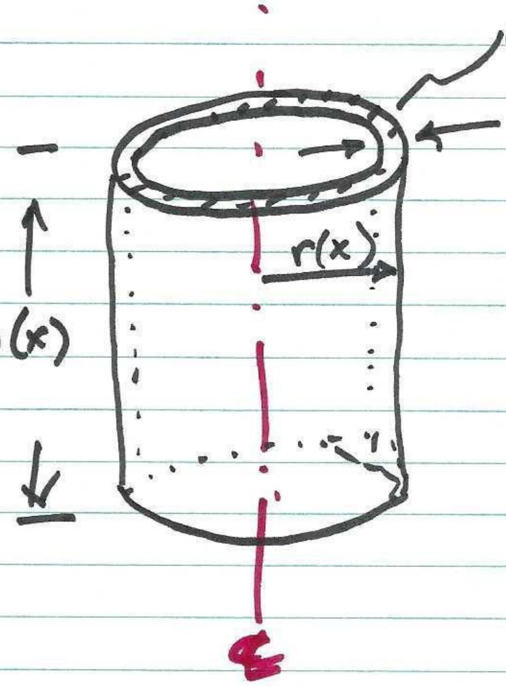
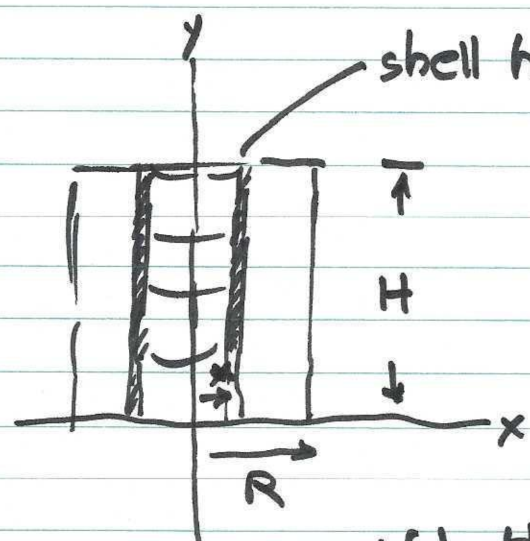
Volumes via Method of Shells



$$(2\pi r \cdot dr \cdot \text{height}) = dV$$

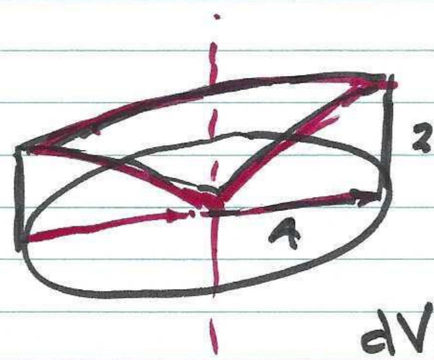
Simple Case: Find volume of cylinder of radius R and height H.



