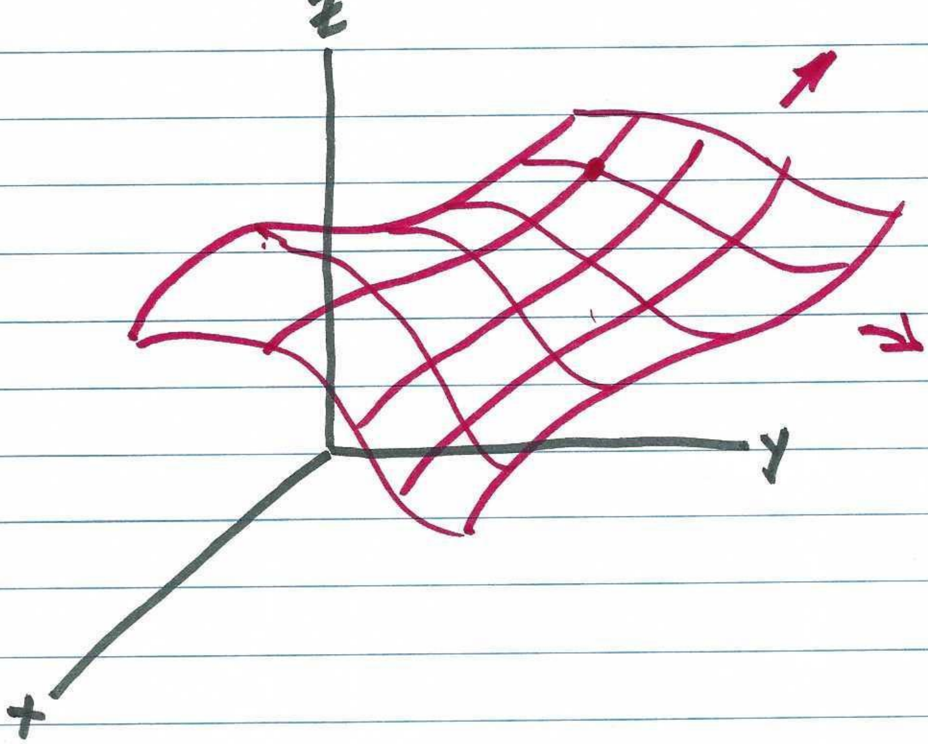
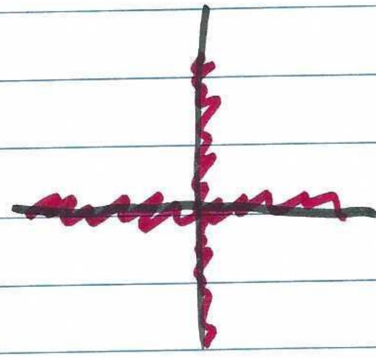
$f(x_1, x_2, \cdots, x_n)$



