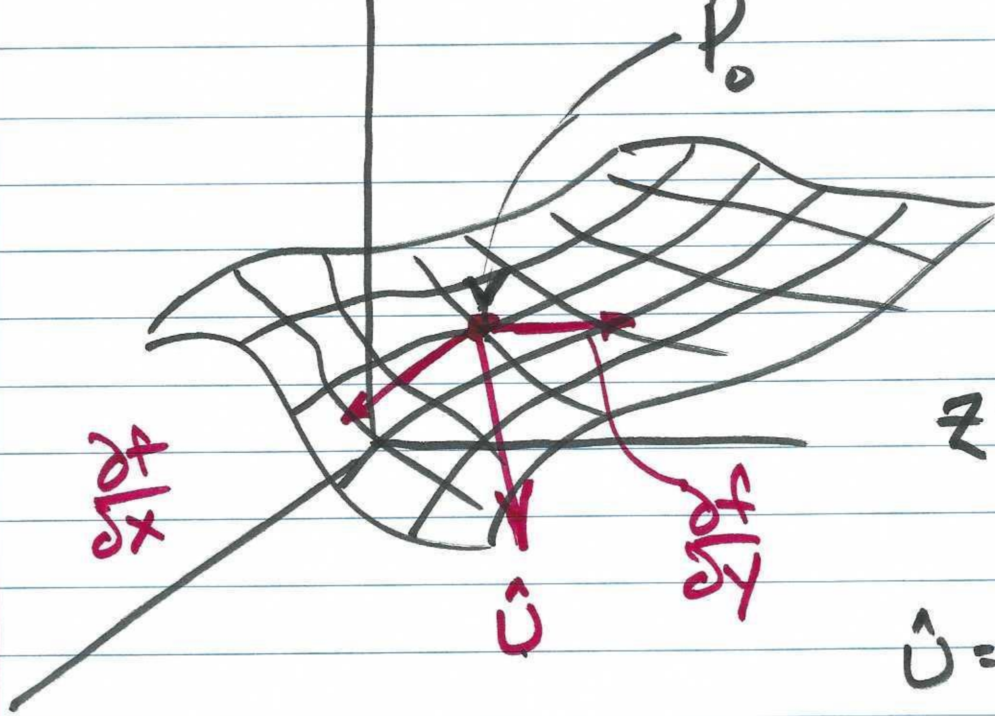
Problems for next exam:

1. Find gradient
2. Find tangent plane
3. Finding higher order partial derivs.
4. Chain Rule - find derivative
5. Chain Rule - draw dependency diag.



$z = f(x,y)$

$\hat{u} = u_1 \hat{i} + u_2 \hat{j}$

$$\left(\frac{df}{ds}\right)_{u,P_0} = \lim_{s \to 0} \left(\frac{f(x_0 + s u_1, y_0 + s u_2) - f(x_0, y_0)}{s} \right)$$

Define ∇f $\qquad$ ∇ is called "del" (nabla)

$$\nabla f = \hat{i} \frac{\partial}{\partial x} f + \hat{j} \frac{\partial}{\partial y} f + \hat{k} \frac{\partial}{\partial z} f$$

$$\nabla = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) \text{——}$$

vector ;
differential
operator

$D_u(f) = \nabla f \cdot \hat{u}$

#7 Finding ∇f if $f(x,y,z) = x^2 + y^2 - 2z^2 + z\ln x$

Eval @ $(1,1,1)$

$$\nabla f = \hat{i}\left(2x + \frac{z}{x}\right) + \hat{j}\left(2y\right) + \hat{k}\left(-4z + \ln x\right)$$

$$\nabla f_{(1,1,1)} = \hat{i}(3) + \hat{j}(2) + \hat{k}(-4)$$

$$= \underline{\langle 3, 2, -4 \rangle}$$

#9 $f(x,y,z) = (x^2+y^2+z^2)^{-1/2} + \ln(xyz)$

Eval @ $(-1, 2, -2)$

$$\frac{\cancel{yz}}{x\cancel{yz}}$$

$$\frac{\partial f}{\partial x} = -\frac{1}{2}(x^2+y^2+z^2)^{-3/2}(2x) + \frac{\cancel{yz}}{x}$$

$$= -\frac{x}{(x^2+y^2+z^2)^{3/2}} + \frac{1}{x}$$

③

$$\frac{\partial f}{\partial y} = -\frac{y}{(x^2+y^2+z^2)^{3/2}} + \frac{1}{y}$$

$$\frac{\partial f}{\partial z} = -\frac{z}{(x^2+y^2+z^2)^{3/2}} + \frac{1}{z}$$

$$\nabla f = \frac{-1}{(x^2+y^2+z^2)^{3/2}}\left[x\hat{i}+y\hat{j}+z\hat{k}\right] + \left[\hat{i}\frac{1}{x}+\hat{j}\frac{1}{y}+\hat{k}\frac{1}{z}\right]$$

