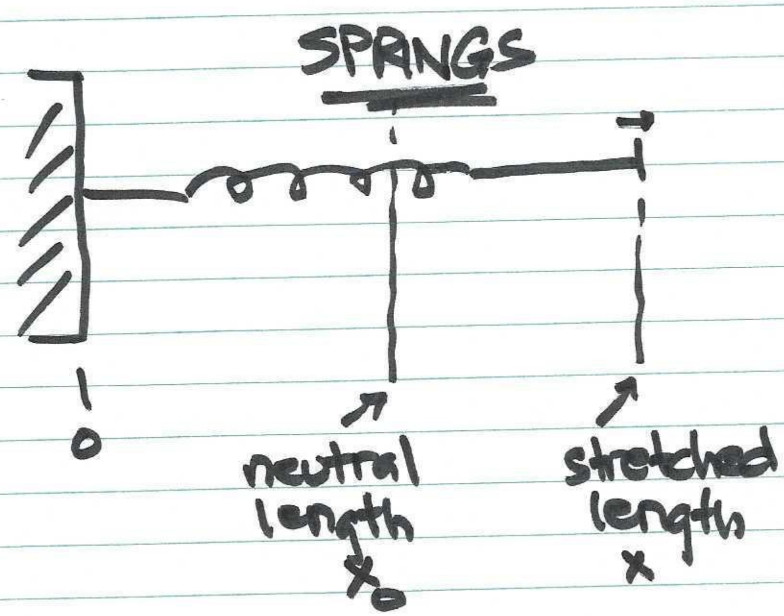
Work & Energy

Two situations:

(i) Elastic problems

(ii) Pumping liquid problems

Elastic



Spring has a "spring constant" k that tells how hard it is pushing or pulling

$$F(x) = -k(x-x_0) \text{ by spring}$$

our force is $-F(x)$

To stretch spring from length x_0 (neutral) to length x, we exert $+k(x-x_0)$

How much work (on the spring) do we do to extend it from x to x+dx?

$$dW(x) = F(x)\,dx = k(x-x_0)\,dx$$

How much <u>total</u> work is required to extend spring from x_0 to x?

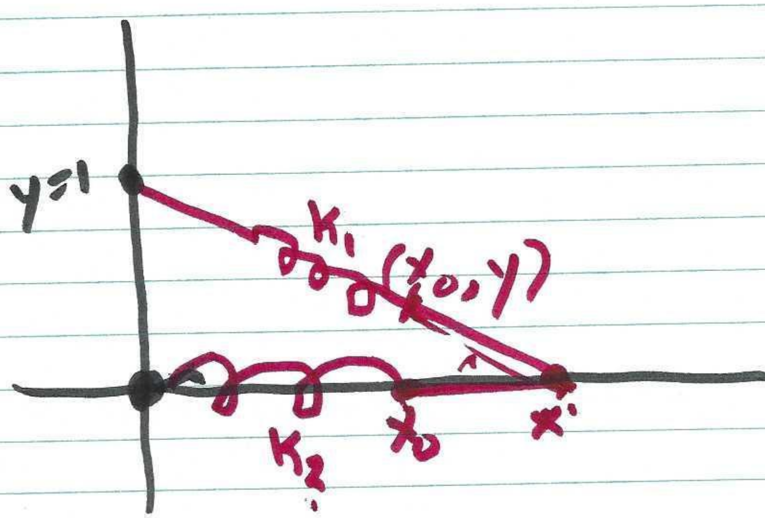
$$\text{Total work} = \int_{\text{start}}^{\text{stop}} dW(x)$$

$$W = \int_{x_0}^{\hat{x}} k(x-x_0)\,dx = \left[\frac{1}{2}kx^2 - kxx_0\right]_{x_0}^{\hat{x}}$$

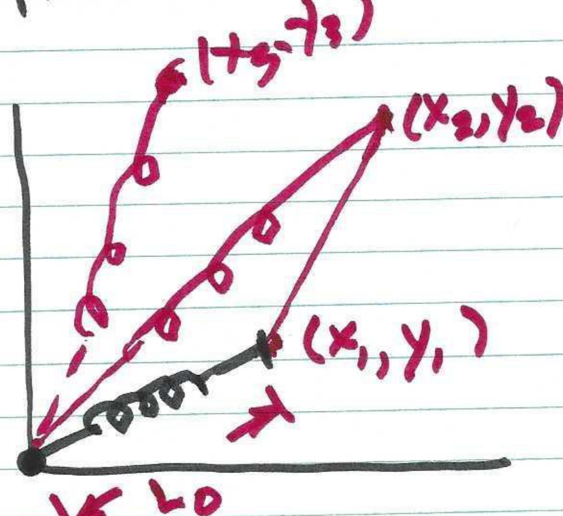
$$= \left[\frac{1}{2}k\hat{x}^2 - k\hat{x}x_0\right] - \left[\frac{1}{2}kx_0^2 - kx_0^2\right]$$

