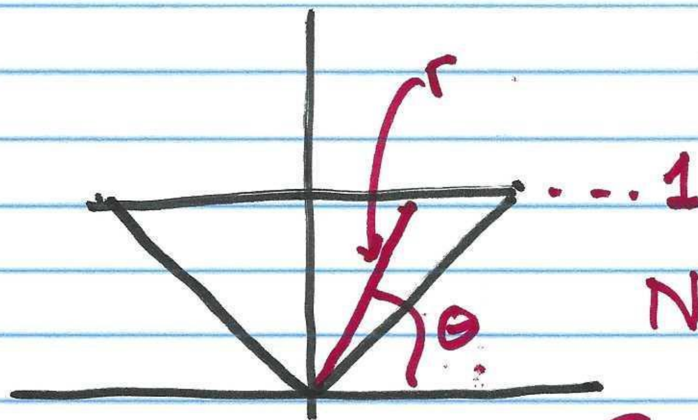
0

10/31

3

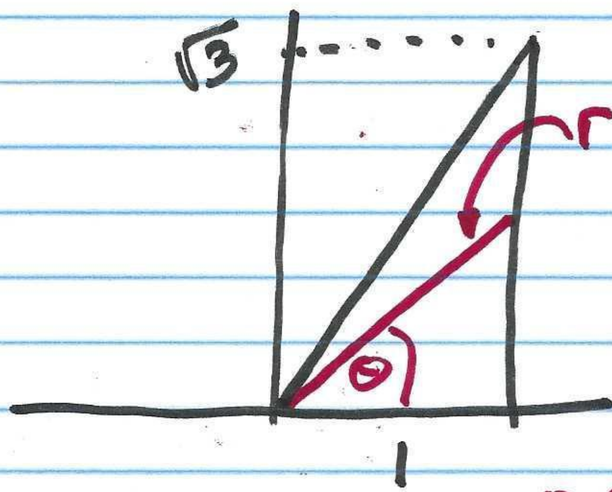


Note $r\sin\theta = 1$

So.. $r = \frac{1}{\sin\theta}$

$\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$

4

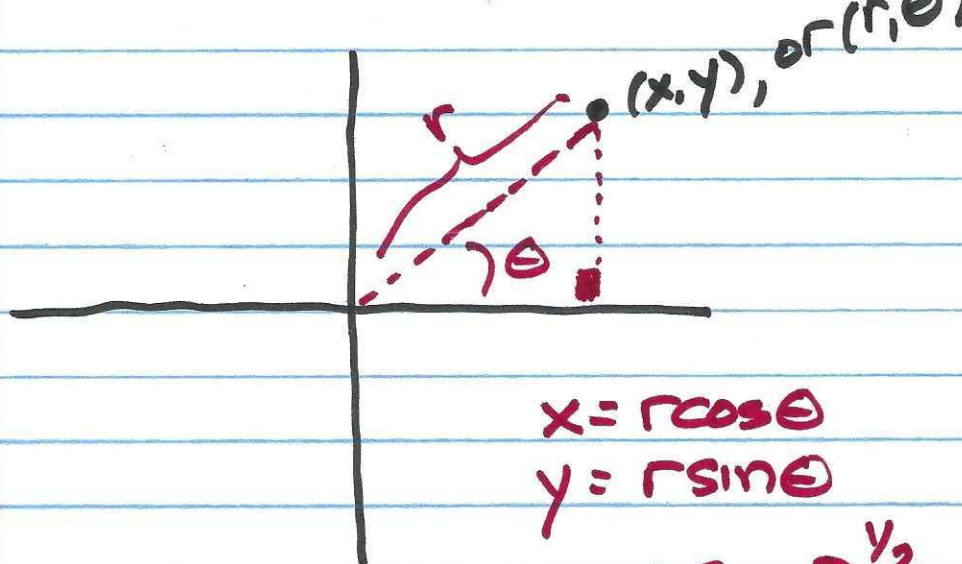


Note $r\cos\theta = 1$

So... $r = \frac{1}{\cos\theta}$

$0 \le \theta \le 60^\circ$

Polar Integrals

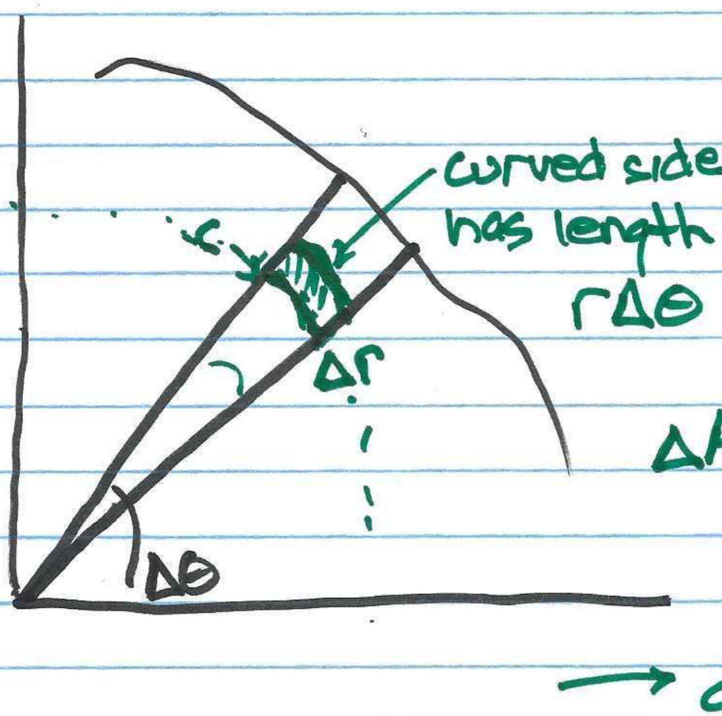


$x = r\cos\theta$

$y = r\sin\theta$

$r = (x^2+y^2)^{1/2}$

$\theta = \arctan\left(\frac{y}{x}\right)$



$\underline{\Delta A = \Delta x \Delta y}$

$\neq \Delta r \Delta\theta$

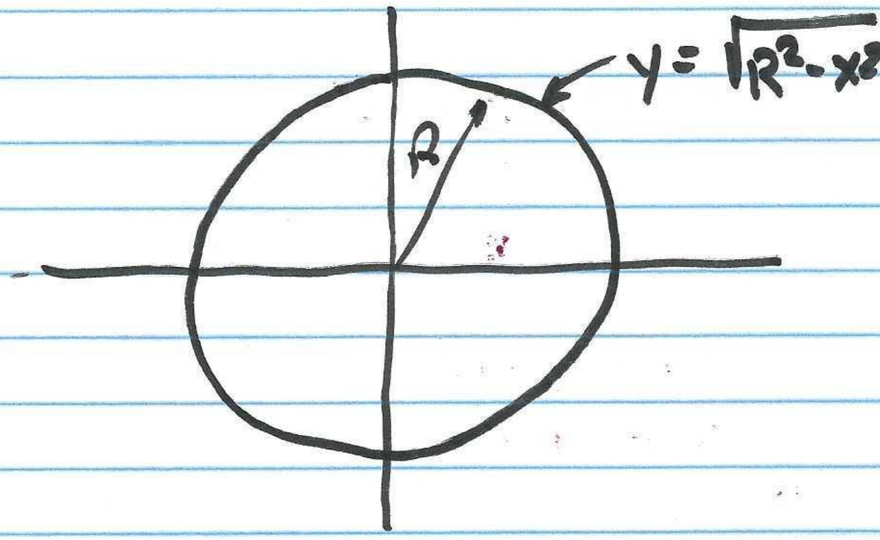
$\Delta A = \Delta r (r\Delta\theta)$

$= r\Delta r\Delta\theta$

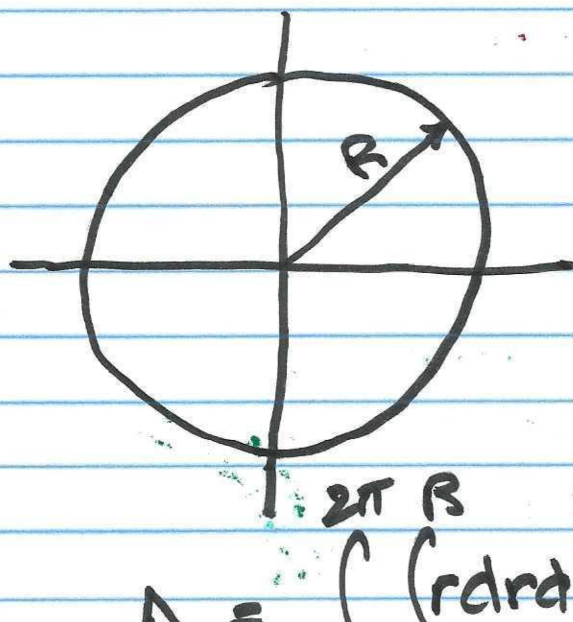
$\longrightarrow dA = r\,dr\,d\theta$

2

Given $f(r,\theta) = r(\theta)$



$r(\theta) = R$, constant