$$2\pi x dx \cdot H = dV(x)$$

$$Vol = H\int_0^R 2\pi x dx = \pi H \int_0^R 2x dx$$

$$\pi H\int_0^R 2x\,dx = \pi H\left[x^2\right]_0^R = \boxed{\pi R^2 H}$$



Circumference $= 2\pi x$
Thickness $= dx$
Height $= \sqrt{x}$

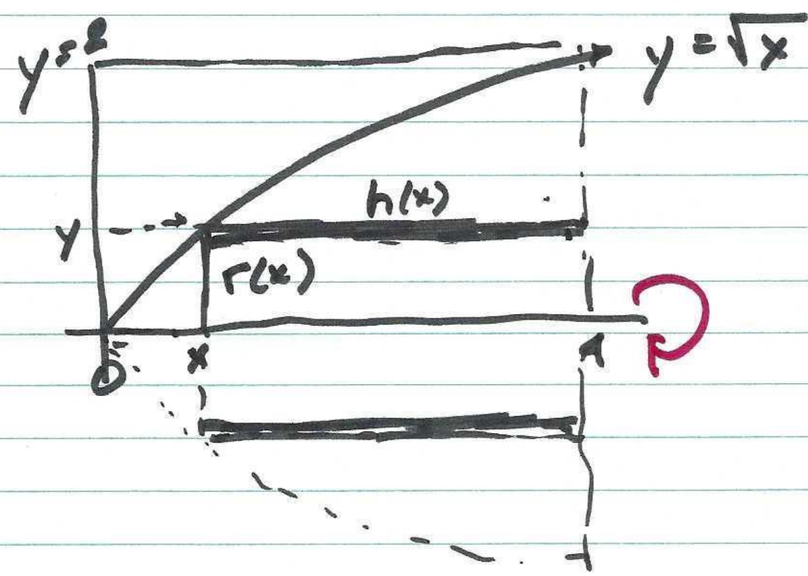
$$dV(x) = 2\pi x\sqrt{x}\,dx$$

$$= 2\pi x^{3/2}\,dx$$

$$\text{Volume} = \int_0^4 2\pi x^{3/2}\,dx$$

$$= 2\pi\int_0^4 x^{3/2}\,dx = 2\pi\cdot\frac{2}{5}\cdot\left[x^{5/2}\right]_0^4$$