If we set $W(x_0) = 0$

$$W(\hat{x}) = \frac{1}{2}k(x-x_0)^2$$



Possible problem:



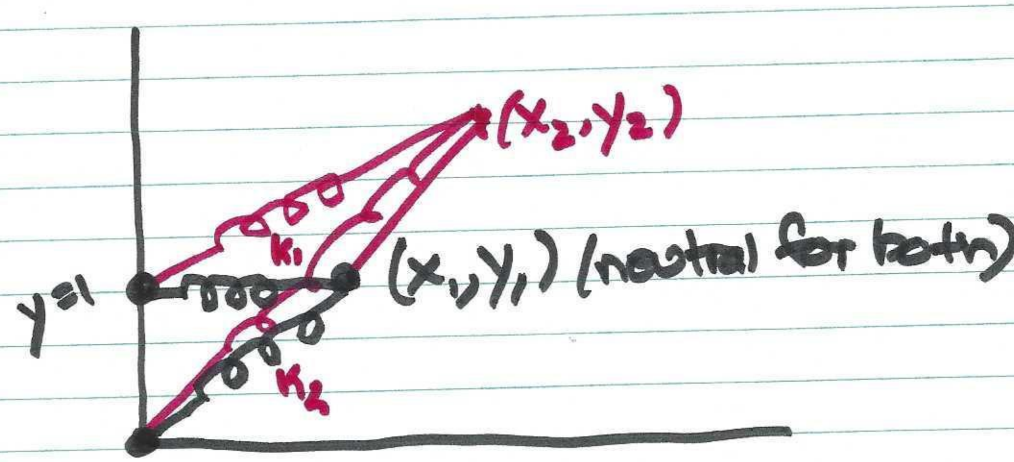
$L = \sqrt{x_2^2 + y_2^2}$

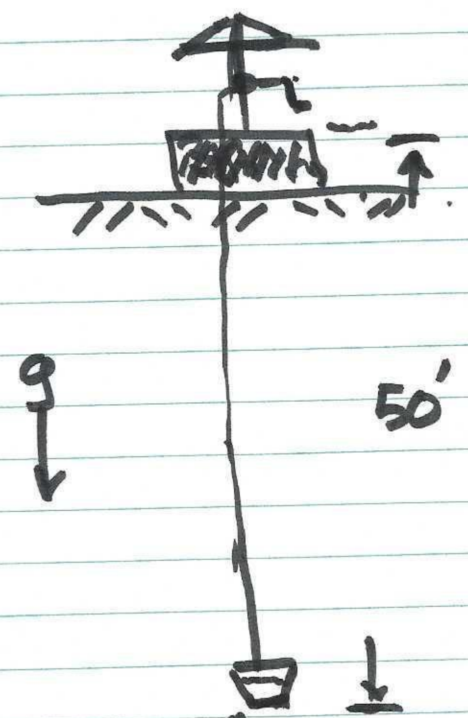
stretch $= L - L_0$

$L_0 = \sqrt{x_1^2 + y_1^2}$

If constant is k

Work = Energy = $\frac{1}{2} k (L - L_0)^2$





(weight)
vol·density = force req'd

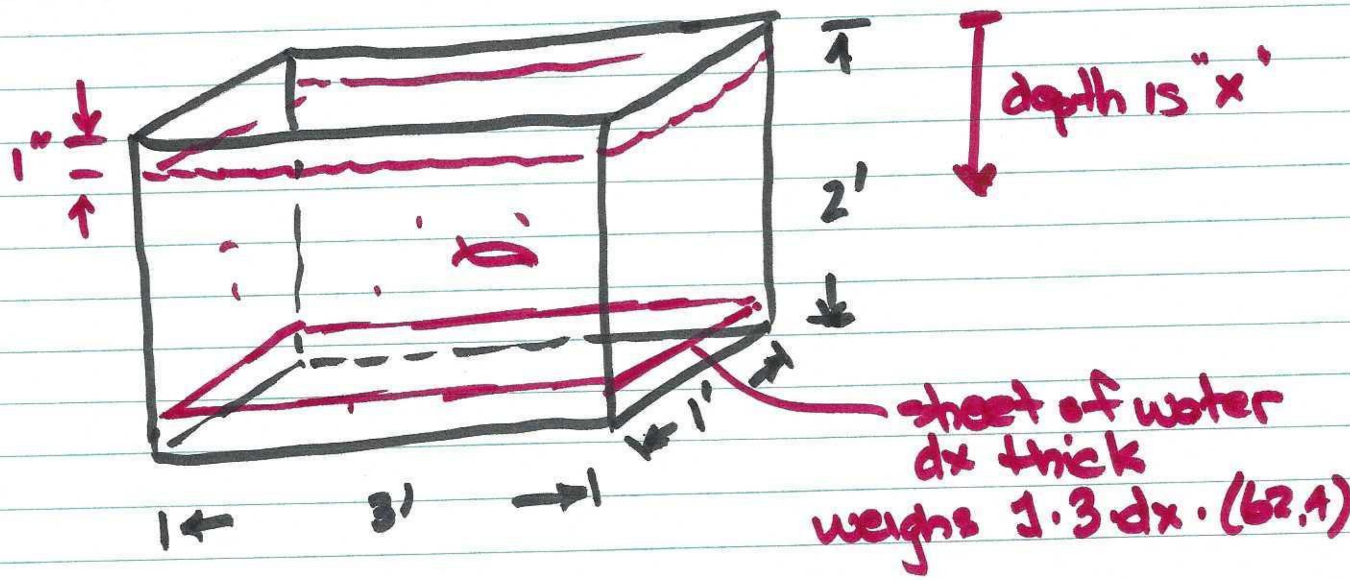
1500± ft-lbs work to lift bucket

5 gal pail (H_2O)

↳ 231 in^3 1155 in^3