Domain Restrictions

$z = \sqrt{y - x^2} \qquad y - x^2 \geq 0 \qquad y \geq x^2$

$z = \frac{1}{xy}$



no axes in domain

$\mathbb{R} \to \mathbb{R}^2 \to \mathbb{R}^3 \to \cdots \mathbb{R}^n$

$z = \sin(xy)$ dom $z = \mathbb{R}^2$

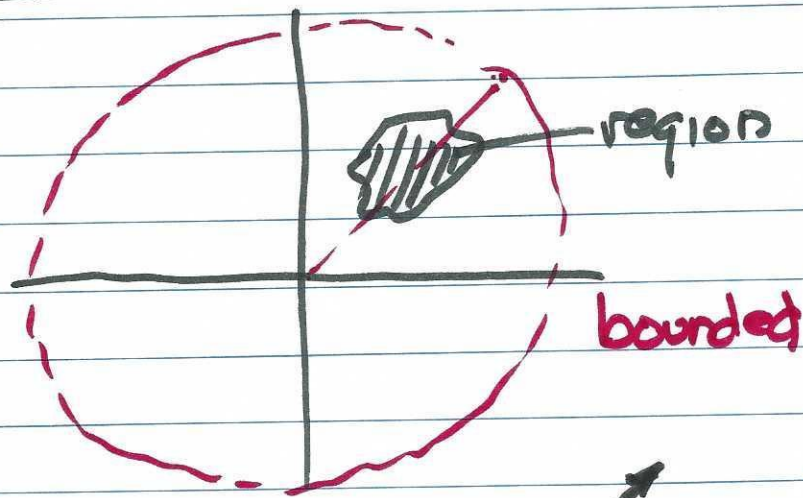
$w = \sqrt{x^2+y^2+z^2}$ dom $w = \mathbb{R}^3$ ran $w = \mathbb{R}_{0+}$

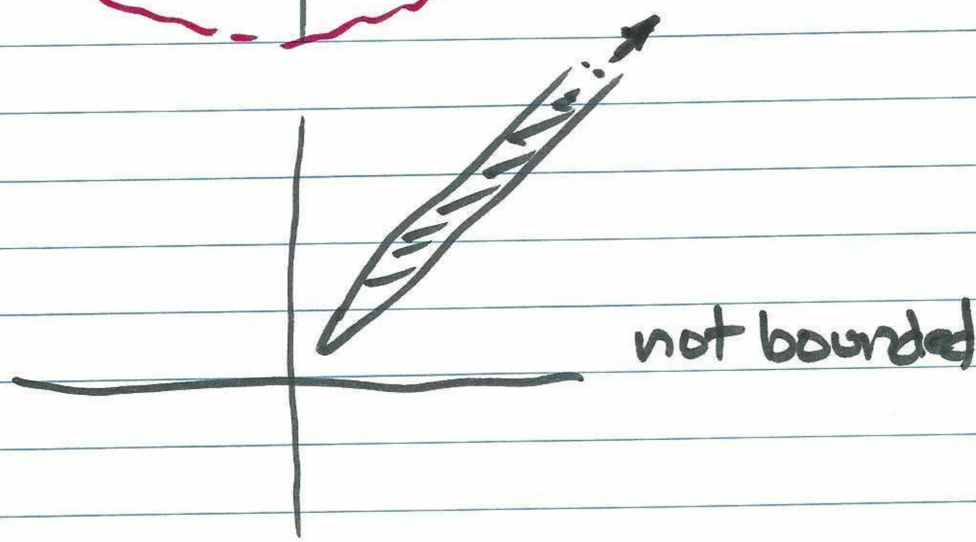
$w = \frac{1}{x^2+y^2+z^2}$ dom $w = \mathbb{R}^3 \setminus \{0,0,0\}$