$$A = \int_0^{2\pi}\int_0^{R} r\,dr\,d\theta$$

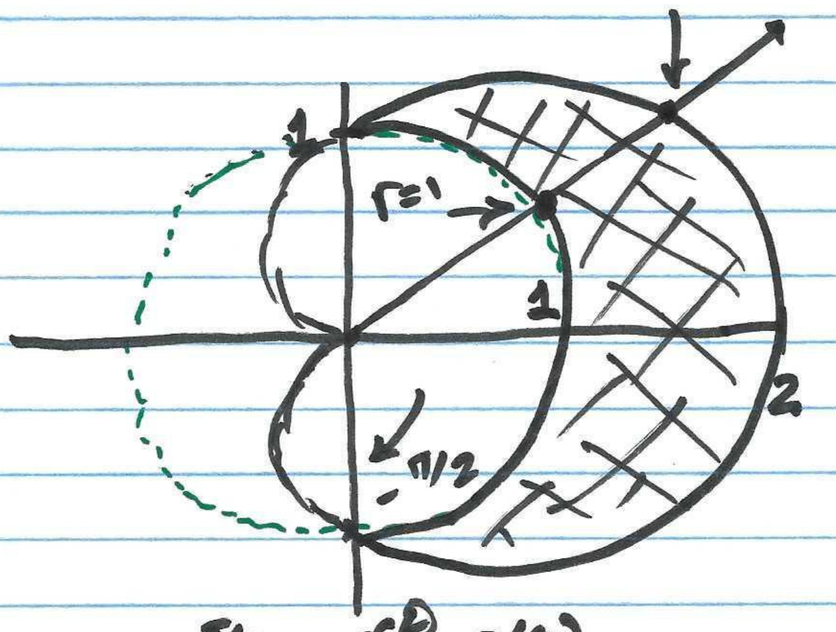
$$= \int_0^{2\pi}\left[\frac{r^2}{2}\right]_0^{R} d\theta$$

③

$$A = \int_0^{2\pi} \frac{R^2}{2}\,d\theta = \frac{R^2}{2}\int_0^{2\pi} d\theta = (2\pi)\left(\frac{R^2}{2}\right) =$$

πR^2 area of circle

$r(\theta) = 1 + \cos\theta$



$\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\theta \in \cancel{[0, 2\pi]}$

$r \in \cancel{[0, 1]}$

$r \in [1, 1+\cos\theta]$

$$\text{So... } A = \int_{-\frac{\pi}{2}}^{+\pi/2}\int_{1}^{1+\cos\theta} r\,\overbrace{(1+\cos\theta)}^{r(\theta)}\,dr\,d\theta$$

$$\textcircled{A}$$

$$\cos^2\left(\frac{x}{2}\right) = \frac{1+\cos x}{2}$$

$$A = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\frac{r^2}{2}\right]_1^{1+\cos\theta} d\theta$$

$$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\pi/2} (1+2\cos\theta+\cos^2\theta)\, d\theta$$

$$= \frac{1}{2}\left[[\theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + 2[\sin\theta]_{-\frac{\pi}{2}}^{\pi/2} + \right]$$

$$\left(\frac{1}{2}\int_{-\frac{\pi}{2}}^{\pi/2} (1+\cos 2\theta)\, d\theta \right)$$

↓ this part only

$$\frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{\pi}{2} + 0$$

5

$$A = \frac{1}{2}\left[\pi + 4 + \frac{\pi}{2}\right] = \frac{1}{2}\left(\frac{3\pi}{2}+4\right) =$$

$$\boxed{\frac{3\pi}{4}+2}$$

---

(2) r goes from 1 to 4

θ " " $-\frac{\pi}{2}$ to $\frac{\pi}{2}$

$$\iint \overset{f(r,\theta)}{\swarrow} r\,dr\,d\theta$$

$$A = \int_{-\frac{\pi}{2}}^{\pi/2}\int_{1}^{4}(1)\,r\,dr\,d\theta$$

$$= \int_{-\frac{\pi}{2}}^{\pi/2}\left[\frac{r^2}{2}\right]_{1}^{4} d\theta = \left(8-\frac{1}{2}\right)\int_{-\frac{\pi}{2}}^{\pi/2} d\theta =$$

