LN3MW

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Space curve - working in $\mathbb{R}^3$

$$\mathbf{r}(t) = x(t)\,\hat{i} + y(t)\,\hat{j} + z(t)\,\hat{k}$$

t is time | Velocity is $\mathbf{r}'(t)$

Acceleration is $\mathbf{r}''(t)$

If $\mathbf{r}'(t)$ is continuous and $\mathbf{r}'(t) \neq 0$

Find velocity of particle w/ path function

$$\mathbf{r}(t) = 2\cos t\,\hat{i} + 2\sin t\,\hat{j} + 5\cos^2 t\,\hat{k}$$

Velocity: $\mathbf{r}'(t) = -2\sin t\,\hat{i} + 2\cos t\,\hat{j} + 10\cos t(-\sin t)\hat{k}$

$-10\underbrace{\sin t\cos t}_{\frac{1}{2}\sin 2t}\,\hat{k}$

$$\mathbf{r}''(t) = -2\cos t\,\hat{i} - 2\sin t\,\hat{j} + (-5\cdot 2\cdot \cos 2t)\hat{k}$$

$$= -2\cos t\,\hat{i} - 2\sin t\,\hat{j} - 10\cos 2t\,\hat{k}$$

speed = $|\mathbf{r}'(t)| = \sqrt{\underbrace{4\sin^2 t + 4\cos^2 t}_{} + 100\sin^2 t\cos^2 t}$

$$= 2\sqrt{1 + 25\sin^2 t\cos^2 t}$$

Let v be velocity

$$\mathbf{v}(t) = \mathbf{r}'(t) = |\mathbf{v}|\left(\frac{\mathbf{v}}{|\mathbf{v}|}\right)$$

What happens when you differentiate a vector of <u>constant</u> <u>length</u>?

Given $\mathbf{r}(t)$ and $|\mathbf{r}(t)| = c$

What is relation between $\mathbf{r}(t)$ & $\mathbf{r}'(t)$

$$\mathbf{r}(t)\cdot\mathbf{r}(t) = |\mathbf{r}(t)|^2 = c^2$$

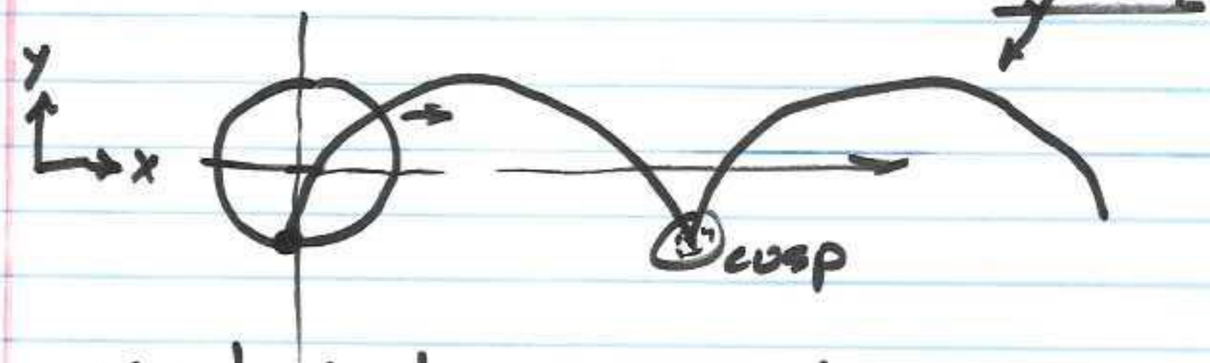
Take derivative: $\frac{d}{dt}\left(\mathbf{r}(t)\cdot\mathbf{r}(t)\right) = ?$

$$|\mathbf{r}(t)|^2 = \mathbf{r}'(t)\cdot\mathbf{r}(t) + \mathbf{r}(t)\cdot\mathbf{r}'(t)$$

