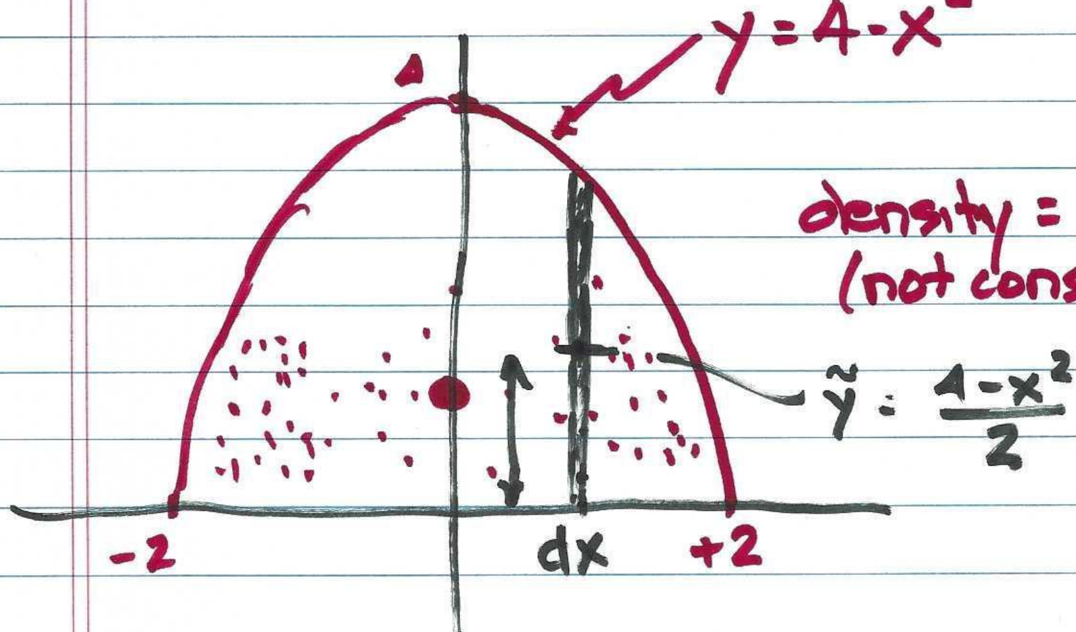
6.6 - Finish moments / COM / centroids & Theorems of Pappus



Where is COM? $\bar{x} = \underline{0}$

Slice @ x has:

width dx
height $4-x^2$
$dA = (4-x^2)\,dx$
$dm = (2x^2)(4-x^2)\,dx$

distance to x-axis $\tilde{y} = \frac{4-x^2}{2}$

②

$$\text{So. } M_x = \int_{-2}^{+2} \tilde{y}\, dm = \int_{-2}^{+2} \left(\frac{4-x^2}{\cancel{2}}\right)(\cancel{2}x^2)(4-x^2)\,dx$$

$$M_x = \int_{-2}^{2} (4-x^2)^2 \cdot x^2 dx = \int_{-2}^{2} (16-8x^2+x^4)x^2 dx$$

$$= \int_{-2}^{2} (16x^2 - 8x^4 + x^6)\,dx$$

$$= \left[16\frac{x^3}{3} - \frac{8x^5}{5} + \frac{x^7}{7}\right]_{-2}^{2}$$

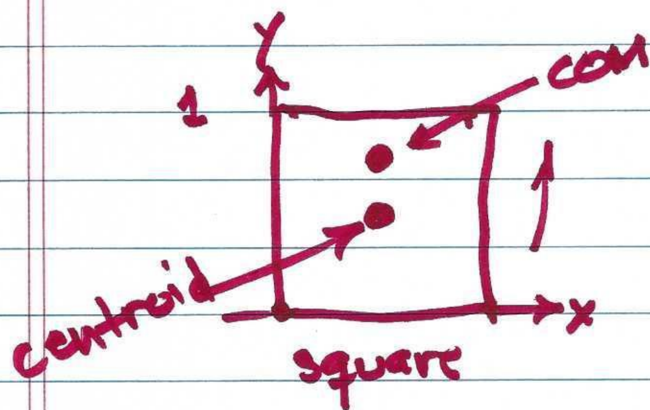
$$= \frac{2048}{105} = M_x$$

$$\bar{y} = \frac{M_x}{M} = \frac{2048}{105} \cdot \frac{15}{256} = \frac{8}{7}$$

