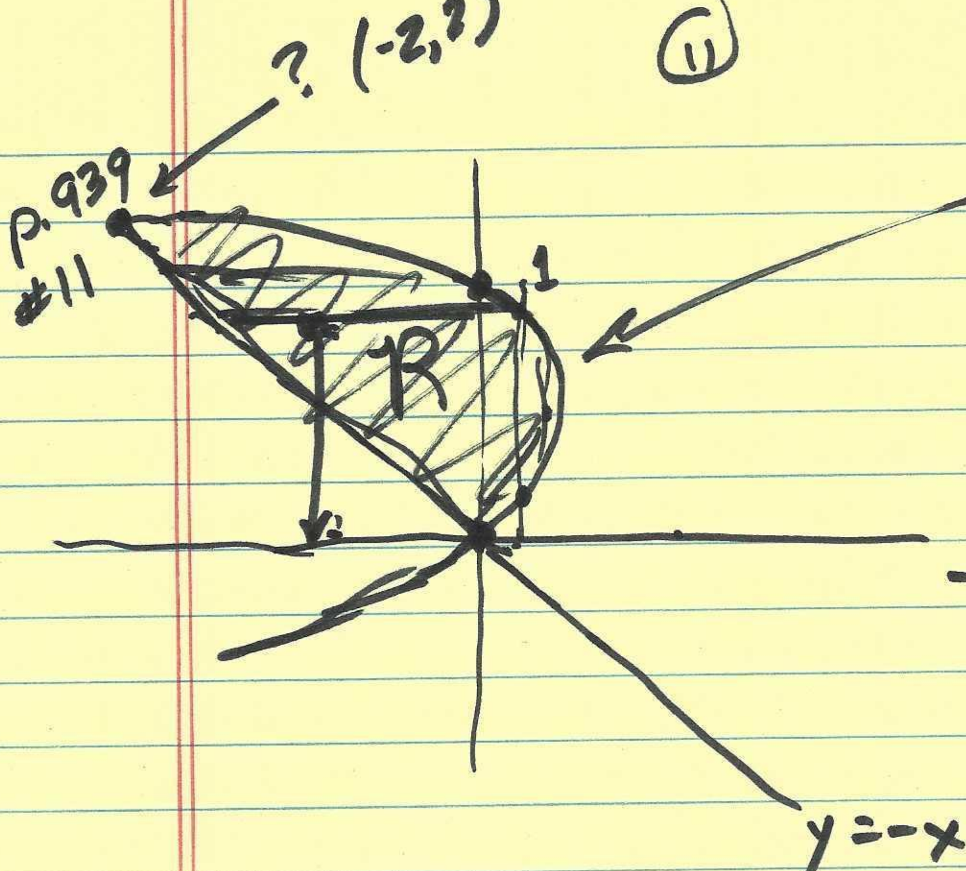
$$M_x = \int_0^1 \int_{x^2}^{x} y\,dy\,dx$$

$$= \int_0^1 \left[\frac{y^2}{2}\right]_{x^2}^{x} dx = \frac{1}{2}\int_0^1 (x^2 - x^4)\,dx$$

$$M_x = \frac{1}{2}\left[\frac{x^3}{3} - \frac{x^5}{5}\right]_0^1 = \frac{1}{2}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{1}{15}$$

$$\bar{y} = \frac{1/15}{1/6} = \frac{6}{15} = \boxed{\frac{2}{5}}$$

(11)



$-y = y - y^2$

$y^2 = 2y$

density is $\sigma(x,y) = x + y$.

