95CCM ①

1/12

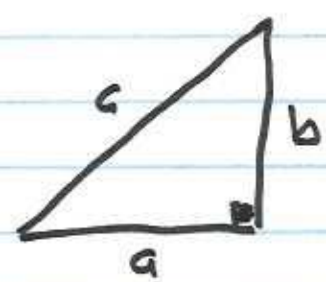
Dot Product: in $\mathbb{R}^3$

$$A = \langle a_1, a_2, a_3 \rangle,\quad B = \langle b_1, b_2, b_3 \rangle$$

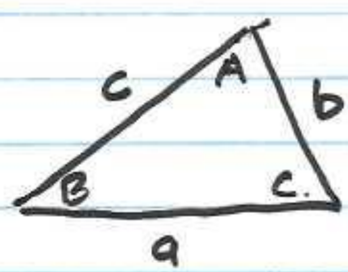
$$A \cdot B \overset{\text{def}}{=} a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$A \cdot B = |A|\,|B| \cos \theta_{ab}$$

Law of Cosines

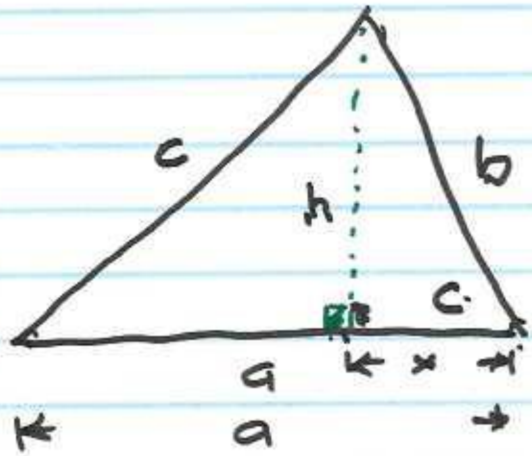


$a^2 + b^2 = c^2$



What is c in general?

(2)



note:

$\frac{x}{b} = \cos C$

$$b^2 = x^2 + h^2$$

$$h^2 = c^2 - (a-x)^2$$

$$h^2 = c^2 - (a^2 - 2ax + x^2)$$

$$b^2 = \cancel{x^2} + c^2 - a^2 + 2ax \cancel{- x^2}$$

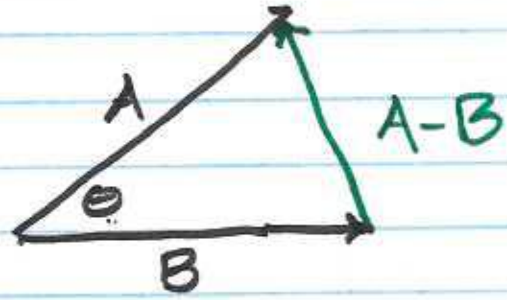
$$a^2 + b^2 - 2ax = c^2$$

$$a^2 + b^2 - 2ab\cos C = c^2$$

Law of Cosines

LOC

$$|x|^2 = x \cdot x$$



$$|A-B|^2 = (A-B)\cdot(A-B) =$$

$$A\cdot A - B\cdot A - A\cdot B + B\cdot B$$

$$|A|^2 - 2A\cdot B \qquad |B|^2$$

= Using LOC

$$|A|^2+|B|^2-2|A||B|\cos\theta = |A-B|^2$$

$$\cancel{|A|^2}+\cancel{|B|^2}-2|A||B|\cos\theta = \cancel{|A|^2}-2A\cdot B+\cancel{|B|^2}$$

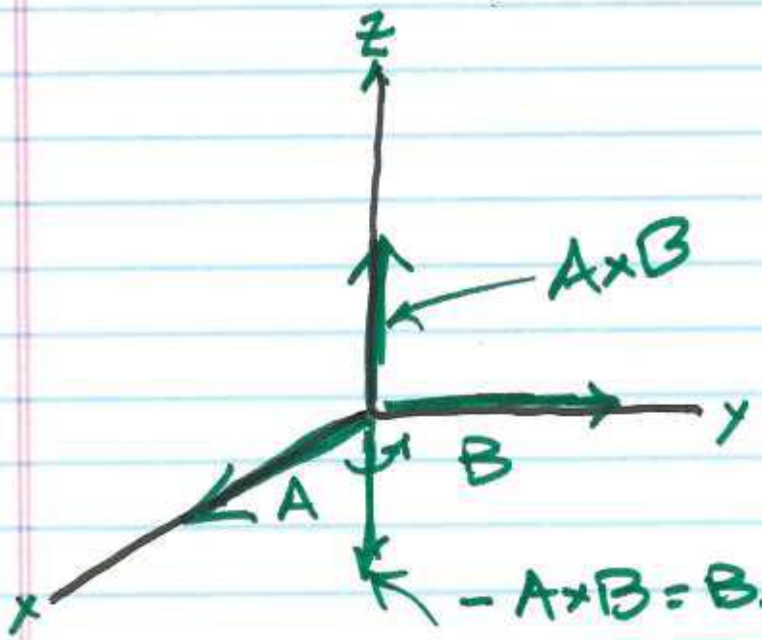
$$-2|A||B|\cos\theta = -2A\cdot B$$

$$A\cdot B = |A||B|\cos\theta$$

Cross · Product

perpendicular unit vector to A : B

$$A \times B = |A||B| \sin\theta \, e_{\perp}$$



$-A \times B = B \times A$ (anticommutative)