ran $w = \mathbb{R}_+$

$w = xy \ln z$ dom $w = \{(x,y,z) \in \mathbb{R}^3 : z > 0\}$

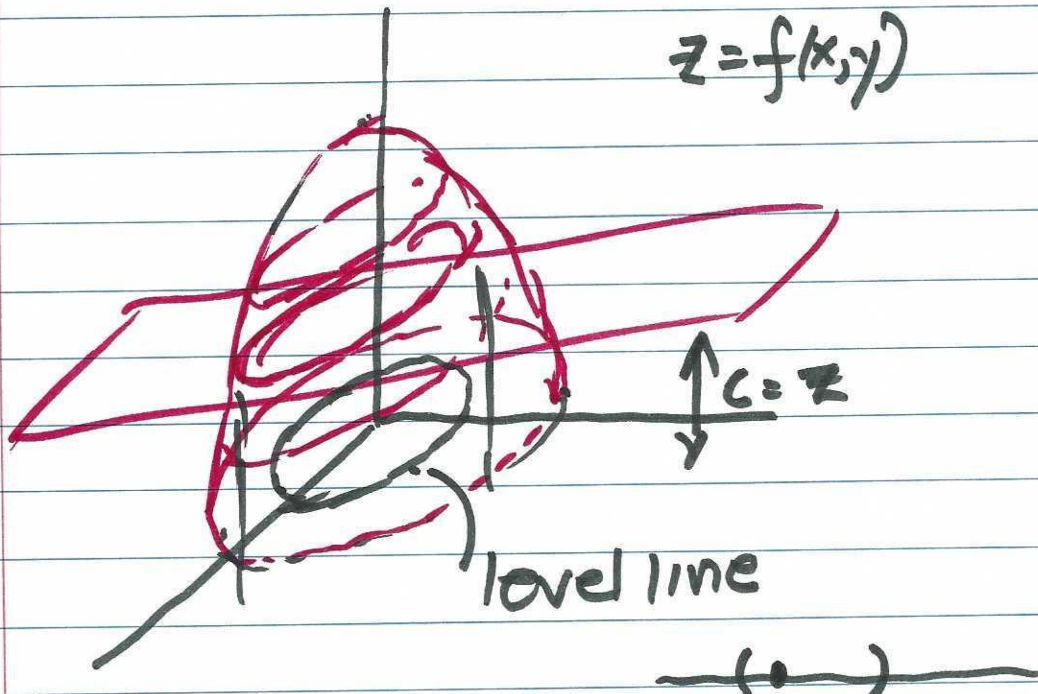
2-variables

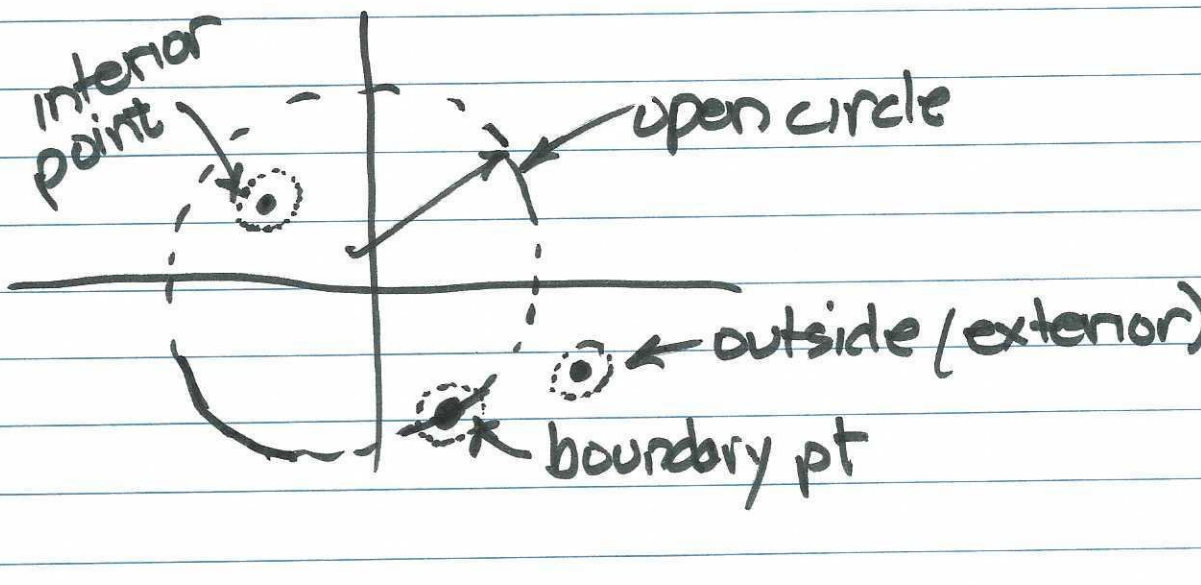




Set of points where $f(x,y) = c$, constant

is called a level curve

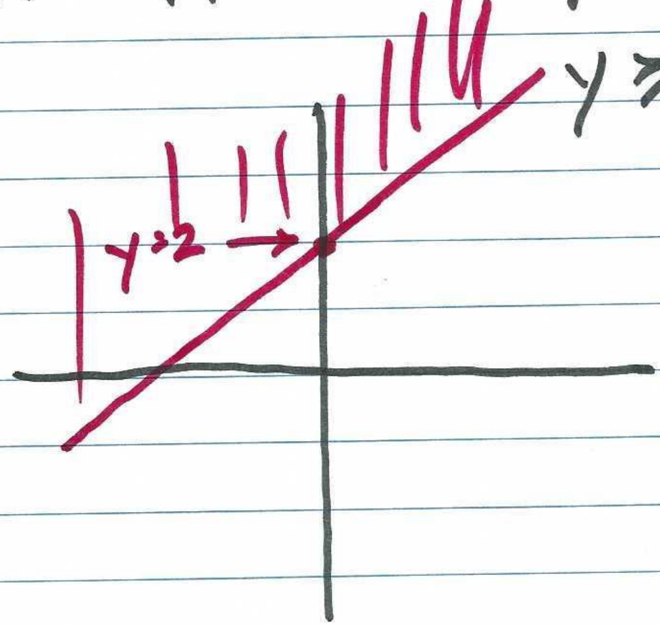

z = f(x,y)
c = z
level line


interior point
open circle