$$= 2\,\mathbf{r}'(t)\cdot\mathbf{r}(t) = \frac{d}{dt}c^2 = 0$$

So... <u>$\mathbf{r}(t) \perp \mathbf{r}'(t)$</u>

(11) Analyze motion on a cycloid



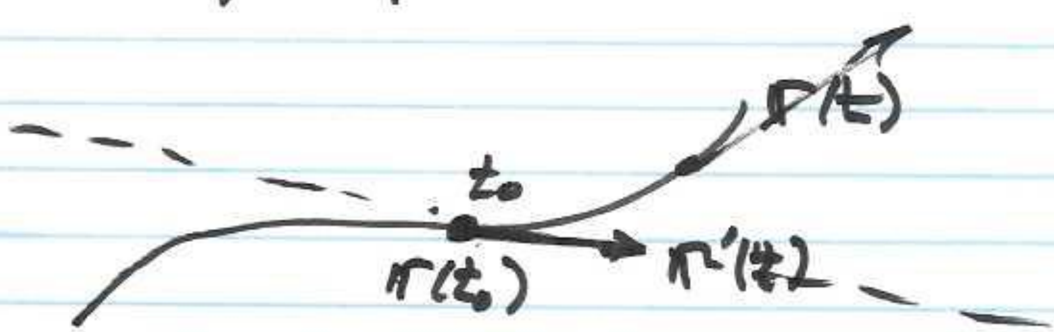
$x = t - \sin t,\ y = 1 - \cos t$

$\mathbf{r}(t) = (t - \sin t)\,\hat{\imath} + (1 - \cos t)\,\hat{\jmath}$

$\mathbf{r}'(t) = (1 - \cos t)\,\hat{\imath} + (\sin t)\,\hat{\jmath}$

We can say @ $t = 0$, $\mathbf{r}'(0) = 0$

Is $\mathbf{r}'(t)$ tangent to $\mathbf{r}(t)$?



@ t_0 tangent line is $\mathbf{r}(t_0) + u \cdot \underline{\mathbf{r}'(t_0)})$

④

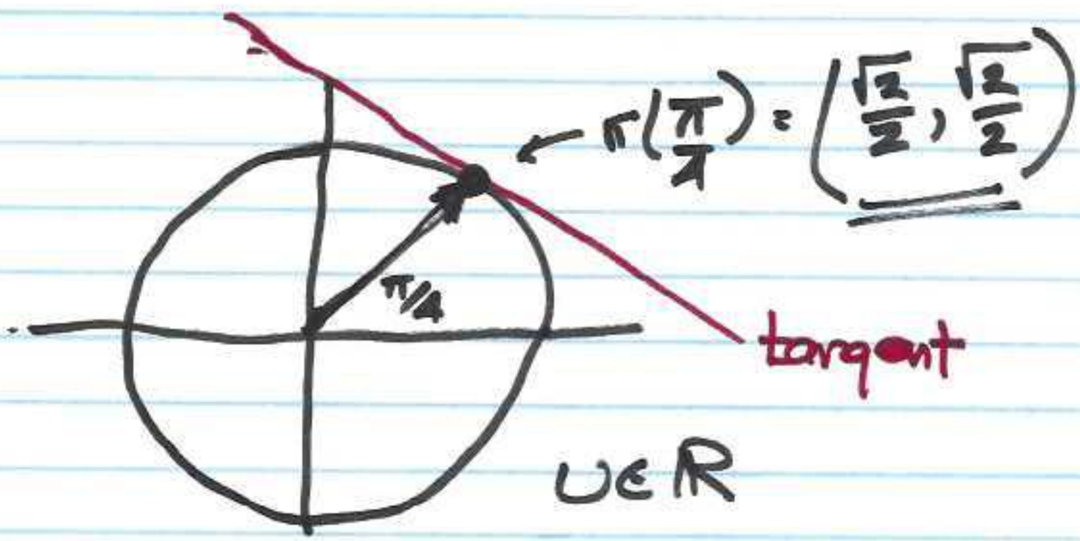
$$\ell(u) = \mathbf{r}(t_0) + u \underbrace{\frac{\mathbf{r}'(t_0)}{|\mathbf{r}'(t_0)|}}_{\text{unit vector tangent to curve}}$$

initial vector to get onto straight

Example $\mathbf{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j}$

What is eqn of tangent line (to circle) when

$t = \frac{\pi}{4}$



$u \in \mathbb{R}$

$$\ell(u) = \mathbf{r}\left(\frac{\pi}{4}\right) + u$$

direction unit vector $\dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)| \leftarrow 1} = \dfrac{-\sin t\,\hat{i} + \cos t\,\hat{j}}{1} = \underbrace{-\sin t\,\hat{i} + \cos t\,\hat{j}}_{\text{unit direction vector}}$