62.4 lbs/cf

What if bucket leaks? Fix it.



How much does it weigh full?

$6\ ft^3 \cdot 62.4\ \frac{lbs}{ft^3}$

$dW(x)$ = differential work for sheet @ x

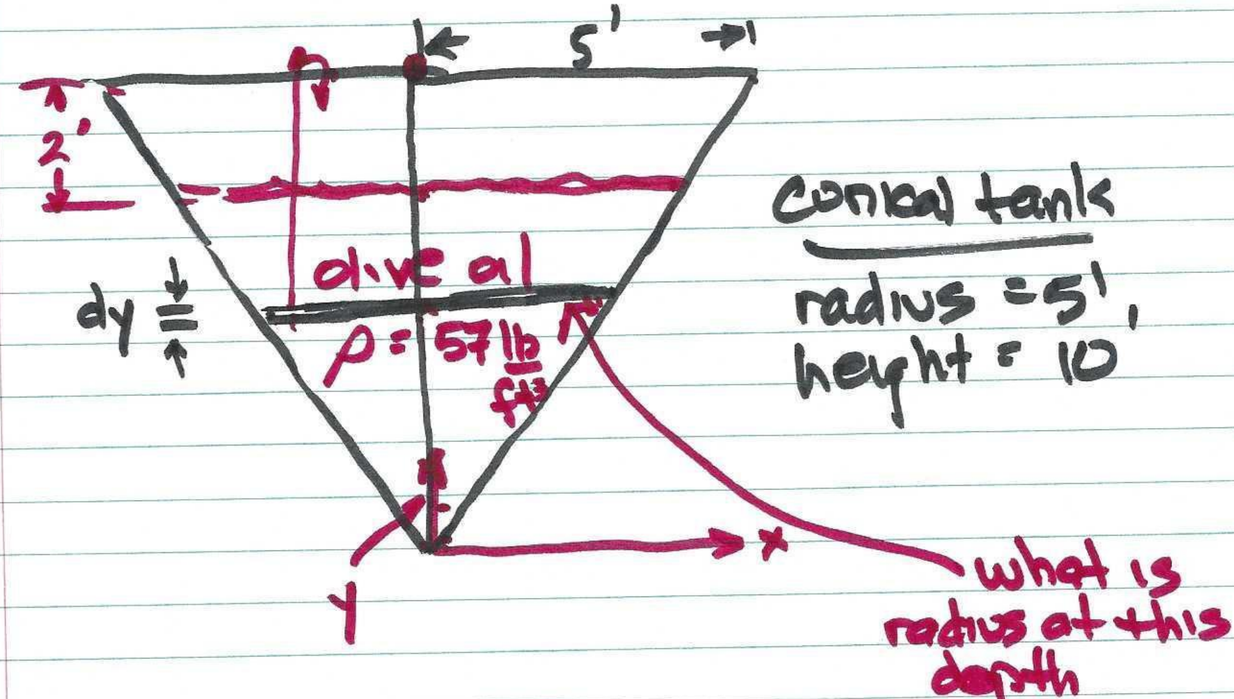
$= (3)(62.4)(dx)\cdot x$

$= 187.2\, x\, dx$

$$W(\text{empty}) = \int_{x=\frac{1}{12}}^{x=2} 187.2\, x\, dx$$

$$W = \left[\frac{187.2\,x^2}{2}\right]_{.085}^{2} \cong \quad \cong \underline{375} \text{ ft-lbs}$$

To empty in 10 min., need to exert 37.5 ftlbs per min.