$$= \frac{4\pi}{5}\cdot 32 = \boxed{\frac{128}{5}\pi}$$



Radius of shell @ y = y

Thickness = dy

So area of ring (cross-section of shell) = $2\pi y$

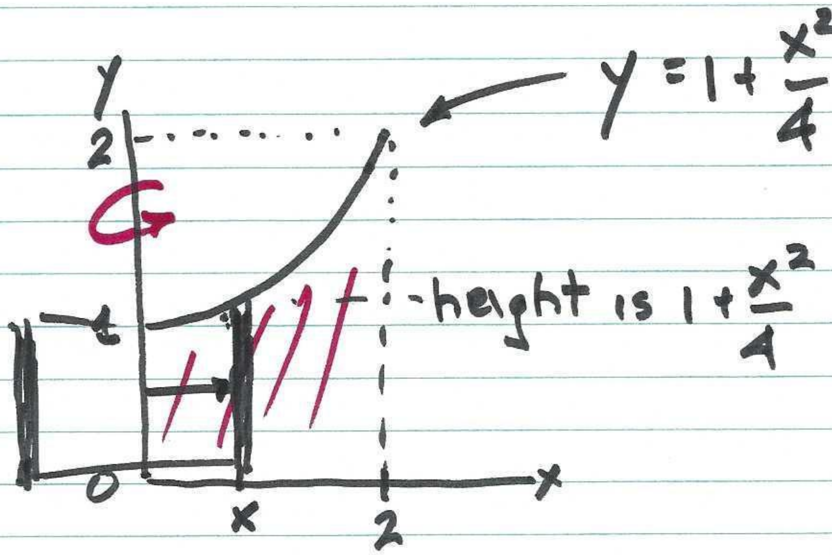
Height = $4 - y^2$

$$dV(y) = 2\pi y(4-y^2)\,dy$$

$$Vol = 2\pi\int_0^2 y(4-y^2)\,dy = 2\pi\int_0^2 4y\,dy - 2\pi\int_0^2 y^3\,dy$$

$$= 2\pi\Big[2y^2\Big]_0^2 - 2\pi\left[\frac{y^4}{4}\right]_0^2$$

$$= 16\pi - 8\pi = \boxed{8\pi}$$



$$dV(x) = 2\pi x \cdot \left(1 + \frac{x^2}{4}\right) dx$$

$$\text{Volume} = \int_0^2 2\pi x \left(1 + \frac{x^2}{4}\right) dx$$

$$= 2\pi \int_0^2 x\,dx + 2\pi \int_0^2 \frac{x^3\,dx}{4}$$

$$= 2\pi \left[\frac{x^2}{2}\right]_0^2 + 2\pi \left[\frac{x^4}{16}\right]_0^2$$

$$= 4\pi + 2\pi = 6\pi$$

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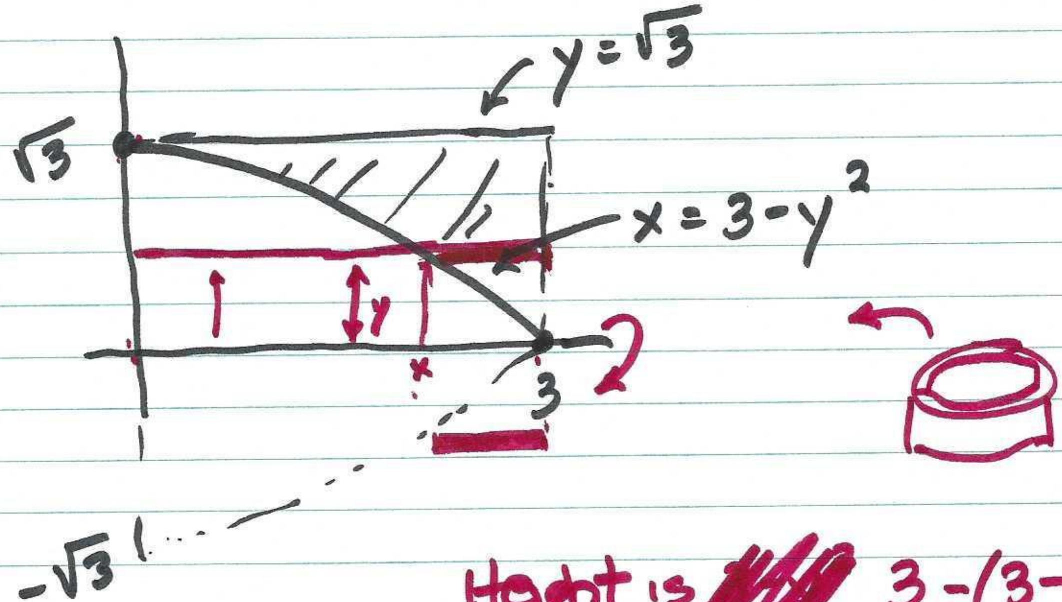


Height is $3-(3-y^2)=y^2$

Radius is y

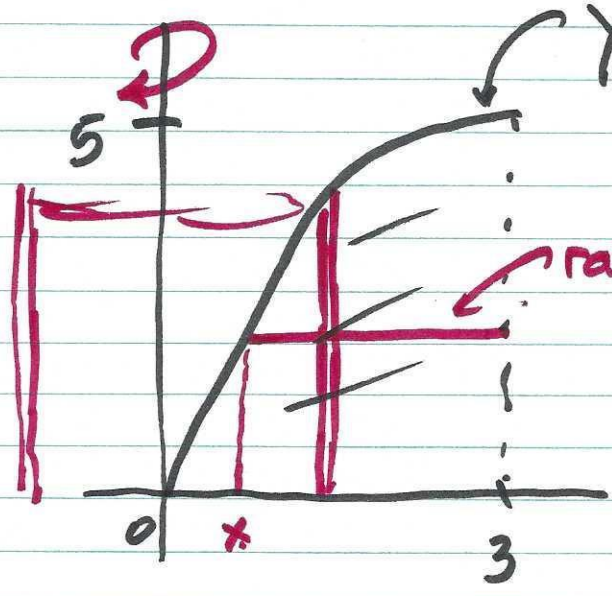
Thickness is dy

$$dV(y) = 2\pi y \cdot y^2\,dy = 2\pi y^3\,dy$$

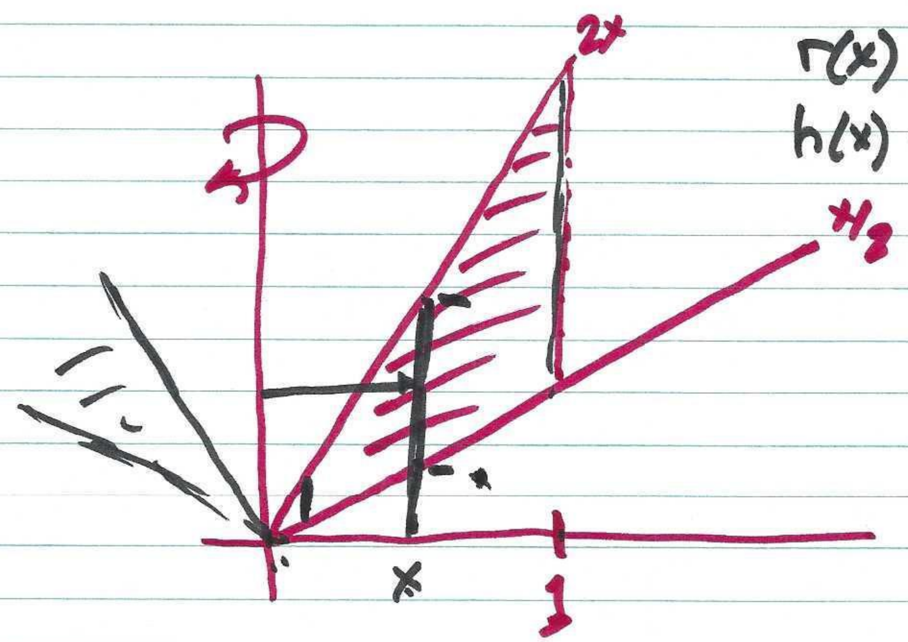
$$\text{Vol} = 2\pi \int_0^{\sqrt{3}} y^3\,dy = 2\pi\left[\frac{y^4}{4}\right]_0^{\sqrt{3}} =$$