outside (exterior)
boundary pt

#5 p.812

$f(x,y) = \sqrt{y-x-2}$ $\quad y-x-2 \geq 0$

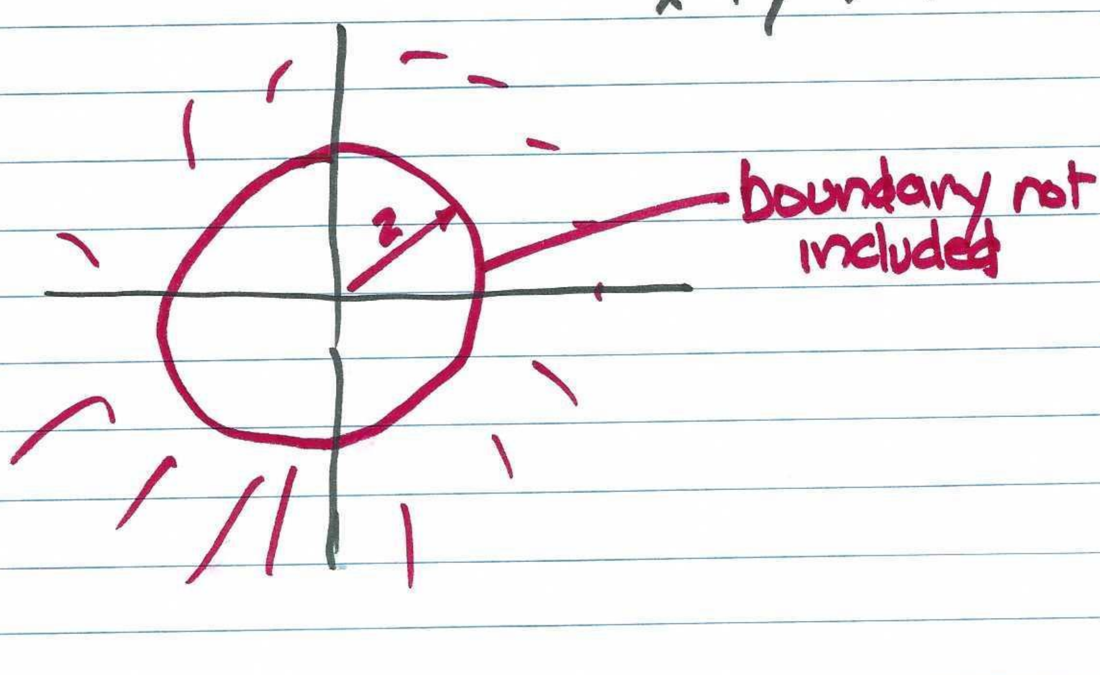
$y \geq x+2$



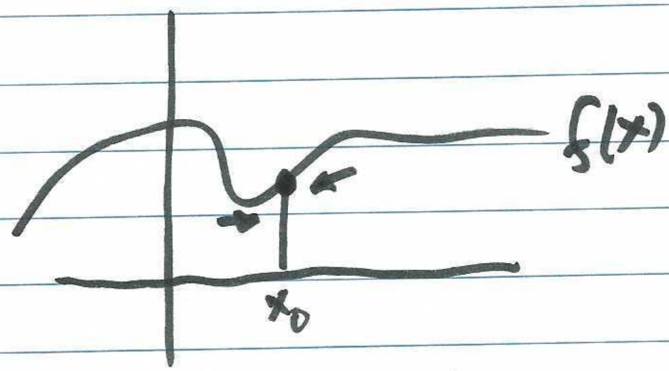
#6

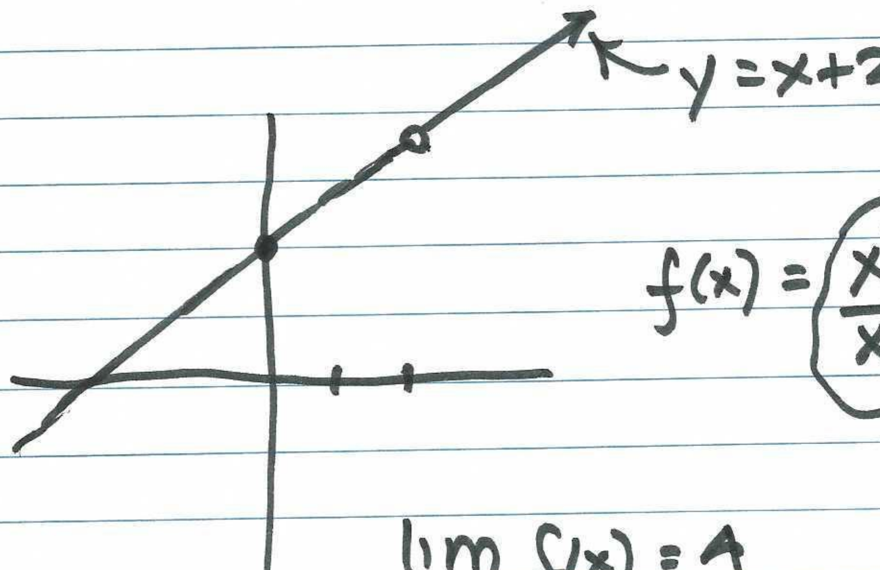
$f(x,y) = \ln(x^2+y^2-4)$ $\quad x^2+y^2-4 > 0$

$x^2+y^2 > 4$



Define limit of $f(x)$ as x approaches x_0





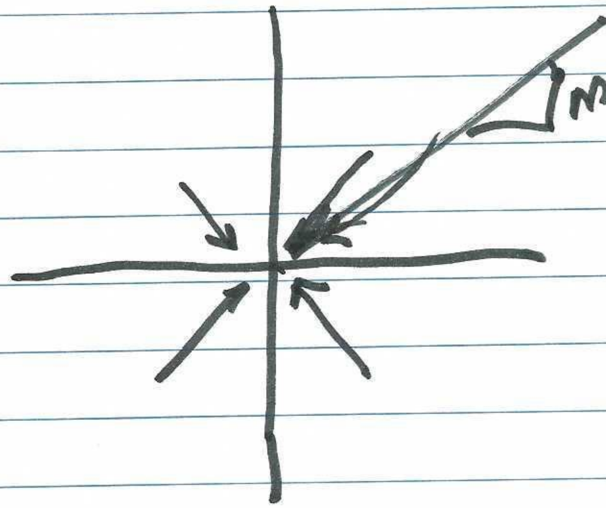
$$f(x) = \frac{x^2-4}{x-2} \sim \underline{x+2}$$

$$\lim_{x \to 2} f(x) = 4$$

<u>$f(2)$ DNE</u>

8

$$f(x,y) = \frac{xy}{x^2+y^2}$$



Suppose we approach origin along the line $y = mx$

$$f(x,y) = \frac{x(mx)}{x^2+m^2x^2} = \frac{m\cancel{x^2}}{\cancel{x^2}(1+m^2)} \quad \frac{m}{1+m^2}$$

is the limit as $(x,y) \to (0,0)$

Rules for Limits:

