①

Exponential Growth/Decay

(*) $\frac{dy}{dx} = f(y) \cdot g(x)$

↙ separation of variables

$\frac{dy}{f(y)} = g(x)\,dx$

$\int \frac{dy}{f(y)} = \int g(x)\,dx$

$\ln f(y) = G(x) + C$

$f(y) = e^{G(x)+C} = e^{C} e^{G(x)} = Ae^{G(x)}$

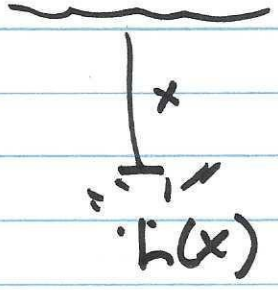
#27

$L(x)$

L = light intensity
x = depth

$\frac{dL(x)}{dx} = -kL$

$\frac{dL(x)}{L} = -k\,dx$



$$\int \frac{dL}{L} = \ln L$$ same

$$\int -k\,dx = -kx + C$$

$$\ln L = -kx + C$$

L_0

$$L = e^{-kx+C} = e^{C}e^{-kx}$$

$$L(x) = L_0 e^{-kx}$$

$$L(18) = \frac{L_0}{2}$$

$$\frac{L_0}{2} = L_0 e^{-k(18)}$$

$$\frac{1}{2} = e^{-18k}$$

$$.693 = -18k \Rightarrow k = -\frac{.693}{18}$$

$$k = -.0385$$

$$L(x) = L_0 e^{-.0385x}$$

$$\frac{L_0}{10} = L_0 e^{-.0385x} \Rightarrow x = 59.7 \text{ ft}$$

#30 $P(t) = P_0 e^{rt}$

$P(3) = P_0 e^{3r} = 10^4$

(1) $\ln P_0 + 3r = \ln(10^4) = 9.21$

$P(5) = P_0 e^{5r} = 4 \times 10^4$

(2) $\ln P_0 + 5r = \ln(4 \times 10^4) = 10.59$

5·(1) $5 \ln P_0 + 15r = 5 \cdot 9.21 = 46.05$

3·(2) $3 \ln P_0 + 15r = 3 \cdot 10.59 = 31.77$

$2 \ln P_0 = (46.05 - 31.77) = 14.28$

$\ln P_0 = 7.14 \Rightarrow P_0 = e^{7.14} = \underline{1261}$

#33 1147 @ t=0 $r = -.39$

$P(13) = 1147 e^{-.39(13)} = 1147 e^{-.507}$

$= 1147 (.0063) = 7.2$ indiv

$W(t) = W_0 e^{-.10t}$ oil output

$\frac{W(t)}{W_0} = \frac{1}{10} = e^{-.10t}$

$\ln\left(\frac{1}{10}\right) = -.10t \Rightarrow t = \frac{\ln\left(\frac{1}{10}\right)}{-.10}$

$t = \frac{-2.3}{-.10} = 23$ yrs.

#37 ^{239}Pu half-life is 24,360 yrs.

10g released in accident

How long for 80% of 10g to decay?

(5)

$Q(t) = Q_0 e^{-kt}$ time in yrs

$Q_0 = 10$ g.

$$\frac{Q(24{,}360)}{Q_0} = \frac{1}{2} = e^{-k(24{,}360)}$$

$$-.69315 = -24{,}360k$$

$$k = \frac{.69315}{24{,}360} = .0000285 \quad (2.85 \times 10^{-5})$$

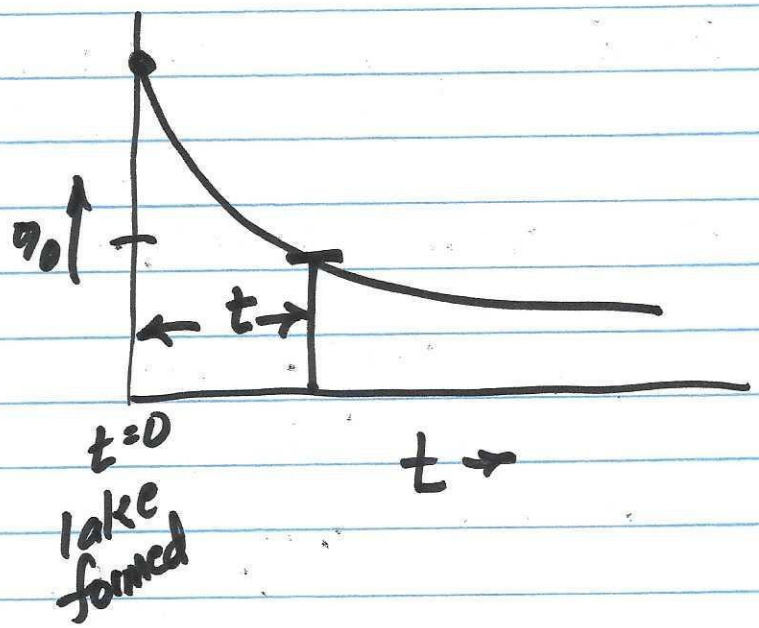
$$2 = 10e^{-(.0000285)t}$$

$$\ln\left(\frac{1}{5}\right) = -.0000285t$$

$$-1.609 = -.0000285t$$

$$t = \frac{1.609}{.0000285} = \underline{56471}$$

#45 Sample has 44.5% of ^{14}C compared to some material today.



$$Q(t) = Q(0)\, e^{-.00012t}$$

$$.445 = \frac{Q(t)}{Q(0)} = e^{-.00012t}$$

$$\ln(.445) = -.809 = -.00012t$$

$$t = \frac{.809}{.00012} \overset{?}{=} 6741 \text{ yrs}$$

$\sinh x := \frac{e^x - e^{-x}}{2}$ $\quad (\sinh x)' = \cosh x$

$\cosh x := \frac{e^x + e^{-x}}{2}$ $\quad (\cosh x)' = \sinh x$

$\tanh x := \frac{\sinh x}{\cosh x}$ $\quad (\tanh x)' = \operatorname{sech}^2 x$

$\operatorname{csch} x := \frac{1}{\sinh x}$

$\operatorname{sech} x := \frac{1}{\cosh x}$

$\coth x := \frac{1}{\tanh x}$

$\cosh^2 x = 1 + \sinh^2 x$