① $\int e^x \cos x \, dx$

$\frac{d}{dx}$		$\int dx$
$\cos x$	$+$	e^x
$-\sin x$	$-$	e^x
$-\cos x$	$+$	e^x
$\sin x$		e^x

$$\int e^x \cos x \, dx = e^x \cos x + e^x \sin x - \int \cos x \cdot e^x \, dx$$

$$2 \int e^x \cos x \, dx = e^x (\cos x + \sin x)$$

$$\int e^x \cos x \, dx = \frac{1}{2} e^x (\cos x + \sin x)$$

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③ $\int t^2 \cos t \, dt$

2

d/dx		∫dx
t^2		$\cos t$
	+	
$2t$		$\sin t$
	−	
2		$-\cos t$
	+	
0		$-\sin t$

$$\int t^2 \cos t\,dt = t^2 \sin t + 2t\cos t - 2\sin t$$

6 $\displaystyle\int_1^e x^3 \ln x\,dx$ dv

$$\boxed{\int u\,dv = uv - \int v\,du}$$

$$= \left(\frac{x^4}{4}\cdot \ln x\right) - \int \left(\frac{x^4}{4}\right)\frac{dx}{x}$$

$$= \frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3\,dx$$

$$= \left.\boxed{\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C}\,\right|_1^e$$

17 $\int (x^2 - 5x) e^x \, dx$

d/dx		$\int dx$
$x^2 - 5x$		e^x
	$+$	
$2x - 5$		e^x
	$-$	
2		e^x
	$+$	
0		e^x

$$= \boxed{e^x(x^2 - 5x) - e^x(2x - 5) + 2e^x + C}$$

13 $\int x \sec^2 x \, dx = ?$

$$= x \tan x - \int \tan x \, dx$$

$$= x \tan x - \int \frac{\sin x}{\cos x} dx$$

$$+ \int \frac{d(\cos x)}{\cos x} \rightarrow \ln \cos x$$

$$\boxed{\int x \sec^2 x \, dx = x \tan x + \ln \cos x + C}$$

31 $\int x \sec x^2\,dx \xrightarrow[du=2x\,dx]{u=x^2} \int \underbrace{x \sec u \frac{du}{2x}}_{\text{cancels}} = $

$\frac{1}{2}\int \sec u\,du = \frac{1}{2}\ln|\sec u+\tan u|$

$\boxed{= \frac{1}{2}\ln|\sec x^2+\tan x^2| + C}$

33 $\int x(\ln x)^2\,dx \xrightarrow[du=\frac{dx}{x}]{u=\ln x} \int \cancel{x}\, u^2 \frac{du}{\cancel{x}} =$

$= \frac{u^3}{3} \rightarrow \boxed{\frac{(\ln x)^3}{3} + C}$

35 $\int \frac{\ln x}{x^2}\,dx \xrightarrow[du=\frac{dx}{x}]{u=\ln x} \int \frac{u}{x^2}\frac{du}{x}$

implies $x = e^u$

$\int u e^{-3u}\,du$

$= u\frac{e^{-3u}}{(-3)} - \int \frac{e^{-3u}}{-3}\,du$

$$= -\frac{1}{3}ue^{-3u} + \frac{1}{3}\left(-\frac{1}{3}e^{-3u}\right)$$

$$\boxed{= -\frac{1}{3}e^{-3u}\left[u + \frac{1}{3}\right] + C}$$

$$= -\frac{1}{3}\left(\frac{1}{x^3}\right)\left[\ln x + \frac{1}{3}\right] + C$$

41 $\int \sin 3x \cos 2x \, dx$

d/dx	$\int dx$
$\sin 3x$	$\cos 2x$
$3\cos 3x$	$\frac{1}{2}\sin 2x$
$-9\sin 3x$	$-\frac{1}{4}\cos 2x$

$+$ $-$ $+$

$$\underline{\int \sin 3x \cos 2x \, dx} = -2\sin 3x \sin 2x + 12\cos 3x \cos 2x + 36\underline{\int \sin 3x \cos 2x \, dx}$$