$$= 2\pi \cdot \frac{9}{4} = \frac{18}{4}\pi$$

⑥



⑧ $y_1 = 2x \quad y_2 = \frac{x}{2} \quad x = 1$



$r(x) = x$

$h(x) = 2x - \frac{x}{2}$

$$dV(x) = 2\pi x \cdot \left(2x - \frac{x}{2}\right) dx$$

$$\text{Vol} = 2\pi \int_0^1 \left(2x^2 - \frac{x^2}{2}\right) dx$$

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$$\text{Vol} = 2\pi\left[\frac{2x^3}{3} - \frac{x^3}{6}\right]_0^1 = \frac{1}{2}\cdot 2\pi = \pi$$

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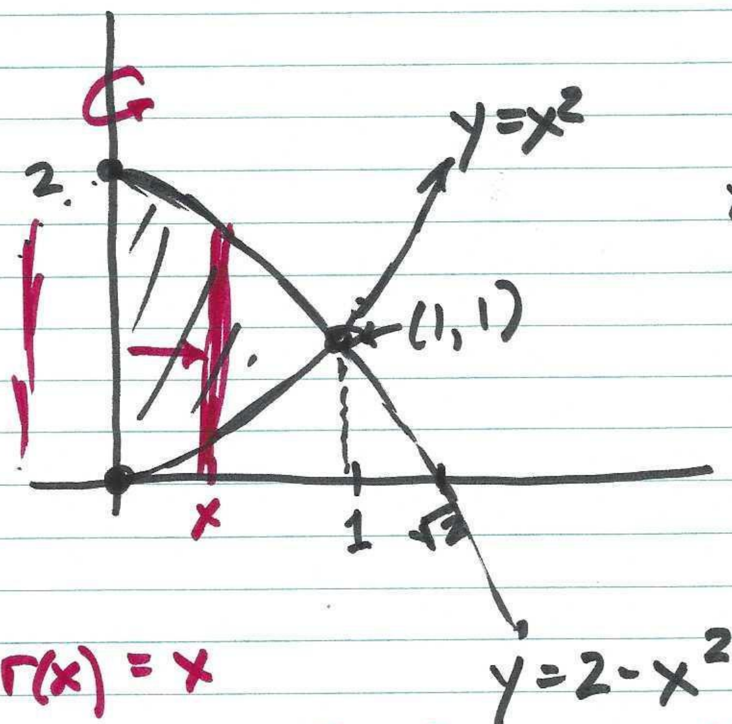
10 $y = 2 - x^2;\ y = x^2;\ x = 0$