$$\frac{15}{2}\pi$$

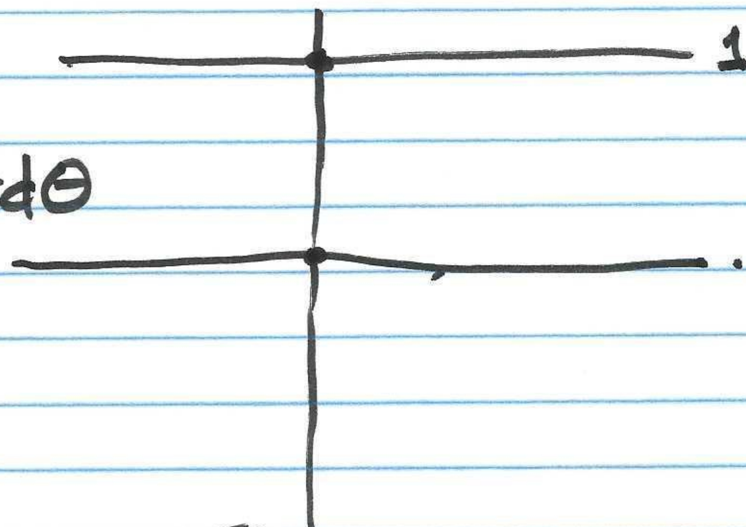
6

p. 919
#10

$$\int_{y=0}^{1}\int_{x=0}^{\sqrt{1-y^2}} (x^2+y^2)\,dx\,dy$$

$x \to r\cos\theta$
$y \to r\sin\theta$

$$\int_{y=0}^{1}\int_{0}^{\sqrt{1-y^2}} r^2\, r\,dr\,d\theta$$



$$\downarrow$$

$$\int_{0}^{\pi/2}\int_{0}^{\cos\theta} r^3\,dr\,d\theta = \int_{0}^{\pi/2}\left[\frac{r^4}{4}\right]_0^{\cos\theta} d\theta$$

$$= \frac{1}{4}\int_{0}^{\pi/2} \cos^4\theta\,d\theta$$

$$I_x = \int_0^\infty e^{-x^2}\,dx$$ - can't do in elem. functions

$$I_y = \int_0^\infty e^{-y^2}\,dy$$

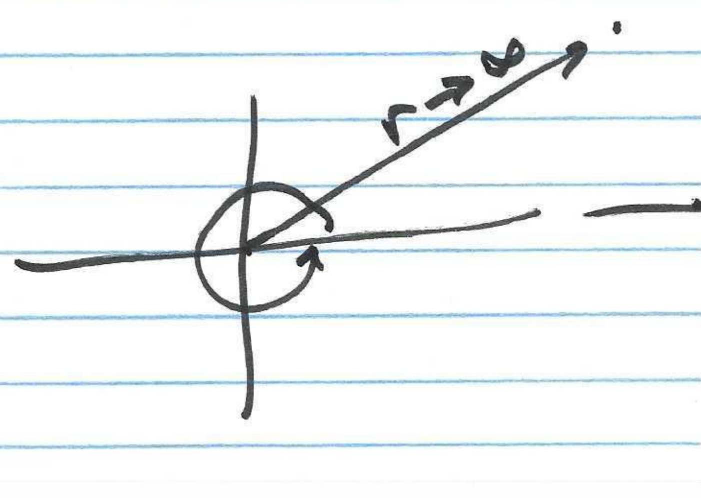
$$\iint f(x)\,g(y)\,dx\,dy$$

$$\int f(x)\,dx \cdot \int g(y)\,dy$$

$$I^2 = I_x I_y = \int_0^\infty \int_0^\infty e^{-x^2} e^{-y^2}\,dx\,dy$$

$$\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy \xrightarrow[\text{to polar}]{\text{transform}}$$

$$\frac{1}{2}\int_0^{2\pi} \int_0^\infty e^{-r^2}\, r\,dr\,d\theta$$



8

$$I^2 = \frac{1}{2}\int_{\theta=0}^{2\pi}\int_0^{\infty} e^{-r^2}\, r\, dr\, d\theta$$