$dI = y^2 dm$

$dm = (x+y)\,dx\,dy$

$$\iint_R y^2 (x+y)\,dx\,dy$$

$$I_x = \int_0^2 \int_{-y}^{y-y^2} y^2 (x+y)\,dx\,dy$$

12

$$I_x = \int_0^2 \int_{-y}^{y-y^2} y^2(x+y)\,dx\,dy$$

$$= \int_0^2 \left[\frac{y^2x^2}{2} + xy^3\right]_{-y}^{y-y^2} dy$$

$$= \int_0^2 \left[\left(\frac{y^2}{2}(y-y^2)^2 + (y-y2)y^3\right) - \left(\frac{y^4}{2} - y^4\right)\right] dy$$

$$= \int_0^2 \left[\frac{y^2}{2}(y^2-2y^3+y^4) + y^4 - y^5 + \frac{y^4}{2}\right) dy$$

$$= \int_0^2 \left(\frac{y^4}{2} - y^5 + \frac{y^6}{2} + y^4 - y^5 + \frac{y^4}{2}\right) dy$$

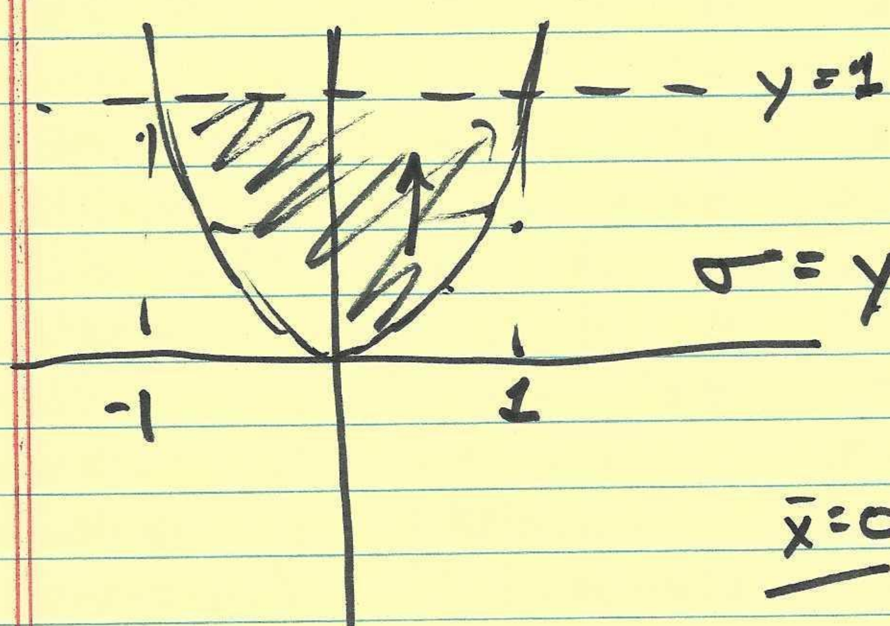
$$= \int_0^2 \left(2y^4 - 2y^5 + \frac{y^6}{2}\right) dy$$

$$\left[\frac{2y^5}{5} - \frac{2y^6}{6} + \frac{y^7}{14}\right]_0^2 =$$

$$\left(\frac{2}{5}\right)(32) - \left(\frac{1}{3}\right)(64) + \left(\frac{128}{14}\right) =$$

$I_y =$

#16



$r = y + 1$

$\bar{x} = 0$

Find centroid & I_y

Mass = M = ?

$$M = \int_{-1}^{+1}\int_{x^2}^{1}(y+1)\,dy\,dx$$

$$= \int_{-1}^{1}\left[\frac{y^2}{2}+y\right]_{x^2}^{1}dx = \int_{-1}^{1}\left(\frac{3}{2}-\frac{x^4}{2}-x^2\right)dx$$

$$= \left[\frac{3}{2}x-\frac{x^5}{2\cdot 5}-\frac{x^3}{3}\right]_{-1}^{1} =$$

$$= \left(\frac{3}{2}-\frac{1}{10}-\frac{1}{3}\right)-\left(-\frac{3}{2}+\frac{1}{10}+\frac{1}{3}\right)$$

$$= 2\left(1.5-0.1-0.33\right) = \underline{\qquad}$$