Suppose $\lim_{(x,y)\to(a,b)} f(x,y) = L$

$\lim_{(x,y)\to(a,b)} g(x,y) = M$ then:

1. $\lim_{(x,y)\to(a,b)} \left[f(x,y) \pm g(x,y)\right] = L \pm M$

2. $\lim_{(x,y)\to(a,b)} \left[f(x,y) \cdot g(x,y)\right] = L \cdot M$

3. $\lim_{(x,y)\to(a,b)} \left[\dfrac{f(x,y)}{g(x,y)}\right] = \dfrac{L}{M}$ if $M \neq 0$

4. $\lim_{(x,y)\to(a,b)} \left[c\, f(x,y)\right] = cL$

5. $\lim_{(x,y)\to(a,b)} \left[(f(x,y))^n\right] = L^n$

6 $\lim_{(x,y)\to(a,b)} \left[\sqrt[n]{f(x,y)}\right] = L^{1/n}$

Def^{n} of Continuity (1-dim)

$f(x)$ is continuous @ x_0 whenever:

(i) $\lim_{x \to x_0} f(x) = L$ exists

(ii) $f(x_0) = M$

(iii) $L = M$

"function constantly attains its limit"

#2 p.820 $\lim_{(x,y)\to(0,4)} \frac{x}{\sqrt{y}} = \frac{0}{2} = 0$

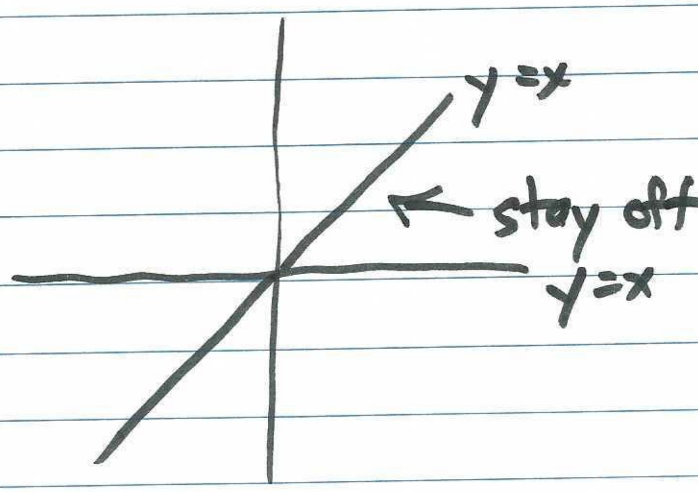
#5 $\lim_{(x,y)\to(0,\frac{\pi}{4})} \sec x \tan y = 1 \cdot 1 = 1$

