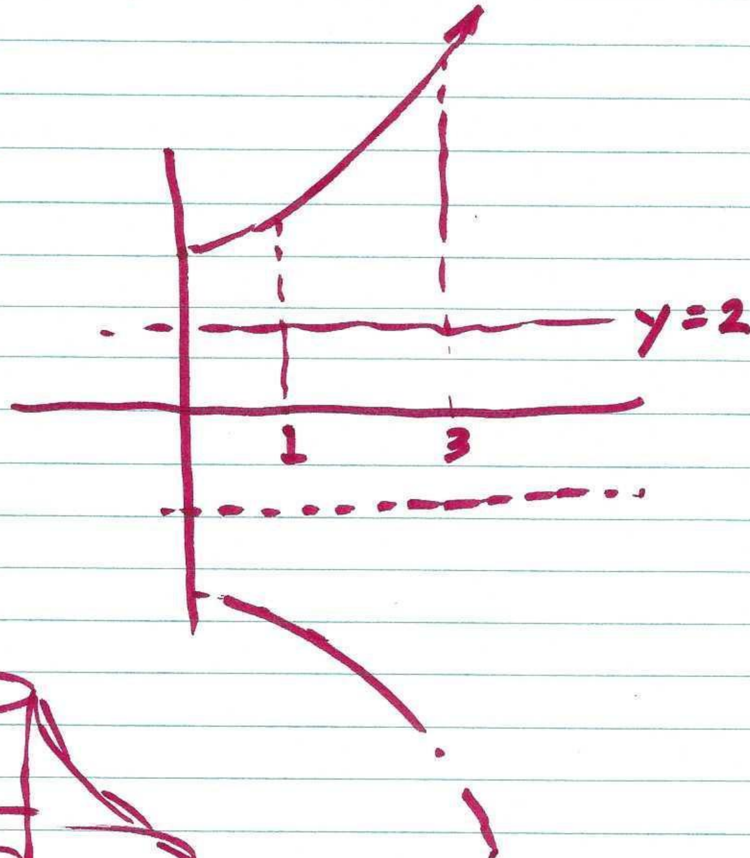
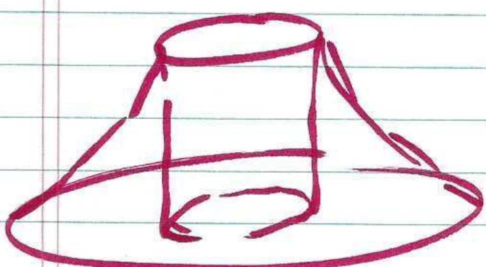
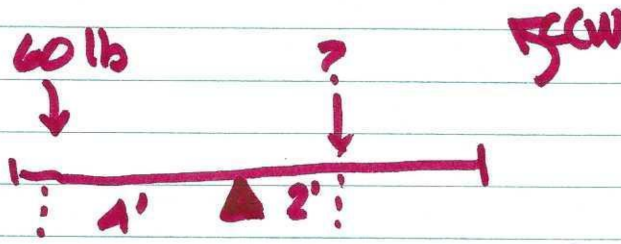
Add 1 to T/F Prob 5

#1 Part B





Chapter 6.6

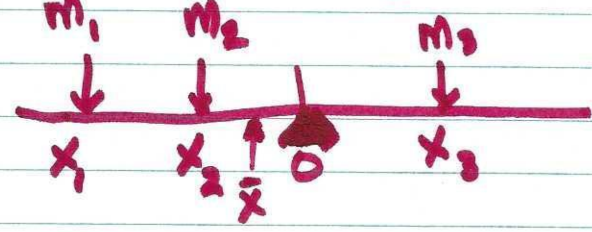


CW clock
CCW counter clock

Left Torque = 60·4 = 240 ft·lbs CCW

Right " = X·2 = 120·2 = 240 ft·lbs CW

"fulcrum"



Where should fulcrum be for balance?

Let $\bar{x}$ be position of fulcrum to achieve balance.