$x^2 = 2 - x^2$

$r(x) = x$

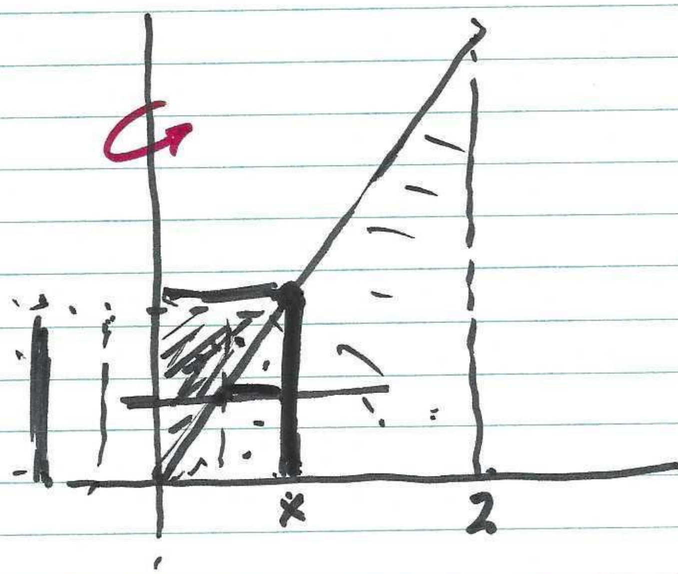
$h(x) = 2 - x^2 - x^2 = 2(1-x^2)$

$dV(x) = 2\pi x\,(2(1-x^2))\,dx$

$$\text{Vol} = 2\pi\int_0^1 2x\,dx - 2\pi\int_0^1 2x^3\,dx$$

$$\text{Vol} = 2\pi\left[x^2 - \frac{x^4}{2}\right]_0^1 = \pi$$

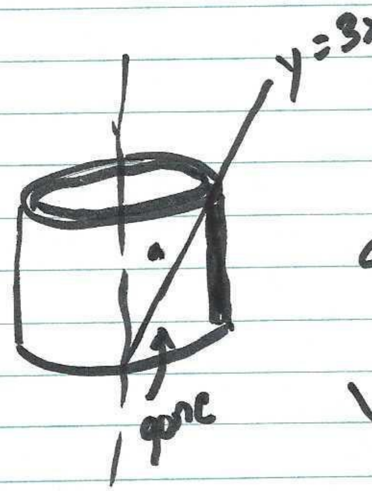
#23 $y = 3x$; $y = 0$; $x = 2$



(a) Use y-axis

height is $3x$

radius is x



$$dV(x) = 2\pi(3x^2)\,dx$$

$$\text{Vol} = \int_0^2 6\pi x^2\,dx$$

$$= \pi\left[\frac{x^3}{2}\right]_0^2 = 4\pi$$

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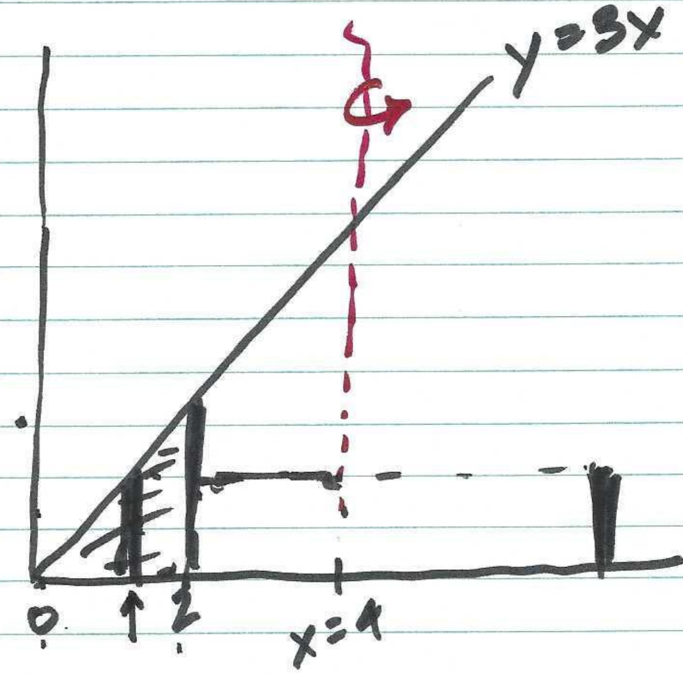