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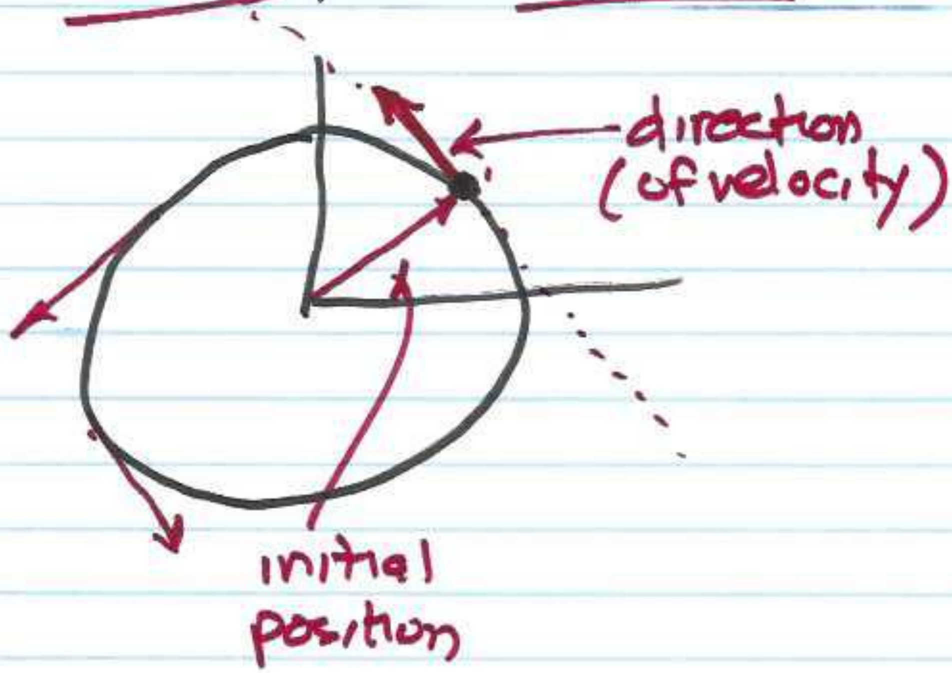
Tangent Line: $l(u) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) + u\left(-\sin t\,\hat{i} + \cos t\,\hat{j}\right)$

$$= \underline{\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)} + u\underline{\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)}$$



Triple Products

A, B, C are vectors ($\cdot$, $\times$)

$(A \times B) \cdot C$ = scalar triple product
→ $A \cdot B \cdot C$ not defined
$A \times B \times C$ = vector triple product

$$\frac{d}{dt}\left(\underbrace{A\times B}_{1^{st}}\cdot C\right) = ?$$

2^{nd}

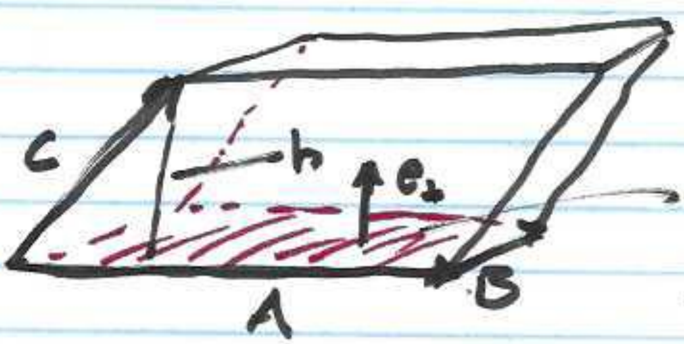
Sidebar $\quad \underbrace{A\cdot B}_{\text{scalar}}\cdot\underbrace{C\cdot D}_{\text{scalar}}$

BAC-CAB

$$(A\times B)\times C = B(A\cdot C) - C(A\cdot B)$$

$A\cdot B\times C = C\cdot A\times B = B\cdot C\times A$ all same

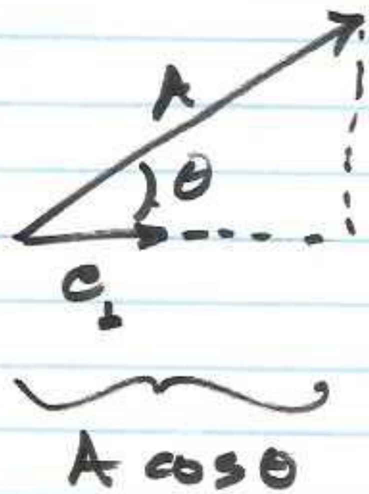
$$A\cdot B\times C = \begin{vmatrix} A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \\ C_1 & C_2 & C_3 \end{vmatrix}$$



$(A\times B)\cdot C = V$

$|A||B|\sin\theta$

$= |A\times B|$



$$A \cdot e_1 = |A||1| \cos\theta$$