z $\sin z$

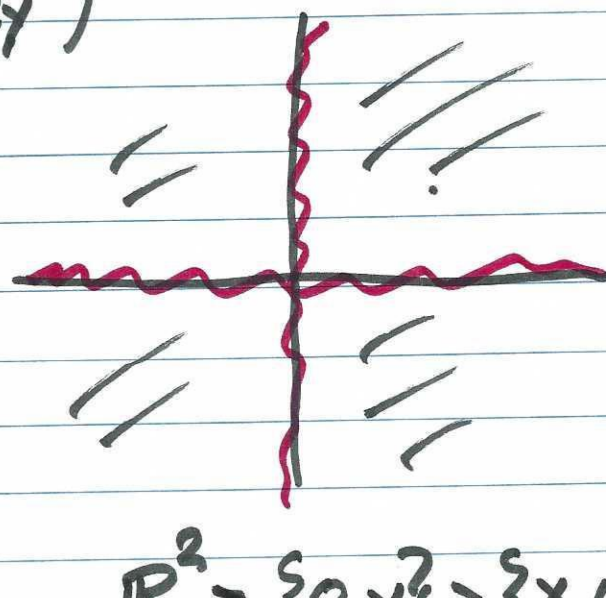
#31 $f(x,y) = \sin(x+y)$

continuous everywhere on $\mathbb{R}^2$

#32 $f(x,y) = \dfrac{x+y}{x-y}$

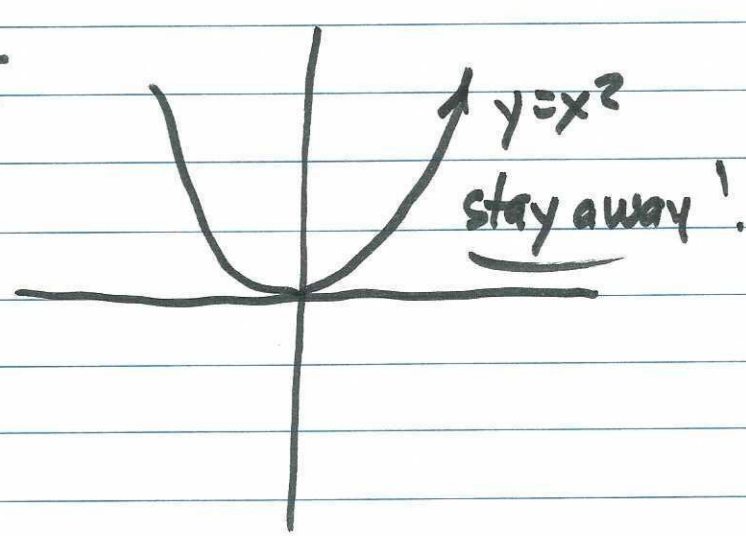


#33 $g(x,y) = \sin\left(\dfrac{1}{xy}\right)$

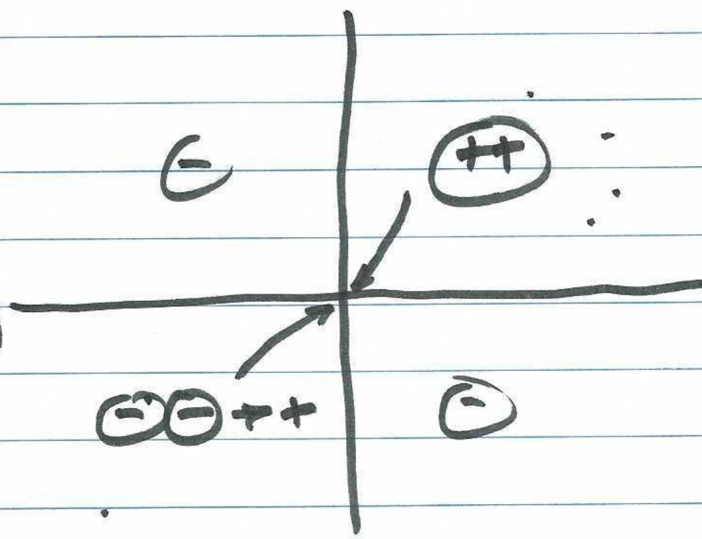


$\mathbb{R}^2 - \{0,y\} - \{x,0\}$

#3A $g(x,y) = \dfrac{1}{x^2 - y}$



#4A $f(x,y) = \dfrac{xy}{|xy|}$



lim @ (0,0) is
1 from 1st or 3rd
quadrants

lim @ (0,0) is
−1 from 2d or 4th quadrant $1 \neq -1$ so

two path test shows no limit @ (0,0)