# Growth Comparison

Given two functions $f(x)$ & $g(x)$, we can compare their rates of growth as $x \to \infty$.

Two ideas:

1. We say $f(x)$ grows more slowly than $g(x)$

if: $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = 0$

equiv: $\lim\limits_{x\to\infty} \dfrac{g(x)}{f(x)} = \infty$

same info

2. We say $f(x)$ grows at the same rate as $g(x)$

if: $\lim\limits_{x\to\infty} \dfrac{f(x)}{g(x)} = L$, a limit

Other ways to say this:

$\underline{f(x) = o(g(x))}$ means $f(x)$ grows more slowly or has smaller order

"Oh"

We write $f(x) = O(g(x))$ if $f(x)$ grows

at a comparable rate as $g(x)$

Ex. 1 Let $f(x) = x + \sin x$

$g(x) = x^2$

Compare these two

$$\frac{f(x)}{g(x)} = \frac{x + \sin x}{x^2} = \underbrace{\frac{1}{x}}_{\to 0} + \underbrace{\frac{\sin x}{x^2}}_{\to 0}$$

Take limit as $x \to \infty$

$(x + \sin x)$ is $o(x^2)$

Ex. 2 Let $f(x) = x + \sin x$

$g(x) = x$

$$\lim_{x \to \infty} \frac{x + \sin x}{x} = \underbrace{1}_{\text{stop}} + \underbrace{\frac{\sin x}{x}}_{\to 0}$$

$(x + \sin x) = O(x)$

Ex. 3 Compare $2^x$ with $e^x$

$$\lim_{x\to\infty} \frac{2^x}{e^x} = \lim_{x\to\infty} \left(\frac{2}{e}\right)^x \quad \text{Note } \frac{2}{e} < 1$$

$$= 0$$

$2^x$ is $o(e^x)$

Ex 4 Compare $x^2$ with $\ln x$

by L'Hôpital Rule

$$\lim_{x\to\infty} \frac{\ln x}{x^2} = \lim_{x\to\infty} \left(\frac{1/x}{2x}\right) = \lim_{x\to\infty} \left(\frac{1}{2x^2}\right)$$

$$= 0.$$

So... $\ln x$ is $o(x^2)$

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(a) $x-3$ $\quad \lim_{x\to\infty} \frac{x-3}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = 0$

(4)

(d) $4^x$ $\lim_{x\to\infty} \frac{e^x}{4^x} = \lim_{x\to\infty} \left(\frac{e}{4}\right)^x \overset{?}{=} 0$

$4^x$ grows faster than $e^x$ or

$e^x$ is $o(4^x)$

(10) T/F $\frac{1}{x+3} = O\left(\frac{1}{x}\right)$ ?

Look @ $\lim_{x\to\infty} \left(\frac{\frac{1}{x+3}}{\frac{1}{x}}\right) = \lim_{x\to\infty} \left(\frac{x}{x+3}\right) = ?$

$= \lim_{x\to\infty} \frac{1}{1} = 1$

So... $\frac{1}{x+3} = O\left(\frac{1}{x}\right)$

T 1. Differential equations involve one or more derivatives.

F 2. Separable (differential) equations involve only <u>one</u> variable

T 3. $\frac{dy}{dx} = Ax$ is a separable diff. eqn.

F 4. $\frac{dy}{dx} = \sin(xy)$ is separable

T 5. $\frac{dy}{dx} = xe^{y}$ is separable

F 6. $\frac{dy}{dx} = xe^{xy}$ is separable

T 7. $\frac{dy}{dx} = xe^{x+y}$ is separable

T 8. $\frac{dy}{dx} = f(x)g(y)$ is separable $\quad \frac{dy}{g(y)} = dx \cdot f(x)$

9. Solutions to separable equations <u>always</u> yield exponential growth/decay

T: 10 $^{14}C$ dating is based on radioactivity

T: 11 polynomial growth is always faster than logarithmic growth

F: 12 $\tanh x = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

$\sinh x := \dfrac{e^x - e^{-x}}{2}$

$\cosh x := \dfrac{e^x + e^{-x}}{2}$

F 13 $x^3 = O(x^2 + \sqrt{x})$ : $(x^2 + \sqrt{x}) = o(x^3)$

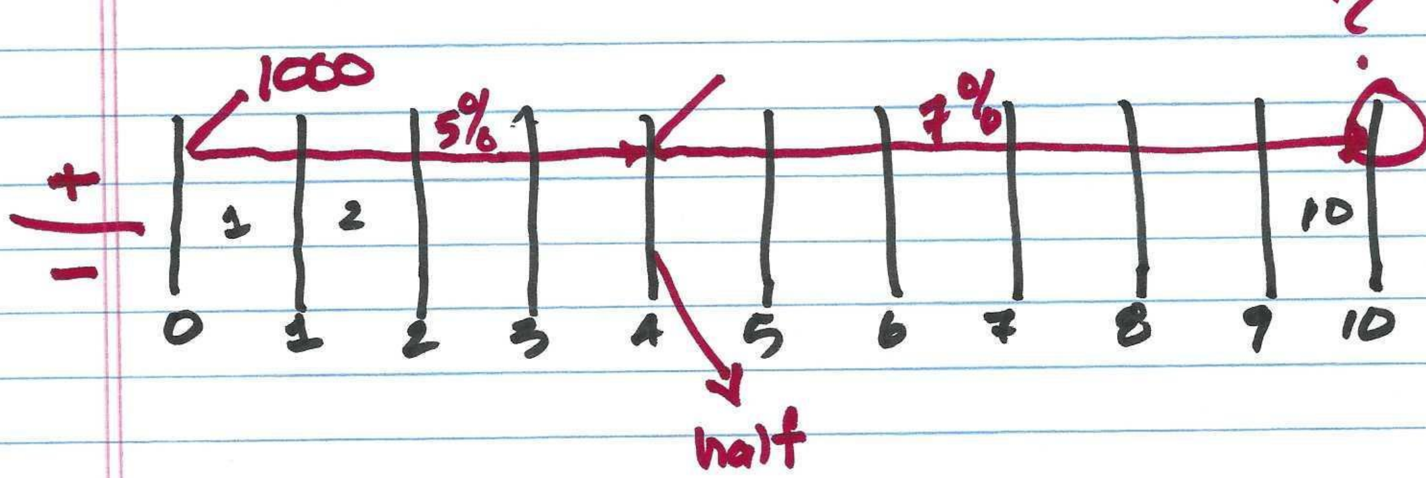
F 14 $\dfrac{x^5 + 4x^3 - 2x^2 + 9x - 3}{x^3 - 4x^2 + 5x - 10} = o(x^2)$

F 15 $P(t) = P(0)e^{-kt}$ could model cont. interest

Four types of word problem:

1. Continuous interest
2. Radioactive decay
3. Law of cooling
4. log substitution integral

If you invest $1000 @ year 1 @ 5% and remove half of the amount that has accrued by end year 4, then change interest rate to 7%. How much do you have on account @ end of year 10.



$$Q(t) = Q(0)e^{-\lambda t} \qquad \lambda > 0$$

$$\frac{Q(t_{1/2})}{Q(0)} = \frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\ln\left(\frac{1}{2}\right) = -\lambda t_{1/2}$$