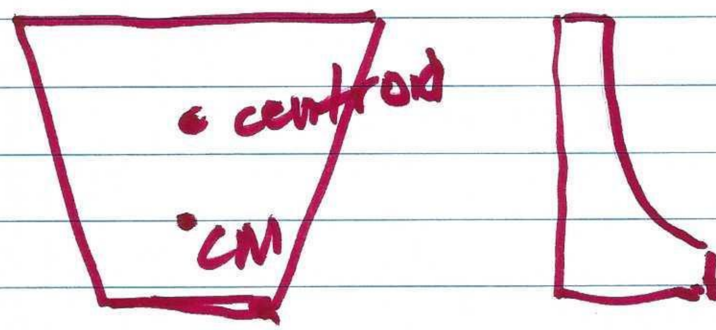
$$M = \int_{-2}^{2} (8x^2 - 2x^4)\,dx = \left[\frac{8}{3}x^3 - \frac{2}{5}x^5\right]_{-2}^{2}$$

$$= \frac{256}{15}$$

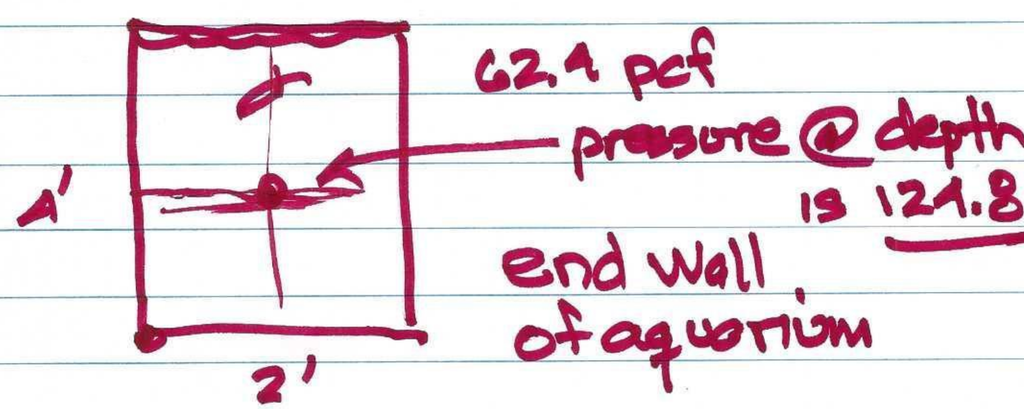
If you set $\delta = 1$ the COM becomes the "centroid" of the figure.



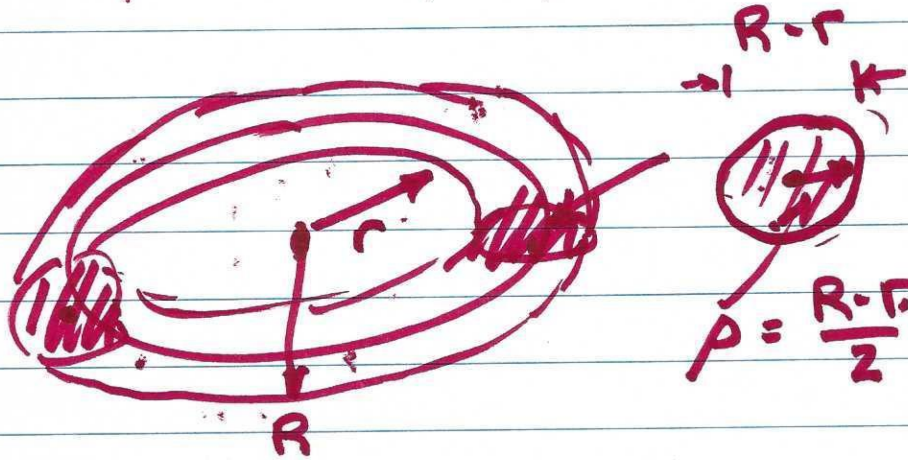
$\delta = y^5$



Total force = (Area of fig.)(pressure @ centroid)