How far is mass m_1 from balance point

$m_1 \not{g}(x_1 - \bar{x})$ is torque from mass m_1

$m_2 \not{g}(x_2 - \bar{x})$ " " " " m_2

$m_3 \not{g}(x_3 - \bar{x})$ " " " " m_3

$+$ ―――――――――

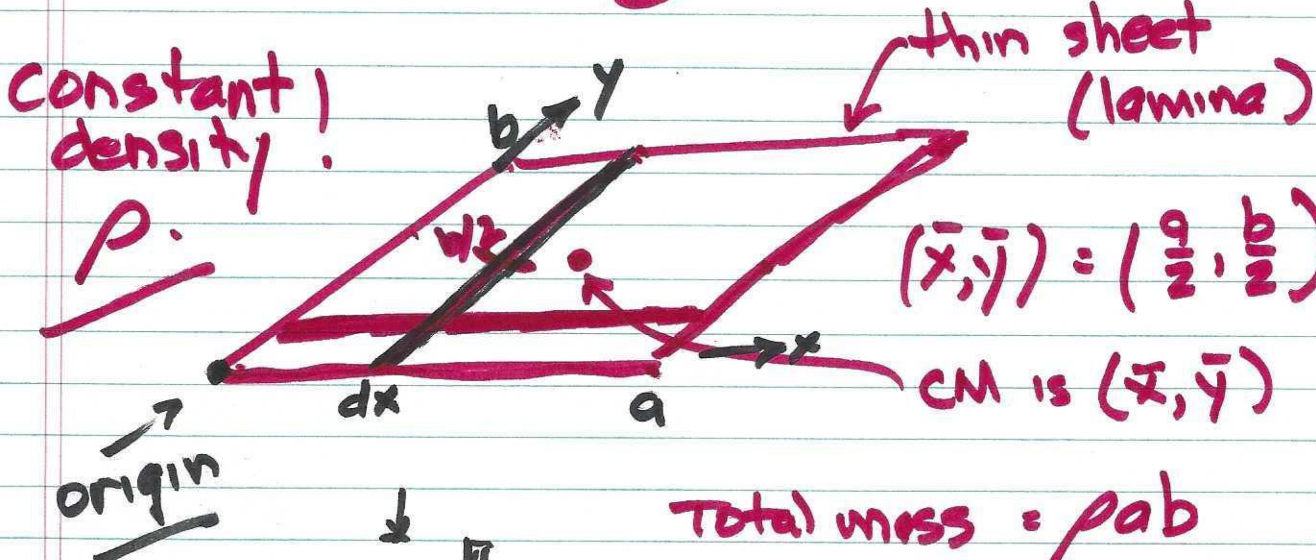
0 (torque is zero for balance)

$$m_1x_1 - m_1\bar{x} + m_2x_2 - m_2\bar{x} + m_3x_3 - m_3\bar{x} = 0$$

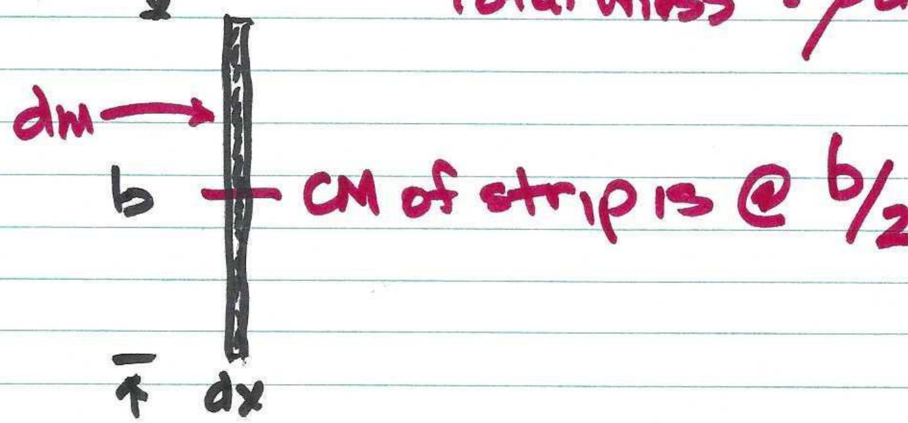
$$m_1x_1 + m_2x_2 + m_3x_3 = (m_1 + m_2 + m_3)\bar{x}$$

So... $$\boxed{\bar{x} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}}$$ $\bar{x}$ is called the center of mass (CM)