$f(x,y) = 2xy - 3y^3 \qquad P_0 = (5,5)$

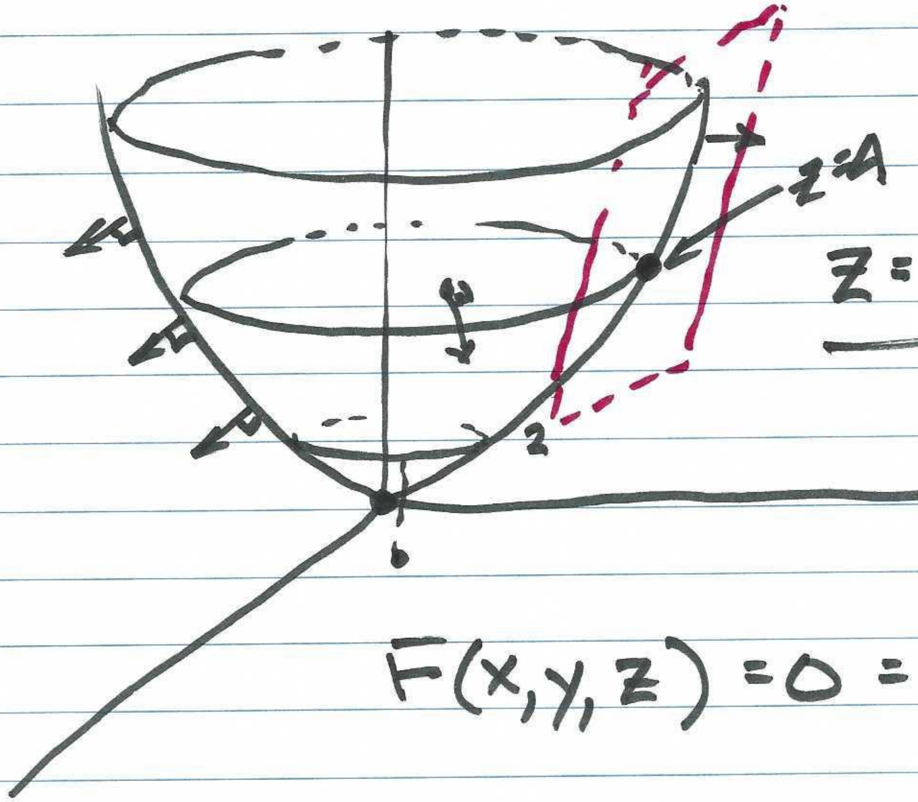
$u = 4\hat{i} + 3\hat{j}$

$$\boxed{\hat{u} = \frac{4}{5}\hat{i} + \frac{3}{5}\hat{j}}$$

$$\nabla f = \hat{i}(2y) + \hat{j}(2x - 9y^2)$$

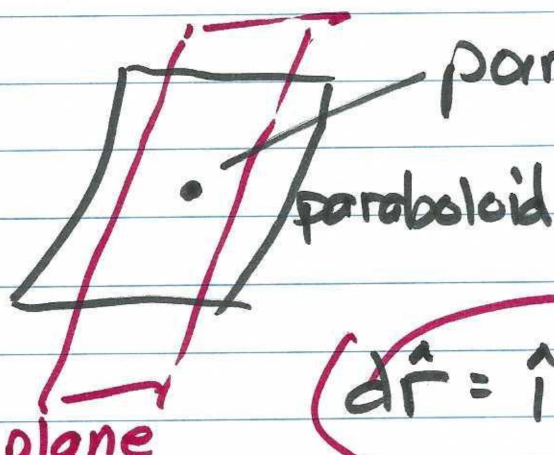
$$\nabla f_{(5,5)} = \boxed{\hat{i}(10) + \hat{j}(-215)}$$

$$D_u f = \nabla f \cdot \hat{u} = 8 - 129 = -121$$



$$z = x^2 + y^2$$

$$F(x,y,z) = 0 = x^2 + y^2 - z$$



$$d\hat{r} = \hat{i}\,dx + \hat{j}\,dy + \hat{k}\,dz$$

$$0 = dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz$$

$$\nabla F = \hat{i}\frac{\partial F}{\partial x} + \hat{j}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z}$$

dot these and get

So... $\nabla F \cdot \underline{d\hat{r}} = 0$ to stay inside the skin of paraboloid.