r integral

Let $u = r^2$ $du = 2r\,dr$

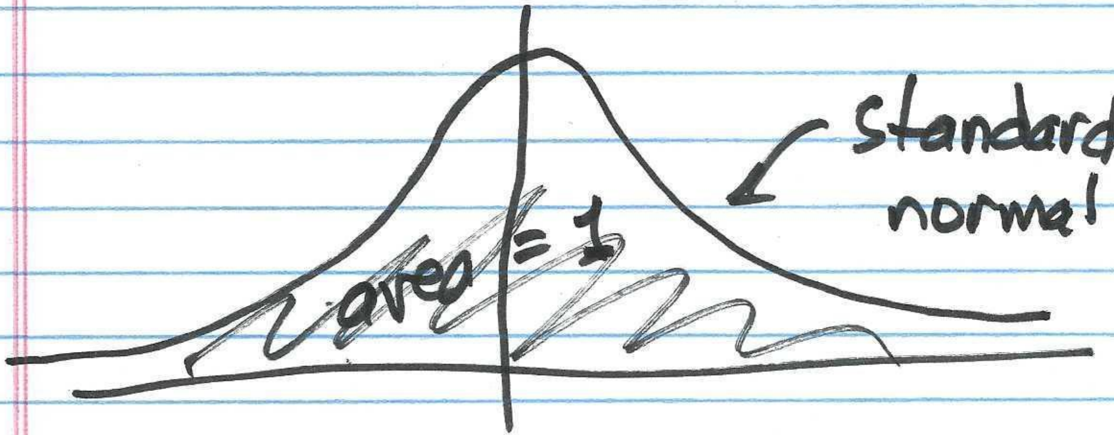
$$\int_0^{\infty} e^{-u}\frac{du}{2} = \frac{1}{2}\left[-e^{-u}\right]_0^{\infty} = \frac{1}{2}\left[0-(-1)\right]$$

$$= \frac{1}{2}$$

$$I^2 = \frac{1}{2}\int_0^{2\pi}\frac{\theta}{2}\,d\theta = \frac{2\pi}{4} = \frac{\pi}{2} \text{ so...}$$

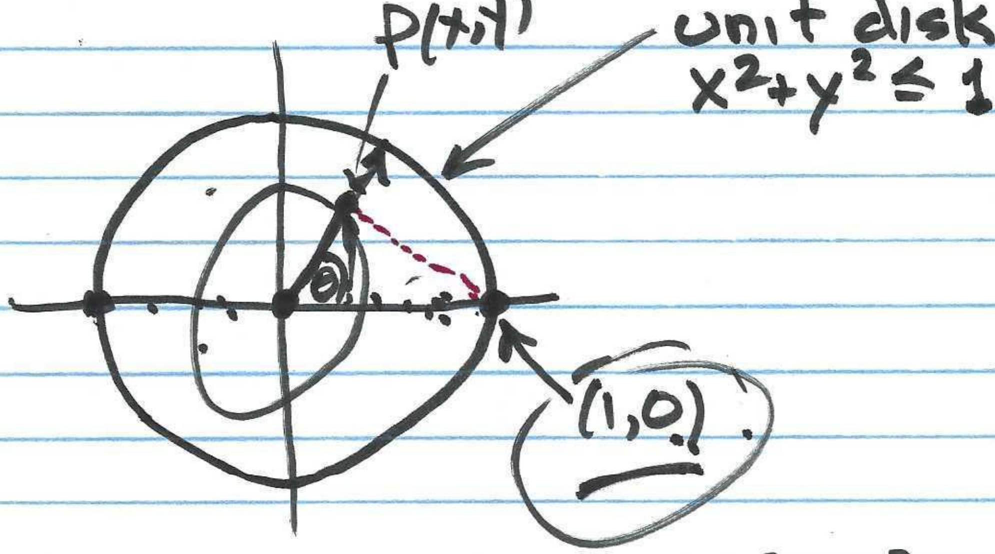
$$I = \frac{\sqrt{\pi}}{2} = \int_0^{\infty} e^{-x^2}\,dx$$

⑨



$$n(x;0,1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

#36



P is @ (r,θ)