$$\bar{x} = \frac{\sum_i m_i x_i}{M} \quad \text{where } M = \sum_i m_i$$



Total mass = ρab

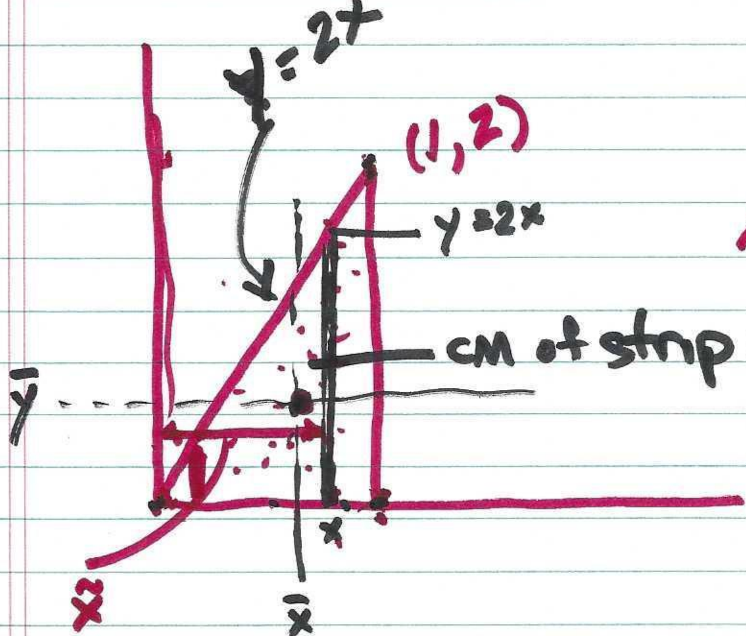


$$\int_0^a dm \cdot \frac{b}{2} = \frac{b}{2}\int_0^a dm = \frac{Mb}{2} = \text{"moment"}$$

of the sheet in y-direction ↑ (M_y) [about y-axis]

$$\bar{y} = \frac{M_x}{M} = \frac{Mb}{2M} = \boxed{\frac{b}{2}}$$

$$\bar{x} = \frac{M_y}{M} = \boxed{\frac{a}{2}}$$



$\rho = 3\ g/cm^2$

What is differential area of strip?

$dA = 2x\,dx$

What is mass of strip?

$dm = \rho\, dA = \rho\, 2x\,dx = 6x\,dx$

$\tilde{x}$ is distance of strip CM from y axis

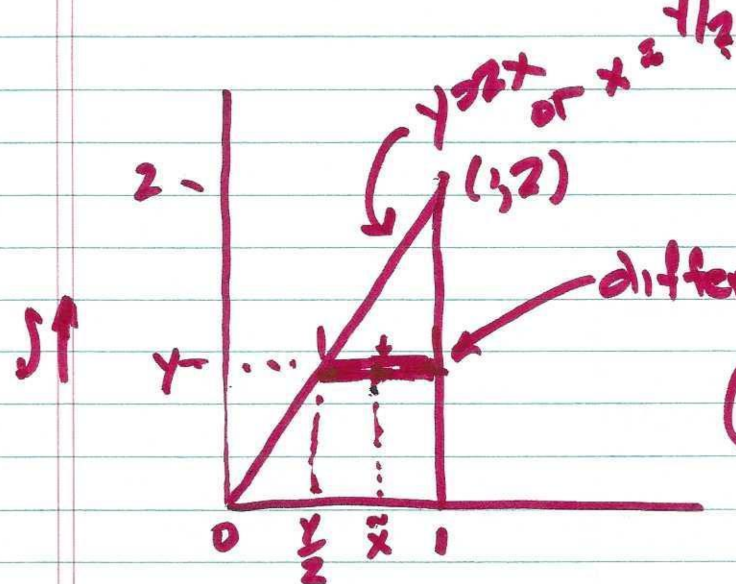
M_y = (moment about y-axis) of all strips

is $\int_0^1 x\,(6x\,dx) = \left[2x^3\right]_0^1 = 2$ gm-cm

(x: lever arm; $6x\,dx$: mass)

Total Mass is 3 g.

$$\bar{x} = \frac{M_y}{M} = \frac{2}{3} = .667$$



$$\left(\frac{y}{2}+1\right)/2 = \frac{y+2}{4}$$

Strip in other direction:

length $1 - \frac{y}{2} = \frac{2-y}{2}$

width dy

diff area $dA = \left(\frac{2-y}{2}\right)dy$

diff mass $dm = 3 \cdot \left(\frac{2-y}{2}\right)dy$

distance of strip CM to y-axis is $\frac{y+2}{4} = \tilde{x}$

diff moment of strip $= dm \cdot \tilde{x}$

$$= 3\left(\frac{2-y}{2}\right)dy \cdot \left(\frac{y+2}{4}\right)$$

$$= 3\,\frac{4-y^2}{8} \cdot dy = \boxed{\frac{3}{8}(4-y^2)dy}$$

$$\text{So } M_y = \int_0^2 \frac{3}{8}(4-y^2)\,dy = ?$$

$$\frac{3}{8}\left[4y - \frac{y^3}{3}\right]_0^2 = \frac{3}{8}\left(8 - \frac{8}{3}\right) = 3 - 1 = \boxed{2}$$