★Conclusion: Gradient of function defining surface is <u>always</u> perpendicular to surface.

Point paraboloid $(\sqrt{2}, \sqrt{2}, 4)$ is on parabolid

$F(x,y,z) = x^2+y^2-z$

$\nabla F = \hat{\imath}\, 2x + \hat{\jmath}\, 2y - \hat{k}$

plane whose direction vector is ∇F

$$2\sqrt{2}\,x + 2\sqrt{2}\cdot y - z = d$$

$$(2\sqrt{2})(\sqrt{2}) + 2\sqrt{2}(\sqrt{2}) - 4 = d.$$

$$4 + 4 - 4 = d \Rightarrow d = 4$$

So tangent plane (to parabolid $x^2+y^2=z$) has eqn. @ $z=4$ $\boxed{2\sqrt{2}x + 2\sqrt{2}y - z = 4}$

6

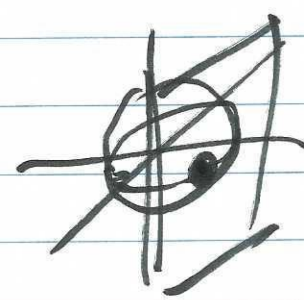
Sphere of radius 1

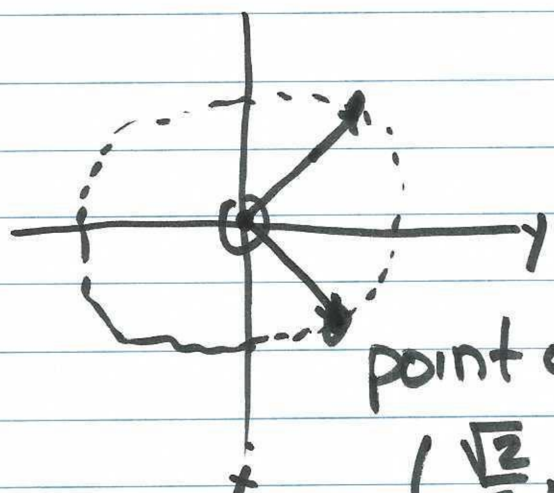
$$x^2+y^2+z^2=1$$

$$F(x,y,z) = x^2+y^2+z^2-1 = 0$$

$$\nabla F = \hat{i}\,2x + \hat{j}\,2y + \hat{k}\,2z$$

$$= 2(x\hat{i}+y\hat{j}+z\hat{k})$$





point on sphere is

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0\right)$

$$\cancel{\frac{\sqrt{2}}{2}x+\frac{\sqrt{2}}{2}y+0z=d}$$

$$\cancel{2}\frac{\sqrt{2}}{\cancel{2}}x+\cancel{2}\frac{\sqrt{2}}{\cancel{2}}y+0=d$$

$$\sqrt{2}x+\sqrt{2}y=d$$

$$1 \quad + 1 \quad = d = 2$$

$\sqrt{2}\,x + \sqrt{2}\,y = 2$ is tangent plane

Sphere pt on sphere is $(1,1,1)$

$R = \sqrt{3}$

Std. Linear Approximation

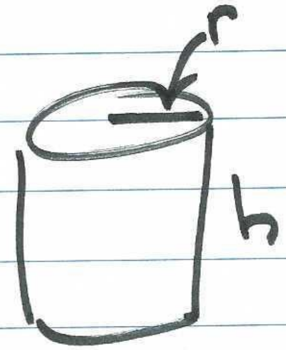
$$f'(x) = \cancel{\lim_{\Delta x \to 0}} \left(\frac{\overbrace{f(x+\Delta x) - f(x)}^{\Delta f}}{\Delta x} \right)$$

$$f'(x) \approx \frac{\Delta f}{\Delta x} \Rightarrow \underbrace{\Delta f = f'(x)\,\Delta x}_{1^{st}\text{ order linear approx}}$$

in 2 & 3 dimensions :

$$\Delta f \approx \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z \ldots$$