radius is $4 - x$
height is $3x$

$$\text{So } dV(x) = 2\pi(4-x)(3x)\,dx$$

$$= 6\pi x(4-x)\,dx$$

$$= (24\pi x - 6\pi x^2)\,dx$$

$$\text{Vol} = 6\pi\int_0^2 (4x - x^2)\,dx$$

$$6\pi\left[2x^2 - \frac{x^3}{3}\right]_0^2 = 6\pi\left(8 - \frac{8}{3}\right)$$

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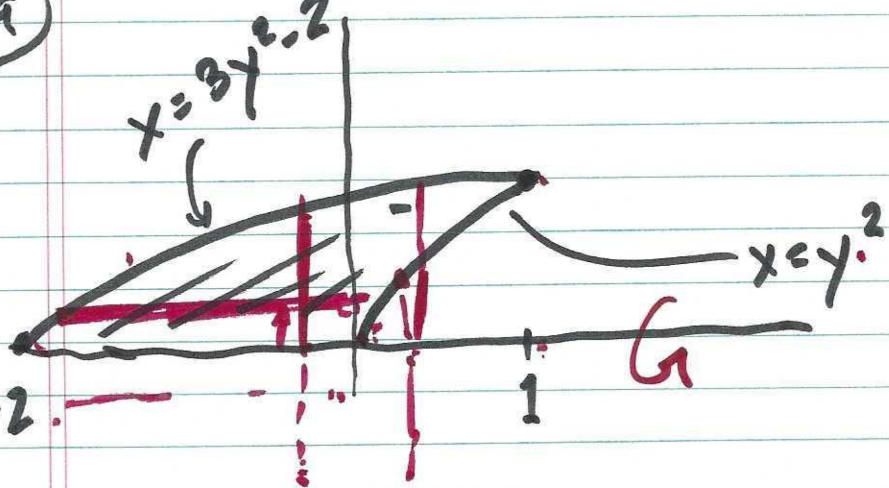


Horizontal Shells $r(y) = y$

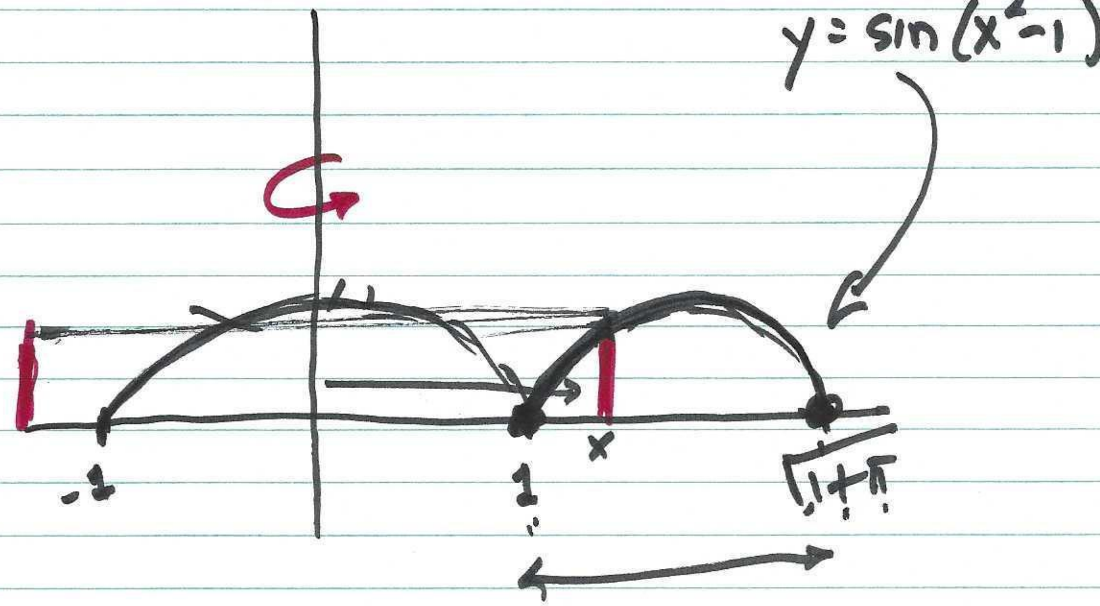
$h(y) = y^2-(3y^2-2)$

$$dV(y) = 2\pi y(2-2y^2)$$

Disks $\perp$ to x-axis

$r(y) = y$ from $x=-2$ to $x=0$

then use washer from $x=0$ to $x=1$



radius $= x$

height $= \sin(x^2-1)$

$$dV(x) = 2\pi x \cdot \sin(x^2-1)\,dx$$

$$\text{Vol} = 2\pi \int_1^{\sqrt{1+\pi}} x \sin(x^2-1)\,dx$$

Let $u = x^2-1 \Rightarrow du = 2x\,dx$

$$\text{Vol} = 2\pi \int_{x=1}^{x=\sqrt{1+\pi}} \sin u \,\frac{du}{2} = \frac{2\pi}{2}\Big[-\cos u\Big]_{x=1}^{x=\sqrt{1+\pi}}$$

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$$\text{Vol} = \pi \left[-\cos\left(x^2 - 1\right) \right]_1^{\sqrt{1+\pi}}$$

$$= \pi\left(1 - (-\cos 0)\right) = 2\pi$$