$$\boxed{\text{So } \bar{x} = \frac{2}{3}}$$ They agree, as expected

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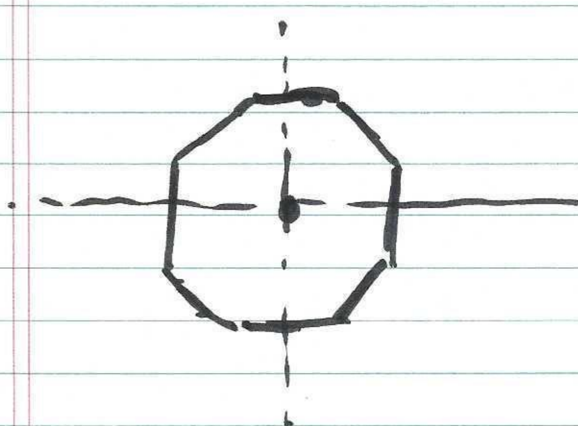
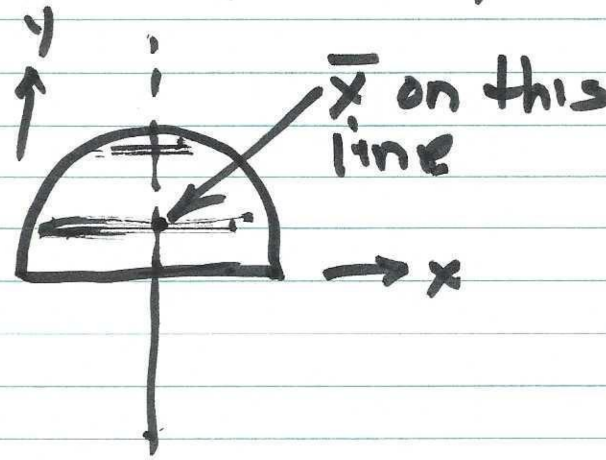
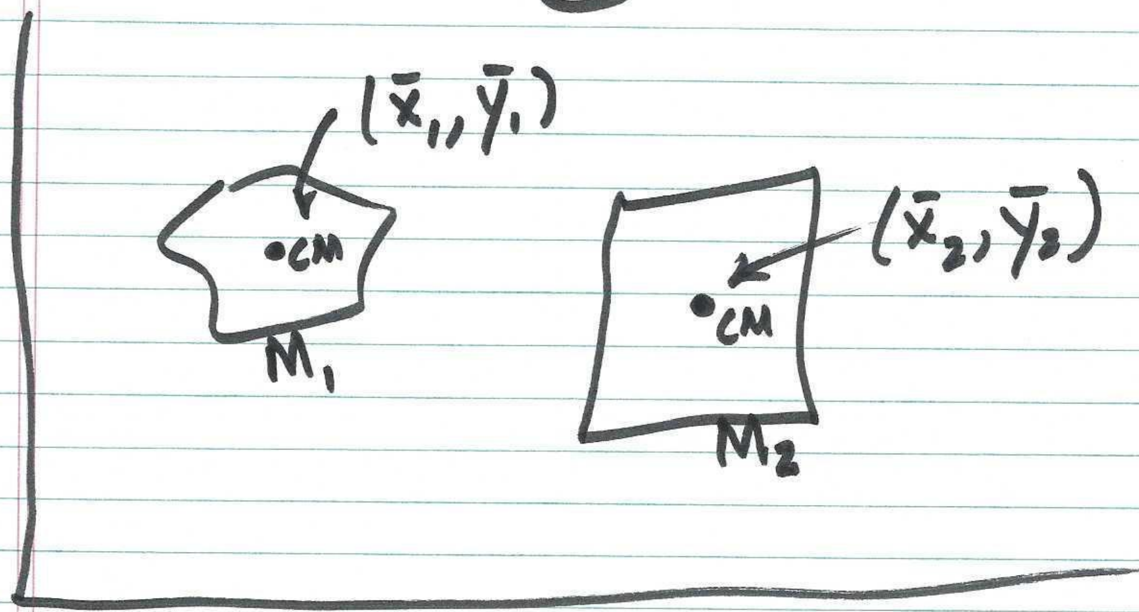
* Good to know: If density is constant, CM is <u>always</u> on an axis of symmetry.







$$\bar{x}_{TOT} = \frac{M_1 \bar{x}_1 + M_2 \bar{x}_2}{M_1 + M_2}$$

$$\bar{y}_{TOT} = \frac{M_1 \bar{y}_1 + M_2 \bar{y}_2}{M_1 + M_2}$$