U456 W

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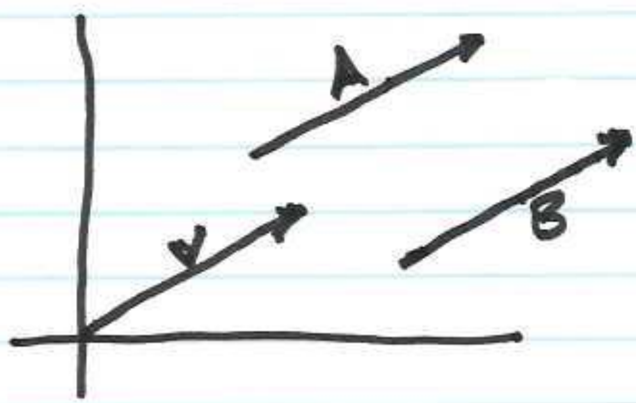
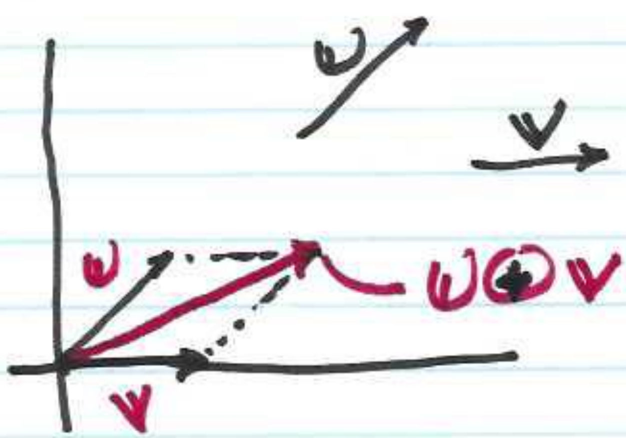
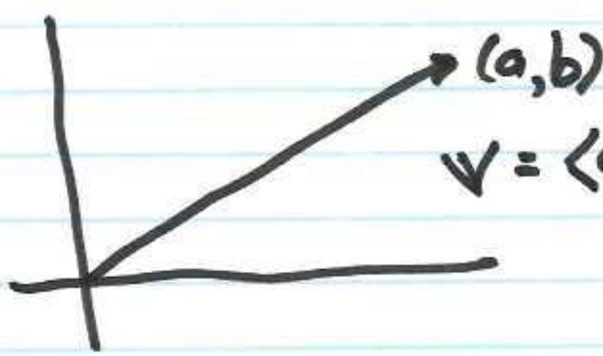
Vecters - line in $\mathbb{R}^2$ or $\mathbb{R}^3$ .. ($\mathbb{R}^n$)

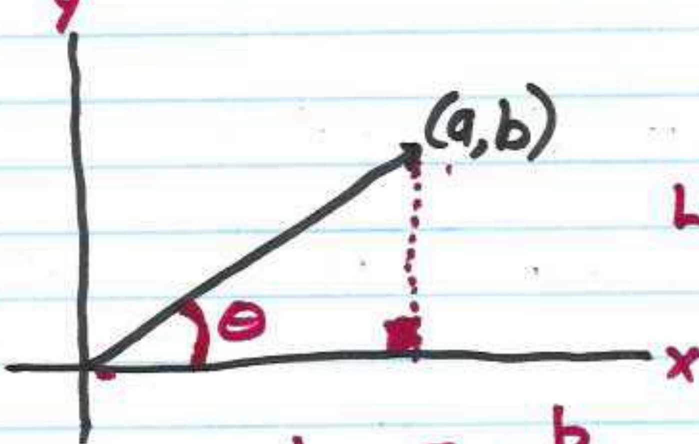
that has length and direction.

u, v, w,









Length $= \sqrt{a^2+b^2}$

$\tan\theta = \frac{b}{a}$

$\theta = \arctan\frac{b}{a}$

don't use $\tan^{-1}\left(\frac{b}{a}\right)$

↑

is this $\frac{1}{\tan\left(\frac{b}{a}\right)}$ or arctan

$\tan^{-1}\left(\frac{b}{a}\right)$

length of v written as $|v|$

some texts use $\|v\|$ for definiteness

another word for length is "norm"

$\left.\begin{array}{l}\langle a,b\rangle\\ \langle -a,b\rangle\\ \langle a,-b\rangle\\ \langle -a,-b\rangle\end{array}\right\}$ same norm

$|v| = |\langle a,b\rangle| = \sqrt{a^2+b^2}$

components

To add $v = \langle a,b \rangle$ to $w = \langle c,d \rangle$

in $\mathbb{R}^2$ $\quad v \oplus w = \langle a+c, b+d \rangle$

$r = \langle a,b,c \rangle \quad s = \langle d,e,f \rangle$

in $\mathbb{R}^3$ $\quad r \oplus s = \langle a+d, b+e, c+f \rangle$

Another operation allowed for vectors is "scalar multiplication"

$c \cdot v = cv = \langle ca, cb \rangle$

Def'n of a vector space

$(V, \mathbb{R})$ for vectors v, u, w

1. $v \oplus u = u \oplus v$ (commutativity)
2. $(u \oplus v) \oplus w = u \oplus (v+w)$

associativity

This means $\mathbb{u} \oplus \mathbb{v} \oplus \mathbb{w}$ is unambiguous

3. There is a vector $\mathbb{0} = \langle 0,0 \rangle$
$\langle 0,0,0 \rangle$

$$\langle a,b,c \rangle + \langle 0,0,0 \rangle = \langle a,b,c \rangle$$

or $\mathbb{v} \oplus \mathbb{0} = \mathbb{0} + \mathbb{v} = \mathbb{v}$

(existence of an additive identity)

4. There is an additive inverse $-\mathbb{v}$ for every vector $\mathbb{v}$ such that $(-\mathbb{v}) \oplus \mathbb{v} = \mathbb{0}$

5. $-\mathbb{v} = (-1)\mathbb{v}$

6. $c\mathbb{v} = \langle ca, cb \rangle$

7. $c(\mathbb{u} \oplus \mathbb{v}) = c\mathbb{u} + c\mathbb{v}$

$\langle a,b\rangle$ times $\langle c,d\rangle$

There is a "multiplication" of vectors called the "dot product".

If $v = \langle a,b\rangle$; $w = \langle c,d\rangle$

Def'n of dot product $\underline{v \cdot w = ac + bd}$

In 3-D $v = \langle a,b,c\rangle$; $w = \langle d,e,f\rangle$

$v \cdot w = ad + be + cf$

$v = \langle -1,2,4\rangle$; $w = \langle 2,-3,6\rangle$

What is $v \cdot w$?

$= -2 - 6 + 24 = 16$

$w \cdot v = |w||v| \cos \theta_{w,v}$

$$\arccos\left[\frac{w \cdot v}{|w||v|}\right] = \theta_{wv}$$

If v is a vector & $|v| = 1$, we say v is a <u>unit</u> vector.

To unitize a vector, form $\frac{v}{|v|} = e_v$

e_v different than $|e_v| = +1$

$\frac{1}{|v|} = c \quad |cv| = +1$