Recall that a series of terms

$a_0 + a_0 r + a_0 r^2 + \cdots a_0 r^k \cdots \to \infty$

If $|r| < 1$

Sum is $\dfrac{a_0}{1-r}$

(98) $1 + e^b + e^{2b} + \cdots e^{kb} + \cdots = 9$

$a_0 = 1 \quad r = e^b$

Sum $= \dfrac{1}{1-e^b} = 9$

$1 = 9(1-e^b) \Rightarrow \frac{1}{9} = 1 - e^b$

$e^b = 8/9$ take logs :

$b = \ln(8/9) = \underline{\ln 8 - \ln 9}$

side of outer square is 2

side of 1^{st} inside square is:

$\sqrt{2}$

side of 2^{nd} inside square is:

$$\sqrt{\left(\frac{\sqrt{2}}{2}\right)^2+\left(\frac{\sqrt{2}}{2}\right)^2} = 1$$

→ So far $2, \sqrt{2}, 1, \frac{\sqrt{2}}{2}$

side of third inside square:

$$\sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{1}{2}\right)^2} = \frac{\sqrt{2}}{2}$$

$4, 2, 1, \frac{1}{2}, \ldots$

This is geometric $a_0 = 4$, $r = \frac{1}{2}$

Sum of all terms (areas) is $\frac{4}{1-\frac{1}{2}} = 8$

3

Stock pays \$2 dividend end of each year.

If money is valued @ 5% per year.

$$2 + 2\left(\frac{1}{1.05}\right) + 2\left(\frac{1}{1.05}\right)^2 + 2\left(\frac{1}{1.05}\right)^3 + \ldots$$

$$NPV = \frac{2}{1-.95} = \frac{2}{.05} = \$40$$

101

$$A(t) = 300e^{-0.12t} \qquad t \text{ in hours}$$

$$A(24) = 300e^{-(0.12)(24)} = 16.8 \text{ mg}$$

% reduction day-to-day is $\frac{16.8}{300} = 5.6\%$

Day 180

$$300\left(1 + .056 + (.056)^2 + (.056)^3 + \ldots\right.$$

$$300\left(\frac{1}{1-.056}\right) = \frac{300}{.944} \qquad (1.059)(300)$$

317.8 mg

(4)

$\rightarrow 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \cdots \longrightarrow \infty$

$1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{3}\right)^2 + \cdots \longrightarrow \frac{\pi^2}{6}$

Integral Test:

$f(x) = \frac{1}{x} \quad n \rightarrow f(n) = \frac{1}{n} = a_n$

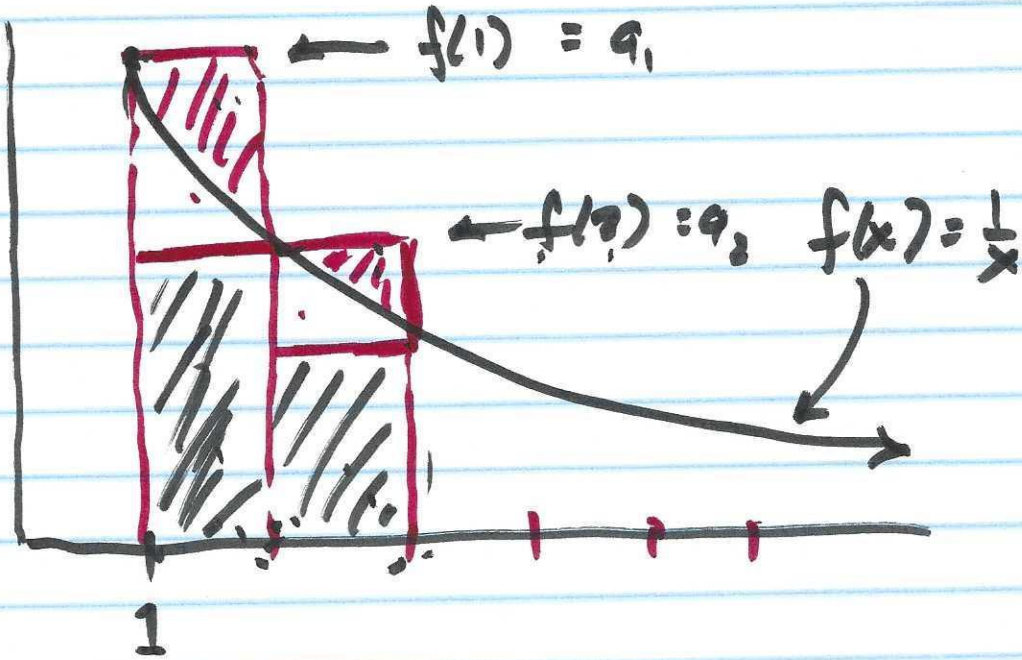
$\frac{1}{x}$ decreases as $x \rightarrow \infty$

$f'(x) < 0 \quad \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$

$\sum_{n=1}^{\infty} \frac{1}{n} = \int_1^{\infty} \frac{1}{x}\,dx = \left[\ln x\right]_1^{\infty} = \underset{\approx}{\infty}$