$(x-1)^2+y^2$

$$f(r,\theta) = (r\cos\theta - 1)^2 + (r\sin\theta)^2$$

10

$$\bar{f} = \frac{1}{\pi}\int_0^{2\pi}\int_0^1 f(r,\theta)\, r\, dr\, d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi}\int_0^1 \left(\underline{r^3\cos^2\theta} - 2r^2\cos\theta + r + \underline{r^3\sin^2\theta}\right) dr\, d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi}\int_0^1 \left(r^3 - 2r^2\cos\theta + r\right) dr\, d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left[\frac{r^4}{4} - \frac{2r^3}{3}\cos\theta + \frac{r^2}{2}\right]_0^1 d\theta$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{1}{4} - \frac{2}{3}\cos\theta + \frac{1}{2}\right) d\theta$$

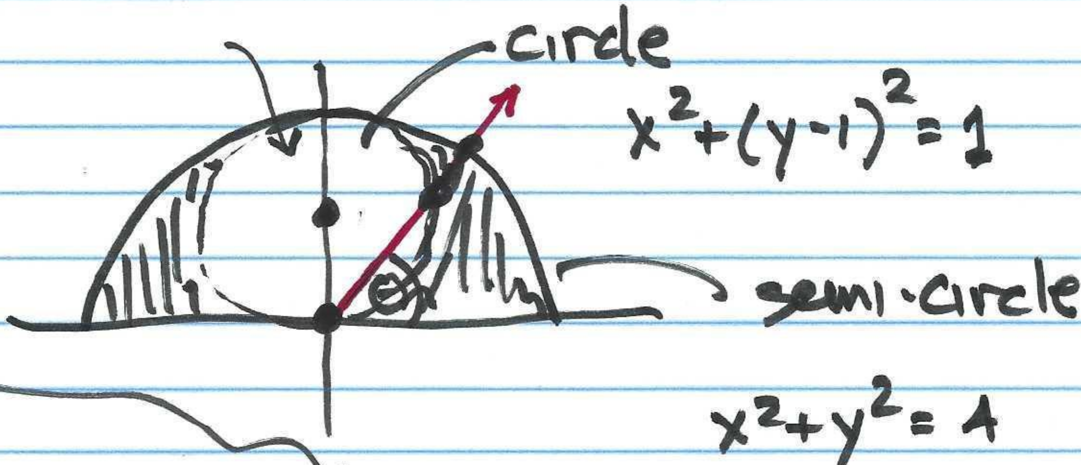
$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{3}{4} - \frac{2}{3}\cos\theta\right) d\theta$$

$$\bar{f} = \frac{1}{\pi}\left[\frac{3\theta}{4} - \frac{2}{3}\sin\theta\right]_0^{2\pi} = \frac{1}{\pi}\left(\frac{3}{4}\cdot 2\pi\right) = $$

$$\frac{3}{2}$$

#A7

$$\iint_R \sqrt{x^2+y^2}\,dA \quad \text{where } R \text{ is region}$$



$$\int_0^{\pi}\int_{2\sin\theta}^{2} r\cdot r\,dr\,d\theta$$

12

$$r^2\cos^2\theta + (r\sin\theta - 1)^2 = 1$$

$$\underbrace{r^2\cos^2\theta + r^2\sin^2\theta} - 2r\sin\theta + \cancel{1} = \cancel{1}$$

$$r^2 - 2r\sin\theta = 0$$

$$r^2 = 2r\sin\theta$$

$$\frac{r}{2} = \sin\theta$$

$$\int_0^{\pi}\int_{2\sin\theta}^{2} r^2\,dr\,d\theta = \int_0^{\pi}\left[\frac{r^3}{3}\right]_{2\sin\theta}^{2} d\theta$$

$$= \int_0^{\pi}\left(\frac{8}{3} - \frac{8}{3}\sin^3\theta\right)d\theta$$

$$= \frac{8}{3}\int_0^{\pi} d\theta - \frac{8}{3}\int_0^{\pi}\sin^3\theta\,d\theta$$

you can do this

$$\iint_R f(x,y)\,dx\,dy = \iint_{R'} f(r\cos\theta, r\sin\theta)\,\overbrace{\frac{\partial(x,y)}{\partial(r,\theta)}}^{\text{local area magnification factor}}\,dr\,d\theta$$

"jacobian"

$$\frac{\partial(x,y)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \end{vmatrix} =$$

$$\begin{vmatrix} \cos\theta & \sin\theta \\ (-r\sin\theta) & r\cos\theta \end{vmatrix} = r\cos^2\theta + r\sin^2\theta$$

$$= r(\cos^2\theta + \sin^2\theta)$$

$$= r.$$