## Derivatives

$f(x)$

To get derivative $f'(x)$, $\frac{df(x)}{dx}$, $D_x f$

$$f'(x) := \lim_{\Delta x \to 0} \left( \frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$$

$f''(x)$ or $\frac{d^2 f}{dx^2}$

↑ Newton ↑ Leibnitz

## Integrals

integrand → $f(x)$; primitive of f → $F(x)$; integrator → $dx$

$$\int f(x)\,dx = F(x) + C \qquad f(x) = F'(x)$$

$$\int_a^b f(x)\,dx = \Big[F(x)\Big]_a^b = F(b) - F(a)$$

| | $f$ | $f'$ |
|---|---|---|
| | $c$ | $0$ |
| | $x$ | $1$ |
| $n \in \mathbb{N}$ | $x^n$ | $nx^{n-1}$ |
| $r \in \mathbb{R}$ | $x^r$ | $rx^{r-1}$ |
| | $e^x$ | $e^x$ |
| $(e^{\ln 2})^x$ | $2^x$ | $\ln 2 \cdot 2^x$ |
| | $b^x$ | $\ln b \cdot b^x$ |
| | $\ln x$ | $\frac{1}{x}$ |
| | $(g \pm h)'$ | $g' \pm h'$ |
| | $gh$ | $g'h + gh'$ |
| | $\frac{g}{h}$ | $\frac{hg' - h'g}{h^2}$ |
| | $g(x)^n$ | $ng(x)^{n-1} \cdot g'(x)$ |
| Chain Rule | $(g \circ h)$ | $[g(h(x))]' = g'(h(x)) \cdot h'(x)$ |

s
c
-s
-c
s

| $f$ | $f'$ |
| --- | --- |
| $\ln g(x)$ | $\frac{1}{g(x)} \cdot g'(x) = \frac{g'(x)}{g(x)}$ ← "relative change" |
| $\sin x$ | $\cos x$ |
| $\cos x$ | $-\sin x$ |
| $\tan x$ | $\sec^2 x$ |
| $\cot x$ | $-\csc^2 x$ |
| $\sec x$ | $\sec x \cdot \tan x$ |
| $\csc x$ | $-\csc x \cdot \cot x$ |
| $\sinh x$ | $\cosh x$ |
| $\cosh x$ | $\sinh x$ |

$\sinh x := \frac{e^x - e^{-x}}{2}$

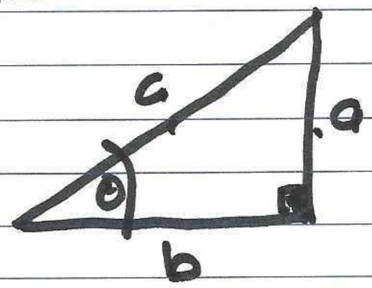
$\cosh x := \frac{e^x + e^x}{2}$

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$

$P'(x) = n a_n x^{n-1} + (n-1) a_{n-1} x^{n-2} + \cdots a_1 + 0$

sohcahtoa

## Trig functions




$a^2+b^2=c^2$ (Pythagoras)

Def^s:

$\frac{a}{c} = \sin\theta$ $\qquad \frac{b}{c} = \cos\theta$

$\frac{a}{b} = \tan\theta$ $\qquad \frac{c}{b} = \sec\theta$

$\frac{c}{a} = \csc\theta$ $\qquad \frac{b}{a} = \cot\theta$

$\sin(90^\circ-\theta) = \cos\theta$ (cofunction identities)

#1

$\frac{a^2+b^2}{c^2} = \frac{c^2}{c^2} \Rightarrow \boxed{\sin^2\theta + \cos^2\theta = 1}$

meh

$\frac{a^2}{a^2} + \frac{b^2}{a^2} = \frac{c^2}{a^2} \Rightarrow \boxed{1+\cot^2\theta = \csc^2\theta}$

#2

$\frac{a^2}{b^2} + \frac{b^2}{b^2} = \frac{c^2}{b^2} \Rightarrow \boxed{\tan^2\theta + 1 = \sec^2\theta}$

| | $f$ | $\int f\,dx$ | $+C$ |
|---|---|---|---|
| | $c$ | $cx$ | |
| | $x$ | $\frac{x^2}{2}$ | |
| | $x^2$ | $\frac{x^3}{3}$ | |
| $n \neq -1$ | $x^n$ | $\frac{x^{n+1}}{n+1}$ | ← $n = -1$ oops! |
| | $\frac{1}{x}$ | $\ln x$ | |
| | $e^x$ | $e^x$ | |
| S ↑ C −S −C ↓ S | $2^x$ | $\frac{2^x}{\ln 2}$ | |
| | $\sin x$ | $-\cos x$ | |
| | $\cos x$ | $\sin x$ | |

⑥

| $f$ | $\int f\,dx$ |
|---|---|
| $x^{\pi}$ | $\dfrac{x^{\pi+1}}{\pi+1}$ |
| $\sec x \tan x$ | $\sec x$ |
| $\sec^2 x$ | $\tan x$ |
| $\sec x$ | $\ln(\tan x + \sec x)$ |
| $\tan x$ | |

$$\int \frac{dx}{\cos x}$$

$$\int \sec x \cdot \left(\frac{\sec x + \tan x}{\sec x + \tan x}\right) dx$$

$d(\tan x)$ $\quad$ $d(\sec x)$

$$\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\, dx$$

$$\int \frac{d(\tan x + \sec x)}{\tan x + \sec x}\, dx$$

$$\int \frac{df}{f}$$

$$\ln(\tan x + \sec x)$$

$$\int \tan x\,dx = \int \frac{\sin x}{\cos x}dx = -\int \frac{-\sin x}{\cos x}dx$$

$$\int \frac{d(\cos x)}{\cos x} = \ln \cos x$$

---

← hard to do

$$\int f(x)\,dx \rightarrow \int f(x(t))\frac{dx}{dt}\,dt$$

so let $x = x(t)$

$$\int 2x e^{x^2} dx = \int e^u du$$

$$\int e^{x^2} dx$$

$$\int \frac{dx}{\ln x}$$

Let $u = x^2$

$e^{x^2} \rightarrow e^u$

$du = 2x\,dx$