5

$$\underbrace{1}_{>\frac{1}{2}} + \underbrace{\frac{1}{2}}_{=\frac{1}{2}} + \underbrace{\frac{1}{3} + \frac{1}{4}}_{>\frac{1}{2}} + \underbrace{\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}}_{>\frac{1}{2}} + \underbrace{\frac{1}{9} \cdots}_{>\frac{1}{2}}$$



Does $a_n = \frac{1}{n^p}$ converge as series if $p > 1$

If $p = 2$ $\quad \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots \frac{1}{n^2} + \cdots$

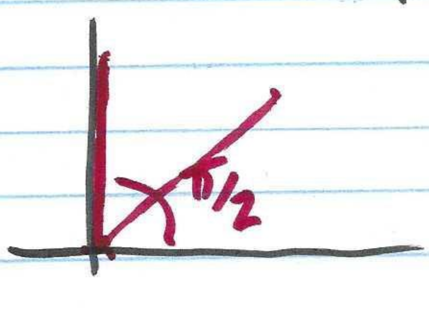
Look @ $\displaystyle\int_1^\infty \frac{dx}{x^2}$ — has same convergence

properties as $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$

$$\int_1^\infty x^{-2}\,dx = \left[-\frac{1}{x}\right]_1^\infty = 0 - \frac{1}{1} = 1$$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ con/div ?

$f(x) = \dfrac{1}{x^2+1}$ $\displaystyle\int_1^\infty \frac{dx}{x^2+1} \stackrel{?}{=} \left[\arctan x\right]_1^\infty$



$\pi/2 - \dfrac{\pi}{4} = \dfrac{\pi}{4}$

Check:

1) $\sum_{n=1}^{\infty} ne^{-n^2} \rightsquigarrow \int_1^{\infty} xe^{-x^2}dx$

Let $u = x^2$ then $du = 2x\,dx$

$$\int_{u=1}^{\infty} e^{-u}\frac{du}{2} = \frac{1}{2}\frac{e^{-u}}{(-1)} = \left[-\frac{1}{2}e^{-u}\right]_1^{\infty}$$

$$0 + \frac{1}{2}\frac{1}{e} = \frac{1}{2e}$$

2) $\sum_{n=1}^{\infty} \frac{1}{2^{\ln n}} \stackrel{?}{=} \int_1^{\infty} \frac{dx}{2^{\ln x}}$

Let $u = \ln x$

$du = \frac{dx}{x}$

$$\int_0^{\infty} \frac{x\,du}{2^u} = \int_0^{\infty} \frac{e^u}{2^u}du$$

e^u

$$= \int_0^\infty \left(\frac{e}{2}\right)^u du = 2\left[\frac{e}{2}\right]_0^\infty = \infty - 1 = \infty$$

NO

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① $$\sum_{n=1}^{\infty} \frac{1}{n^2} \longrightarrow \int_1^\infty \frac{dx}{x^2} < \infty$$

③ $$\sum_{n=1}^{\infty} \frac{1}{n^2+4} \longrightarrow \int_1^\infty \frac{dx}{x^2+4}$$

$$\int_1^\infty \frac{dx}{4\left[\left(\frac{x}{2}\right)^2+1\right]} = \frac{1}{4}\int_1^\infty \frac{dx}{\left(\frac{x}{2}\right)^2+1}$$

Let $u = \frac{x}{2}$ so $du = \frac{1}{2}dx$

$$\frac{1}{4}\int_{x=1}^{\infty} \frac{2\,du}{u^2+1} = \frac{1}{2}\int_{x=1}^{\infty} \frac{du}{u^2+1} = \frac{1}{2}\arctan u \Big|_{x=1,\ u=1/2}^{u,x=\infty}$$

$$\underline{\frac{\pi}{2} - \frac{1}{2}\arctan\frac{1}{2}}$$

8 $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(n^2)}{n} = 2\sum_{n=2}^{\infty} \frac{\ln n}{n}$?

$\displaystyle 2\int_2^{\infty} \frac{\ln x}{x}\,dx$ Let $u = \ln x$

$du = \dfrac{dx}{x}$

$\displaystyle 2\int_{\ln 2}^{\infty} u\,du$ $\cancel{2} \cdot \left[\dfrac{u^2}{\cancel{2}}\right]_{\ln 2}^{\infty}$

$\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}} \longrightarrow \int_2^{\infty} \frac{\ln x\,dx}{\sqrt{x}}$

Note: $\dfrac{\ln x}{\sqrt{x}} \geq \dfrac{\ln x}{x}$

Look @ $\displaystyle\int_2^{\infty} \frac{\ln x\,dx}{x} = \left[\ln x\right]_2^{\infty} = \infty$ NO

63 $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\cdots\frac{1}{n} - \ln n = a_n$

Can show this converges to

$\gamma = 0.5772156649015 33 \ldots$

$\ln n = \int_1^n \frac{dx}{x}$

Also know $\ln(n+1) \leq 1 + \ln n$

$0 < \ln(n+1) - \ln n \leq 1 + \frac{1}{2} + \cdots \frac{1}{n} - \ln n \leq 1$

So sequence above $\sum a_n$ is bounded above