So Total force $= \underline{8 \cdot 121.8}$ lbs



$\rightarrow T_c = 2\pi\left(\frac{r+R}{2}\right)$ = distance centroid travels

To get area of donut

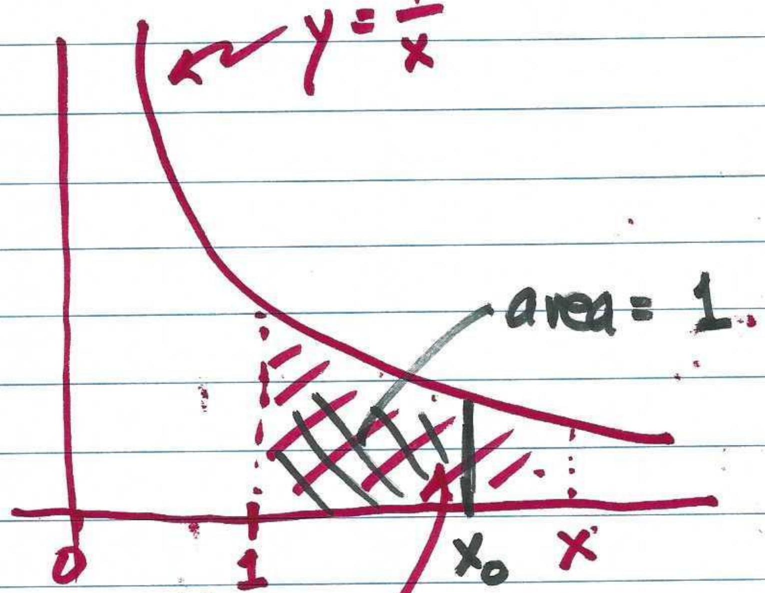
Circumference of slice is $2\pi\left(\frac{R-r}{2}\right)$

Area of donut is $\cancel{2}\pi\left(\frac{r+R}{\cancel{2}}\right)\cdot\cancel{2}\pi\left(\frac{R-r}{\cancel{2}}\right)$

$= \pi^2(R^2-r^2)$ ✓

Vol of donut is $\pi(r+R)\,\pi\left(\frac{R-r}{2}\right)^2$

$= \frac{\pi^2}{4}(R+r)(R-r)^2 = \frac{\pi^2}{4}(R^2-r^2)(R-r)$



area defines ln x

$$\int_1^x \frac{1}{t}\,dt = \Big[\ln t\Big]_1^x = \ln x - 0 = \ln x$$

$$\int_1^{x_0} \frac{1}{t}\,dt = \underline{\ln x_0 = 1}$$

e

Given a base b, we know:

(i) $b^x \cdot b^y = b^{x+y}$

(ii) $b^x / b^y = b^{x-y}$

(iii) $(b^x)^y = b^{xy} = b^{yx} = (b^y)^x$

(iv) $\sqrt[y]{b^x} = b^{x/y} = (b^{1/y})^x = \left[\sqrt[y]{b}\right]^x$

Rule: $b > 0$ & $b \neq 1$

$$\boxed{b^x = b^y \iff x = y}$$ Principle of exponential equations

$2^3 = 4^x$ what is x?

$2^{3} = (2^2)^x = 2^{2x}$

$\Rightarrow 2x = 3$ or $x = 3/2$

* Let $A = b^x$ & $B = b^y$

we say $\log_b A = x$ & $\log_b B = y$

(i) $\boxed{\log_b(A \cdot B) \stackrel{?}{=} x+y = \log_b A + \log_b B}$

$A \cdot B = b^x \cdot b^y = b^{x+y}$

(ii) $\boxed{\log_b\left(A/B\right) = x-y = \log_b A - \log_b B}$

$A/B = b^x / b^y = b^{x-y}$

(iii) $\boxed{\log_b A^c = cx = c \cdot \log_b A}$

$A^c = (b^x)^c = b^{cx}$

(iv) $\boxed{\log \sqrt[c]{A} = x/c = \frac{1}{c} \log_b A}$

$\sqrt[c]{A} = A^{1/c} = b^{x/c}$

$\ln 2 = .69315$ $\ln\left(\frac{1}{2}\right) = \underline{-.69315}$

1) $\frac{d}{dx}\log_b x = \frac{1}{\ln b}\cdot\frac{1}{x}$

$\frac{d}{dx}\ln x = \frac{1}{x}$

#2 $\int_{-1}^{0}\frac{3dx}{3x-2} \stackrel{?}{=} \int_{x=-1}^{0}\frac{du}{u} = [\ln u]_{x=-1}^{0}$