Assume $\Delta x, \Delta y, \Delta z \underline{\ll 1}$



$$V = \pi r^2 h$$

$r = 2$ in. $\quad \Delta r = .01$ in

$h = 5$ in. $\quad \Delta h = -.02$ in

$$\Delta V = \frac{\partial V}{\partial r}\Delta r + \frac{\partial V}{\partial h}\Delta h$$

$$\Delta V = 2\pi r h\,\Delta r + \pi r^2 \Delta h$$

$$= 2\pi(2)(5)(.01) + \pi(4)(-.02)$$

In limit we get

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

10
p. 860

$$z = ye^x - ze^{y^2}$$

Find tangent plane @ $(0, 0, 1)$

$$F(x,y,z) = ye^x - ze^{y^2} - z = 0$$

$$\nabla F = \hat{i}\, ye^x + \hat{j}(e^x + 2yze^{y^2}) + \hat{k}(e^{y^2} - 1)$$

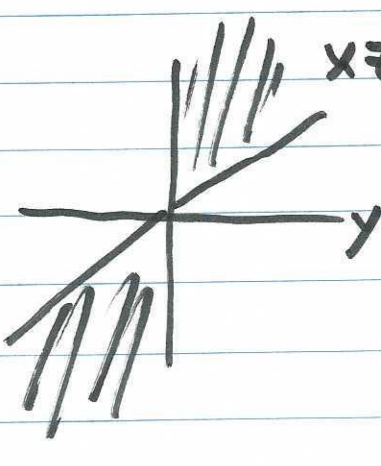
$$\nabla F_{(0,0,1)} = \hat{i}(0) + \hat{j}(1) + \hat{k}(0)$$

$$0x + 1y + 0z = d$$

$$y = d \qquad d = 0$$

$$\underline{y = 0}$$

xz plane



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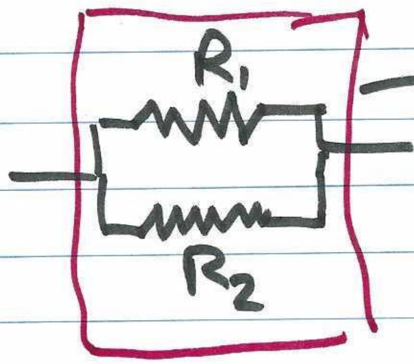
p. 862 # 52

Given resistances R_1, R_2



→ R total

$R(R_1, R_2) =$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} \Rightarrow R = \boxed{\frac{R_1 R_2}{R_1 + R_2}}$$

$$\Delta R = \frac{\partial R}{\partial R_1}\Delta R_1 + \frac{\partial R}{\partial R_2}\Delta R_2$$

$$\frac{\partial R}{\partial R_1} = \frac{(R_1+R_2)R_2 - R_1 R_2 (1)}{(R_1+R_2)^2} = \frac{R_2^2}{(R_1+R_2)^2}$$

$$\frac{\partial R}{\partial R_2} = \frac{R_1^2}{(R_1+R_2)^2}$$

$$\Delta R \approx \frac{1}{(R_1+R_2)^2}\left[R_2^2 \Delta R_1 + R_1^2 \Delta R_2\right]$$

cost of ordering

$$Q = \sqrt{\frac{2KM}{h}}$$ ← items sold/wk

weekly holding cost

quantity

To which variable is Q most sensitive

@ point $(K_0, M_0, h_0) = (2, 20, 0.05)$

$$Q = \left(\frac{2kM}{h}\right)^{1/2}$$

$$\frac{\partial Q}{\partial K} = \frac{1}{2}\left(\frac{2kM}{h}\right)^{-1/2} \cdot \left(\frac{2M}{h}\right)$$

$$= \frac{M}{h} \cdot \frac{\sqrt{h}}{\sqrt{2km}} = \frac{\sqrt{M}}{\sqrt{h}\sqrt{2k}} = \sqrt{\frac{M}{2kh}}$$

$$\frac{\partial Q}{\partial M} = \sqrt{\frac{K}{2Mh}}$$

$$\frac{\partial Q}{\partial h} = -\frac{1}{\cancel{2}}\left(\frac{2kM}{h}\right)^{-1/2} \cdot \cancel{2}km\left(-\frac{1}{h^2}\right)$$

(12)

$$\frac{\partial Q}{\partial h} = \sqrt{\frac{h}{2KM}} \cdot \left(-\frac{KM}{h^2}\right) = \sqrt{\frac{KM}{2h^3}}$$

Summary:

$$Q_M = \sqrt{\frac{K}{2Mh}}$$

$$Q_h = \sqrt{+\frac{KM}{2h^3}}$$

$$Q_K = \sqrt{\frac{M}{2Kh}}$$

Linearize:

$$f(x,y,z) = \frac{\partial f}{\partial x}(x-x_0) + \frac{\partial f}{\partial y}(y-y_0) +$$

$$\frac{\partial f}{\partial z}(z-z_0)$$

Ex. $f(x,y) = e^x \cos y \; ; \; (0,0)$

$$L(f) = (e^x\cos y)(x-0) - (e^x \sin y)(y-0)$$
$$= xe^x\cos y - ye^x \sin y$$