6

$$-35\int \sin 3x\cos 2x\,dx = -2\sin 3x\sin 2x + 12\cos 3x\cos 2x$$

$$\int \sin 3x\cos 2x\,dx = \frac{2}{35}\sin 3x\sin 2x - \frac{12}{35}\cos 3x\cos 2x$$

45 $$\int \cos\sqrt{x}\,dx \xrightarrow[du = \frac{dx}{\sqrt{x}}]{u=\sqrt{x}} = \int \cos u\cdot(u\,du)$$

Start with $$\int u\cos u\,du = +u\sin u - \int \sin u\,du$$

$$= u\sin u + \cos u$$

$$\boxed{\int \cos\sqrt{x}\,dx = \sqrt{x}\sin\sqrt{x} + \cos\sqrt{x} + C}$$

51 $$\int x\arctan x\,dx$$

$$= \frac{x^2}{2}\cdot\arctan x - \int\left(\frac{x^2}{2}\right)\left(\frac{1}{1+x^2}\right)dx$$

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$$\frac{x^2}{2}\arctan x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx$$

$$\begin{cases} u = 1+x^2 & du = 2x\,dx \\ x^2 = u-1 \end{cases}$$

$$\int \frac{u-1}{u}\frac{du}{2x} = \frac{1}{2}\int\frac{(u-1)\,du}{u\sqrt{u-1}} = ?$$

$$\frac{1}{2}\int \frac{\sqrt{u-1}}{u}\,du$$

$$\int \frac{\cancel{u}\,du}{\cancel{u}\sqrt{u-1}} - \int\frac{du}{u\sqrt{u-1}}$$

$$\int \frac{2\cancel{x}\,dx}{\cancel{x}} - \int \frac{2\cancel{x}\,dx}{\cancel{x}(1+x^2)}$$

$$\int 2\,dx - \int \frac{2\,dx}{1+x^2} = 2x - 2\arctan x$$

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$$\int x \arctan x\,dx = \frac{x^2}{2}\arctan x -$$

$$\frac{1}{2}\cdot\frac{\cancel{1}}{\cancel{2}}\left(\cancel{2}x - \cancel{2}\arctan x\right)$$

$$\int x \arctan x\,dx = \frac{1}{2}\left[x^2\arctan x + \arctan x - 2\right]$$

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57 $$\text{Area} = \int_0^{\pi}\left[x\sin x - 0\right]dx$$