$$A \times B = \begin{array}{c l} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \end{vmatrix} & \leftarrow \text{vectors} \\ \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} & \Big\} \text{ real \#s} \end{array}$$

$$\begin{vmatrix} + & - & + \\ - & + & - \\ + & - & + \end{vmatrix}$$

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$\hat{i} = \langle 1, 0, 0 \rangle$ $\hat{j} = \langle 0, 1, 0 \rangle$ $\hat{k} = \langle 0, 0, 1 \rangle$

$$A \times B = \hat{i}(a_2 b_3 - a_3 b_2) - \hat{j}(a_1 b_3 - a_3 b_1) + \hat{k}(a_1 b_2 - a_2 b_1)$$

$$= |A||B| \sin\theta \, e_\perp$$

Prob: $A = \langle 2, 0, 3 \rangle$ $B = \langle 0, 2, 5 \rangle$

Find vector of length 1 $\perp$ to A & B.

So calculate $A \times B$:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 3 \\ 0 & 2 & 5 \end{vmatrix} = ?$$

$$A \times B = \hat{i}(-6) - \hat{j}(10) + \hat{k}(4)$$

$$= \langle -6, -10, 4 \rangle$$

$$|\langle -6, -10, 4 \rangle| = \sqrt{36 + 100 + 16}$$

$$= \sqrt{152} \sim 12.5$$

$$e_{\perp} \frac{1}{12.32} \langle -6, -10, 4 \rangle$$

Convenient Facts: Dot Product detects perpendicularity.

Cross Product detects parallel/antiparallel vectors

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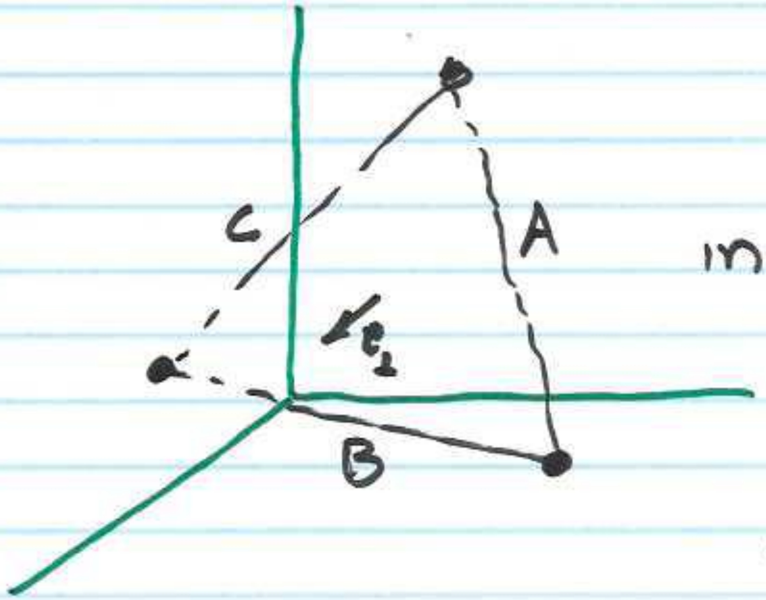
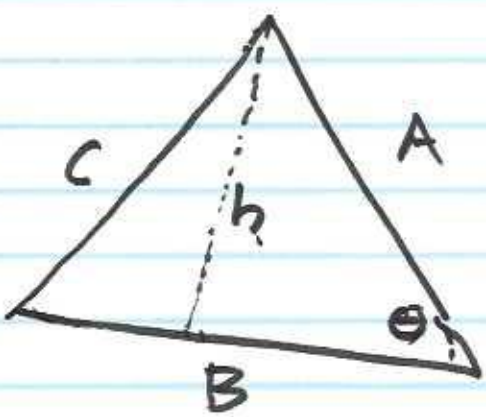


