SPRING 2025 - CALCULUS 2 - TEST #2B - Solutions

1) You have \$1000 invested at 6% per annum continuous interest starting in year 1. At the end of year 3 you withdraw half the money in the account and leave the remainder on deposit. At the beginning of year 4, you get a better rate of 7.5% per annum. How much would you have on deposit at the end of year 10?

Use the Pert formula to find future values. After three years at 6% the \$1000 grows to \$1197.22. Removing half of that leaves \$598.61. Letting that grow for seven years at 7.5% gives \$1011.93.

2) A cup of soup cooled from 90^o C to 60^o C in ten minutes in a room whose temperature was 20^o C. How much longer will it be until the soup is at 35^o C?

Newton's Law of Cooling is $H(t) - H_S = (H_0 - H_S)e^{-kt}$*, where* $H(t)$ *is temperature of an object at time* t *after it has started at temperature* H_0 *to cool off or heat up in an ambient temperature of* H_S*. We need to find* k*, so we plug in our data.* $H(10) = 60$, $H_S = 20$, $H_0 = 90$. *So* $(60-20) = (90-20)e^{-10k}$*. Or* $40 = 70e^{-10k}$*. Solving for* k *we have* $\ln(0.5714) = -0.5596 = -k(10)$*, then* $k = 0.05596$*. We want the time* t *to cool from* 60 *to* 35 *with this cooling constant.* $(35-20) = (60-20)e^{-0.05596t}$*. This gives* $15 = 40e^{-0.05596t}$*, or* $\ln(0.375) = -0.981 = -0.05596t$*. We find* $t = 17.5$ *minutes longer.*

3) Evaluate: $\int_1^4 \frac{2^{\sqrt{x}}}{\sqrt{x}}dx$

Let $u = \sqrt{x}$*. Then* $du = \frac{dx}{2\sqrt{x}}$*. So* $\int_1^4 \frac{2^{\sqrt{x}}}{\sqrt{x}}dx = \int_1^2 2^u(2du) = \left[\frac{2 \cdot 2^u}{\ln 2}\right]_1^2 = \frac{2}{\ln 2}[2^2 - 2] = \frac{4}{\ln 2}$.

4) Plutonium-239 has a half-life of $24{,}360$ days. If 25 [g] of plutonium is released into the atmoshere by a nuclear accident, how many years will it take for 65% of it to decay?

Convert to years: 24360 days = 66.7 years. From the half-life info, $\frac{1}{2} = e^{-\lambda(66.7)}$*, where* λ *is the (positive) decay constant. Solving the exponential equation we get* $-.69315 = -66.7\lambda$*, or* $\lambda = 0.0104$*. Then putting that back into the decay equation, we get* $Q(t) = Q(0)e^{-0.0104t}$*. We want* t *so that* $\frac{Q(t)}{Q(0)} = 0.35$*, so* $0.35 = e^{-0.0104t}$*. Solving this exponential equation we get* $\ln(0.35) = -1.049 = -0.0104t$*, so* $t = 100.9$ *years for 65% to decay or 35% to remain.*

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Integration by Parts

$u(x), v(x)$

$$\frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$$

$$\int \quad " \quad = \quad " \quad + \quad "$$

$$\underset{\text{no-brainer}}{uv} = \underset{\text{easy}}{\int u\,dv} + \underset{\text{difficult}}{\int v\,du}$$

$$\star \quad \boxed{\int v\,du = uv - \int u\,dv}$$

Ex: $\int x \underbrace{\cos x\,dx}_{du} = x\sin x - \int \sin x\,(1)\,dx$

$$= x\sin x - \int \sin x\,dx$$

$$= x\sin x + \cos x + C$$

2

Ex: $\int \ln x\,dx = \int \underbrace{1 \cdot \overset{v}{\ln x}\,dx}_{du} =$

$$x \ln x - \int x \underbrace{\left(\frac{1}{x}\right)}_{\frac{d(\ln x)}{dx}} dx =$$

$$\int \ln x\,dx = x \ln x - x + C$$

$$= x(\ln x - 1) + C$$

Ex: $\int x^2 e^x\,dx = x^2 e^x - \int 2x e^x\,dx$

$u = x^2$

$v = e^x$

$du = 2x\,dx$

$dv = e^x\,dx$

1st pass

$$\int x^2e^x\,dx = x^2e^x - 2\underbrace{\int xe^x\,dx}$$

$$\int xe^x\,dx = xe^x - \int (1)e^x\,dx$$

$$= xe^x - e^x$$

$$\int x^2e^x\,dx = x^2e^x - 2\left[xe^x - e^x\right] + C$$

$$= e^x\left[x^2 - 2x + 1\right] + C$$

Ex. $$\int e^x\cos x\,dx = e^x\sin x - \underbrace{\int \sin x\cdot e^x\,dx}$$

$$\left(-\cos x\cdot e^x + \int (+\cos x)e^x\,dx\right)$$

$$\int e^x\cos x\,dx = e^x\sin x - \left[-\cos x\cdot e^x + \int e^x\cos x\,dx\right.$$

$$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x$$

$$\int e^x \cos x\,dx = \frac{e^x}{2}(\sin x + \cos x)$$

$$\int x^4 e^x\,dx = ?$$

d/dx		$\int dx$
x^4	$+$	e^x
$4x^3$	$-$	e^x
$12x^2$	$+$	e^x
$24x$	$-$	e^x
24	$+$	e^x
0	$-$	e^x

"Tabulor Integration"

$$x^4e^x - 4x^3e^x + 12x^2e^x - 24xe^x + 24e^x$$

⑤

$$\int x^3 \cos x \, dx = ?$$

d/dx		$\int dx$
x^3		$\cos x$
	$+$	
$3x^2$		$\sin x$
	$-$	
$6x$		$-\cos x$
	$+$	
6		$-\sin x$
	$-$	
0		$\cos x$
	$+$	

$$\int x^3 \cos x \, dx = x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6\cos x$$

⑯ $\int p^4 e^{-p} \, dp$

$\frac{d}{dp}$		$\int dp$
p^4	$+$	e^{-p}
$4p^3$		$-e^{-p}$
	$-$	
$12p^2$	$+$	e^{-p}
$24p$	$-$	$-e^{-p}$
24	$+$	e^{-p}
0		$-e^{-p}$
	$-$	

⑥

$$\int p^4 e^{-p}\,dp = -e^{-p}\cdot p^4 - e^{-p}\cdot 4p^3 - e^{-p}\cdot 12p^2$$

$$- e^{-p}\cdot 24p - e^{-p}(24)$$

$$= -e^{-p}\left(p^4 + 4p^3 + 12p^2 + 24p + 24\right)$$