$$\frac{r(y)}{y} = \frac{5}{10}$$

$$r(y) = \frac{y}{2}$$

depth of sheet is $10-y$

weight of oil sheet @ depth $10-y$ is $\longrightarrow$

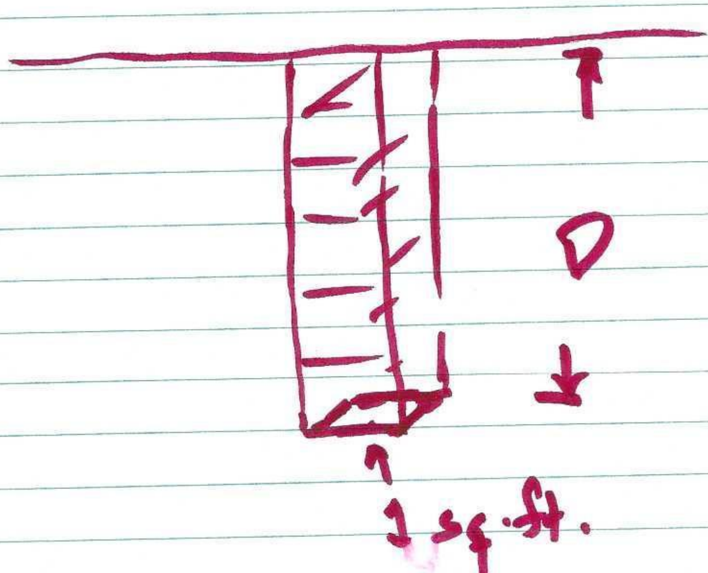
$$dW(y) = (10-y)\left(\pi \frac{y^2}{4}\right)(57)\,dy$$

$$= \pi \frac{57}{4}(10-y)y^2\,dy$$

$$W = \pi\int_0^8 \frac{57}{4}(10-y)y^2\,dy = ?$$

$$\frac{57\pi}{4}\left[\frac{10y^3}{3} - \frac{y^4}{4}\right] = 30{,}561 \text{ ft}\cdot\text{lbs}$$

Pressure is force/area

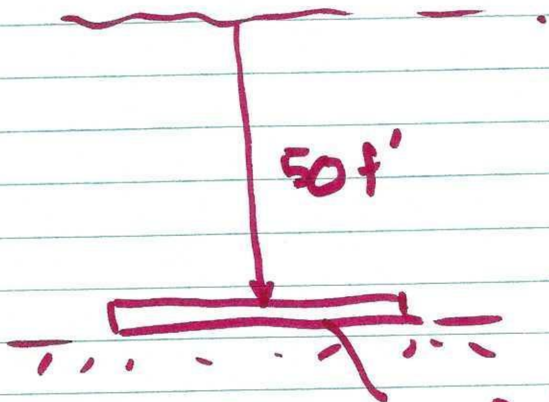


$\underline{D \cdot 1 \cdot 62.4}$

$62.4D \cdot \frac{\text{lbs}}{\text{sq. ft.}}$

Pressure @ depth D

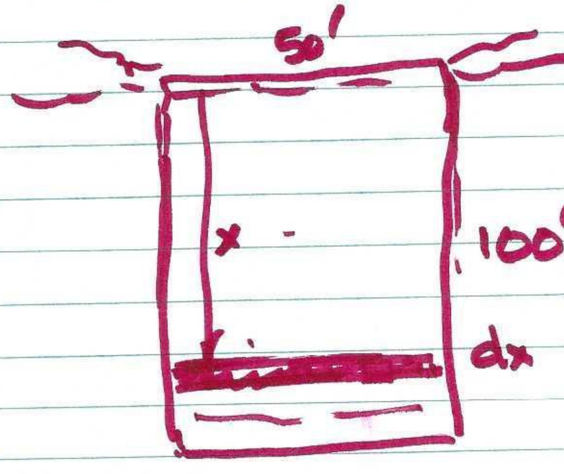
is $62.4D$

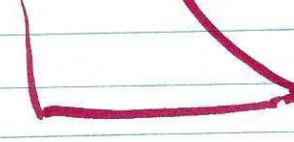


area of slab is 100 sq. ft.

Force = Pressure (@ 50') × Area

$(62.4)(50)(100) = (62.4)(5000) = 312{,}000$ lbs





$$dF(x) = \underbrace{(50\,dx)}_{A(x)} \cdot \underbrace{(62.4x)}_{P(x)}$$

$$dF(x) = (3120)x\,dx$$

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$$F = \int_0^{100} (3120)\,x\,dx = \left[\frac{3120\,x^2}{2}\right]_0^{100}$$

$\frac{3120}{2} \cdot 10^5$ lbs

15.6×10^8

$1.56 \times 10^{3+5} = 1.56 \times 10^8$ lbs

0.78×10^4

7.8×10^3 tons

1) Disk

2) Area of Surface of Rev

3) Work