Let $u = 3x-2 \Rightarrow du = 3dx$

$\left(\ln[3x-2]\right)_{-1}^{0} = \ln|-2| - \ln|-5|$

$= \ln 2 - \ln 5 = \ln\frac{2}{5}$

#6 $\int \frac{\sec y \tan y}{2+\sec y}dy$ Recall: $\sec^2 y = 1+\tan^2 y$

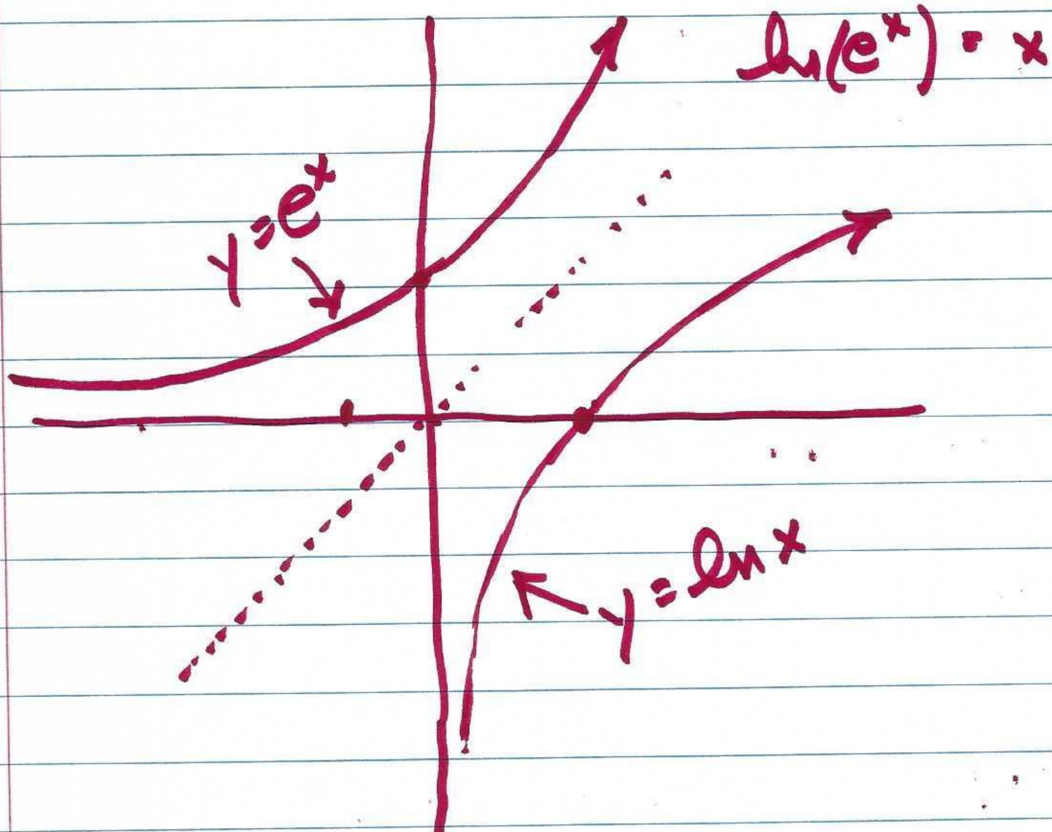
$d(\sec y) = \sec y \tan y\, dy$

Let $u = 2+\sec y$

$du = \sec y \tan y\, dy$

$$= \int \frac{du}{u} = \ln|u| + C$$

$$= \ln|2 + \sec y| + C$$



#12 $\displaystyle\int \frac{\ln(\ln x)\,dx}{x \ln x} \overset{?}{=}$

Try $u = \ln x \Rightarrow du = \frac{dx}{x}$

$= \displaystyle\int \frac{\ln u \, du}{u}$

Try $w = \ln u \Rightarrow dw = \frac{1}{u}du$

$= \displaystyle\int w\,dw = \frac{w^2}{2} + C \Rightarrow \frac{(\ln u)^2}{2} + C \Rightarrow$

$\Rightarrow \dfrac{[\ln(\ln x)]^2}{2} + C$

#19 $\displaystyle\int \frac{e^{1/x}\,dx}{x^2} \overset{?}{=}$ $\quad \cancel{u = \ln\left(\frac{1}{x}\right)}$

$v = \frac{1}{x} \quad dv = \frac{-1}{x^2}dx$

$\displaystyle\int e^v(-dv)$

$$= -\int e^{v}\,dv = -e^{v} + C \Rightarrow \underline{e^{-1/x} + C}$$

#21 $$\int e^{\sec \pi t} \sec \pi t \tan \pi t \, dt$$

$$u = \sec \pi t \quad du = \pi \sec \pi t \tan \pi t \, dt$$

$$\frac{1}{\pi}\int e^{u}\,du = \frac{e^{u}}{\pi} + C = \frac{e^{\sec \pi t}}{\pi} + C$$

#31 $$\int_0^{\pi/2} 7^{\cos t} \sin t \, dt \qquad 7 = e^{\ln 7}$$

$$\int_0^{\pi/2} e^{\ln 7 \cdot \cos t} \underline{\sin t \, dt} =$$

$$u = \ln 7 \cos t \quad du = \ln 7 \underline{(-\sin t) dt}$$

$$\int_0^{\pi/2} e^{u} \frac{du}{-\ln 7} = \frac{1}{-\ln 7}\int_0^{\pi/2} e^{u}\,du = \frac{e^{u}}{-\ln 7} +$$

12

$$= \left[\frac{e^{\ln 7 \cos t}}{-\ln 7}\right]_0^{\pi/2} = \left[\frac{7^{\cos t}}{-\ln 7}\right]_0^{\pi/2}$$

$$-\left(\frac{7^1}{-\ln 7}\right) = \frac{7}{\ln 7}$$