in 3-D

move to



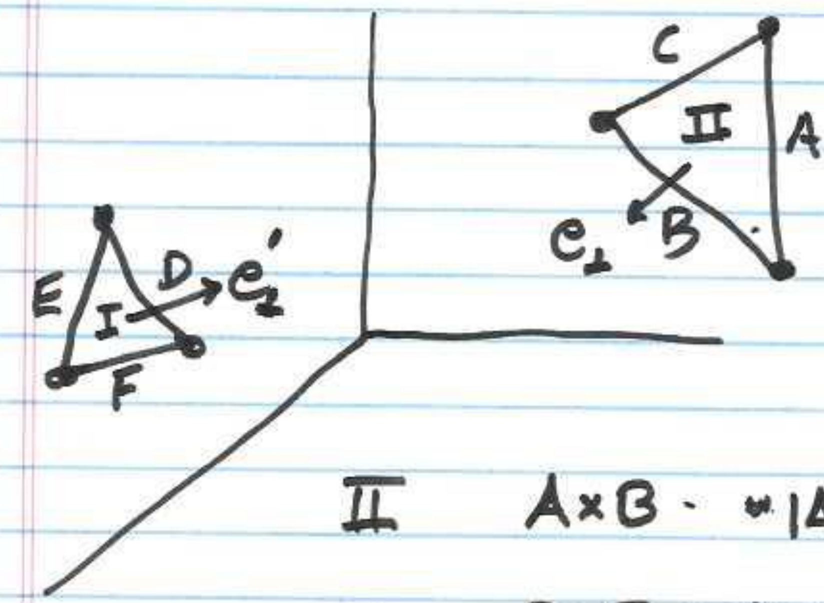
2-D

$$\text{area} = B \cdot \frac{h}{2}$$

<u>but</u> $h = A \, \cancel{\cos}\, \sin\theta$

$$\text{area} = \frac{AB}{2} \sin\theta$$

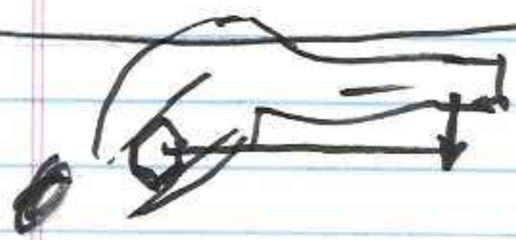
What is $\frac{1}{2}|A \times B| =$



II $\quad A \times B = |A||B| \sin\Theta \, e_\perp$

I $\quad D \times E = |D||E| \sin\varphi \, e'_\perp$

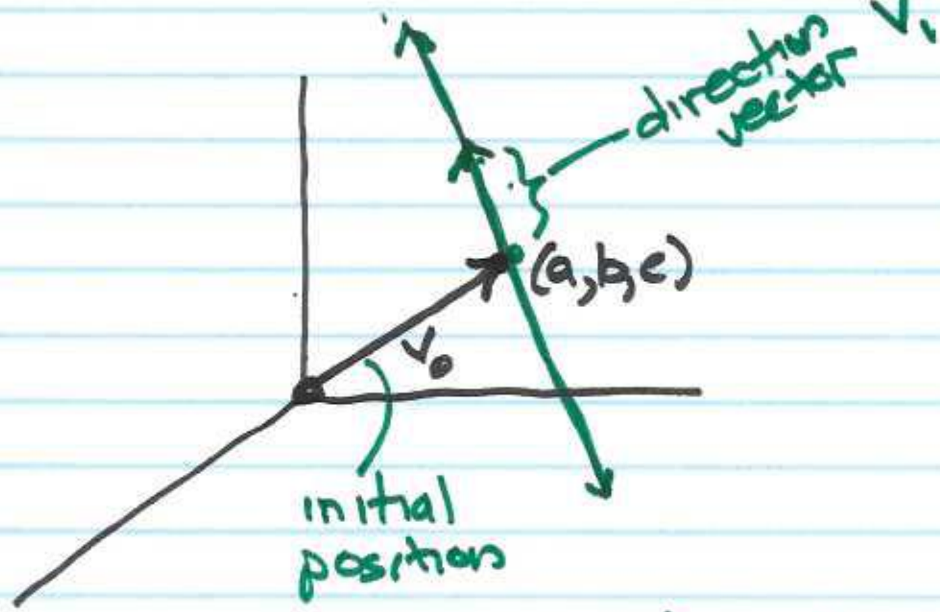
I & II in $\parallel$ planes if $e_\perp = e'_\perp$



$$\tau = r \times F$$

$$F = q\left(E + v \times B\right)$$ Lorentz Force Law

Lines in Space



dir vector

$$L : V_0 + t V_1$$

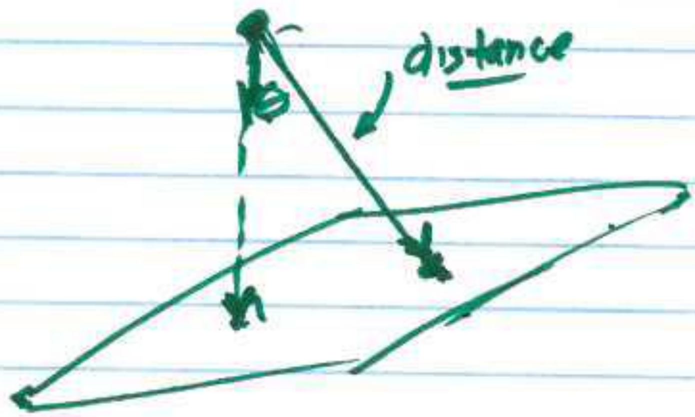
initial

$\in \mathbb{R}$

$$V_0 = \langle a, b, c \rangle$$

$$V_1 = \langle x_1, y_1, z_1 \rangle$$

$$\boxed{\begin{aligned} x(t) &= a + t x \\ y(t) &= b + t y \\ z(t) &= c + t z \end{aligned}}$$


distance
θ
n