$$\int x\sin x\,dx = -x\cos x - \int(-\cos x)\,dx$$

$$= \left[-x\cos x + \sin x\right]_0^{\pi} =$$

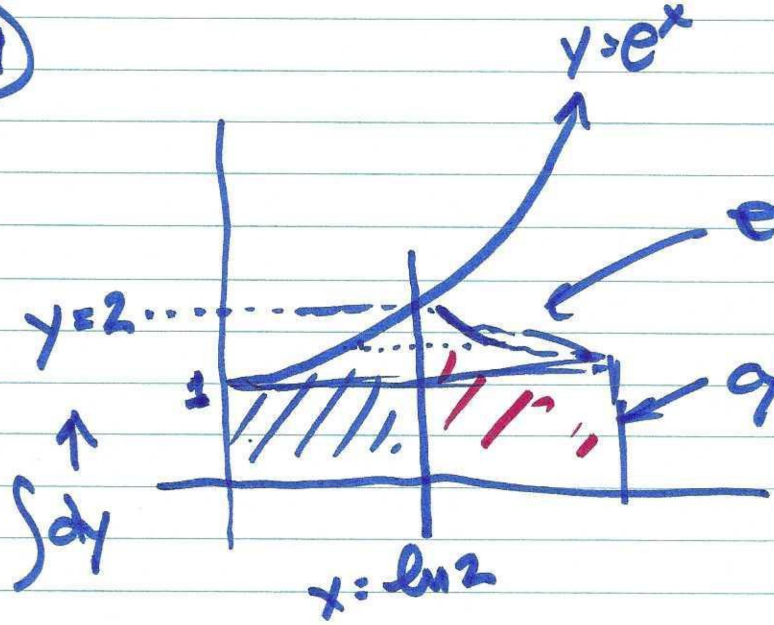
$$(\pi) - (0) = \pi$$

$$\left[-x\cos x + \sin x\right]_{n\pi}^{(n+1)\pi}$$

n even $\quad -(n+1)\pi\cos\left(\underline{(n+1)\pi}\right) + \sin(n+1)\pi$

$(n+1)\pi - 0 \;=\; (n+1)\pi$

59



exponential

cylinder

volume is

$\underline{\pi(\ln 2)^2 \cdot 1}$

$$\int_1^2 \left(\quad \right) dy$$

radius of disk

@ height y is

$\pi(e^x - \ln 2)^2\, dy$

$\pi\left(e^{2x} - 2\ln 2\, e^x + (\ln 2)^2\right) dy$

$\pi\left(y^2 - 2y\ln 2 + (\ln 2)^2\right)$

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$$\text{Vol} = \pi(\ln 2)^2 + \pi\int_1^2 y^2 - 2y\ln 2 + (\ln 2)^2\,dy$$

$$\pi\left[\frac{y^3}{3} - \frac{y^2\ln 2}{2} + (\ln 2)^2 y\right]_1^2$$

$$\text{So Vol} = \pi(\ln 2)^2 + \pi\left(\frac{8}{3} - 2\ln 2 \cdot 2(\ln 2)^2\right) -$$

$$\left(\pi\left(\frac{1}{3} - \frac{1}{2}\ln 2 + (\ln 2)^2\right)\right.$$

66 $y = 4e^{-t}(\sin t - \cos t) \quad t \geq 0$

$$\bar{y} = \frac{1}{2\pi}\int_0^{2\pi} \quad dt$$

$$\bar{y} = \frac{4}{2\pi}\int_0^{2\pi} \underline{e^{-t}(\sin t - \cos t)}\,dt$$

$$\int e^{-t} \sin t \, dt$$

d/dt		$\int dt$
$\sin t$	$+$	e^{-t}
$\cos t$	$-$	$-e^{-t}$
$-\sin t$	$+$	e^{-t}

$$-e^{-t}\sin t - e^{-t}\cos t + \int -\sin t \, e^{-t}\,dt$$

$$\int e^{-t}\sin t\,dt = -e^{-t}\sin t - e^{-t}\cos t - \int e^{-t}\sin t\,dt$$

$$2\int e^{-t}\sin t\,dt = -e^{-t}\left[\sin t + \cos t\right]$$

$$\star \quad \int e^{-t}\sin t\,dt = -\frac{1}{2}e^{-t}\left[\sin t + \cos t\right]$$

$-\int e^{-t}\cos t\,dt$

d/dt		∫dt
$\cos t$	$+$	e^{-t}
$-\sin t$	$-$	$-e^{-t}$
$-\cos t$	$+$	e^{-t}

$$-e^{-t}\cos t + e^{-t}\sin t - \int \cos t \cdot e^{-t}\,dt$$

$$\int e^{-t}\cos t\,dt = e^{-t}\left[\sin t - \cos t\right] - \int e^{-t}\cos t\,dt$$

$$2\int e^{-t}\cos t\,dt = e^{-t}\left[\sin t - \cos t\right]$$

★★ $$\int e^{-t}\cos t\,dt = \frac{1}{2}e^{-t}\left[\sin t - \cos t\right]$$

$$\bar{y} = \frac{2}{\pi}\left[-\frac{e^{-t}}{2}\left[\sin t + \cancel{\cos t}\right] - \frac{e^{-t}}{2}\left[\sin t - \cancel{\cos t}\right]\right]_0^{2\pi}$$

$$\bar{y} = \frac{2}{\pi}\left(-e^{-t}\sin t\right)\Big|_0^{2\pi} = \underline{0}$$