Exponential Growth

Contrast with linear growth:

Two interest formulas:

on deposit

%/yr as fraction

1) Simple interest $I = Pr$

2) Continuously compounded $F = Pe^{rt}$

future value

on deposit

time on deposit

Invest \$1.00 @ 5% per year when Jesus was 21:

1) Simple $2000 \text{ yrs} \cdot \frac{5¢}{\text{yr}} = \$100.00$

2) $F = 1e^{(.05)(2000)} = e^{100} = \$\underline{2.7 \times 10^{43}}$.

Value of solid gold planet Earth:

$R = 4000$ mi
1 mi $= 5280$ ft
1 ft $= 12$ in

So $\frac{4}{3}\pi(4000)^3$ is cubic miles

1 cubic mile is 5280^3 cubic feet

1 cubic feet is 1728 cubic inches

· 1 cubic inch is __ how many oz. of gold

gold is trading @ \$3000/oz spot market

Hallmark of exponential change is:

· [the rate of change of something is proportional to the amount present]

Suppose $x(t)$

$$\frac{dx}{dt} = kx \rightarrow \int \frac{dx}{x} = \int k\,dt$$

$$\ln x = kt + c$$

take antilog $\quad x = e^{kt+c}$

$$x = e^{kt+c} = e^{kt} \cdot e^{c}$$

Suppose $x = A$ when $t = 0$ (initial condition)

$$A = e^{k \cdot 0} e^{c} \Rightarrow \underline{A = e^{c}}$$

Finally: $\underline{x(t) = A e^{kt}}$ growth factor

If $k = -r$

Let $Q(t)$ be am't of radioactive material present @ time t.

$$Q(t) = Q(0) e^{-rt}$$ decay factor

Population Growth

months

$$F(t) = F(0) e^{(.02)t}$$

1000 trout

$$F(t) = 1000 e^{.02t}$$

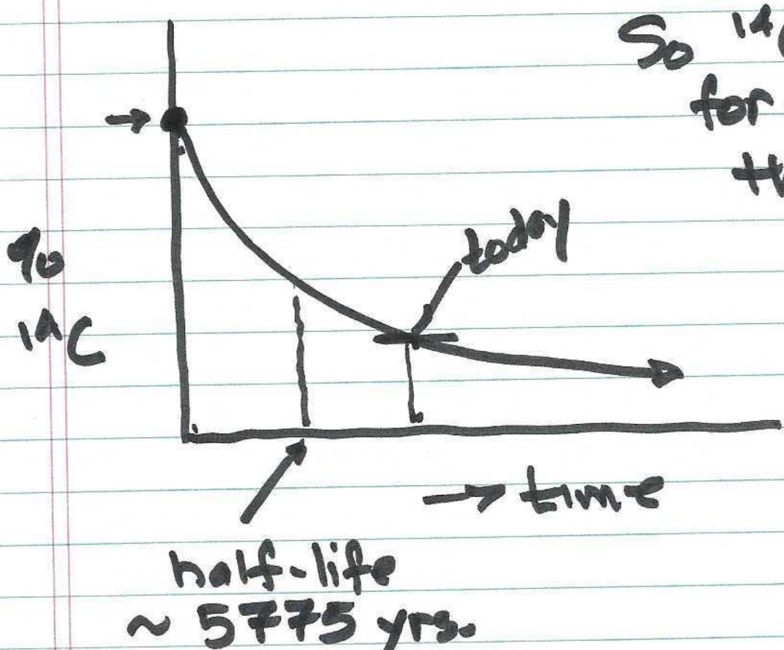
$$50{,}000 = 1000 e^{.02t} \quad ★$$

$$50 = e^{.02t}$$

$$\ln(50) = .02t$$

$\ln 50 = 3.91 = .02t \Rightarrow t = 50 \cdot 3.91$ months

Exponential Decay



So ^{14}C detection works for objects no older than 4 half-lives

Lascaux Cave painting:

~ 15,000 years old

$$Q(t) = Q(0)e^{-.00012t} \leftarrow \text{yrs}$$

$$?? \boxed{\frac{Q(t)}{Q(0)}} = e^{-.00012t}$$

Compute $e^{(-.00012)(1.5\times10^4)}$

~$12\frac{1}{2}$

.165

$$\frac{dT}{dt} = -\overset{+}{k}(T - T_0)$$

Start: egg @ 98C into water @ 18C

$t = 5$ min $\Rightarrow$ egg @ 38C, water @ 18C

how much more time for egg to reach 20C.

★ $H - H_s = (H_0 - H_s)e^{-kt}$

H is object temp (function of time)

H_s is the environment temp

H_0 is starting object temp

$H = 18 + (98 - 18)e^{-kt}$

Find that $k = 0.28$?

$$H(t) = 18 + 80e^{-kt}$$
$$= 18 + 80e^{-(0.28)t}$$
$$\Rightarrow e^{-(0.28t)} = \frac{1}{40}$$
$$-(0.28)t = \ln\frac{1}{40}$$
$$\Rightarrow t = 13 \text{ min.}$$