M8Z32

1

2/9

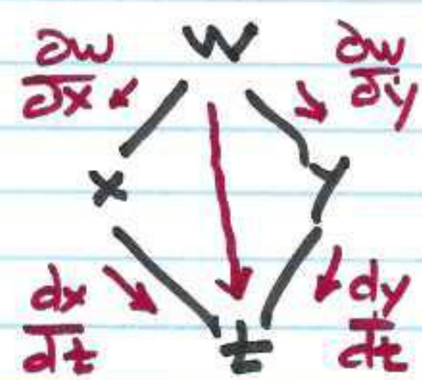
Chain Rule:

Given $f(g(x))$, $[f(g(x))]' = \underbrace{f'(g(x)) \cdot g'(x)}_{\text{2 diff's}}$

In 2-dimensions $w(x(t))$

Want $\frac{dw}{dt}$ "ordinary derivative"

suppose $w = w(x,y)$
& $x = x(t)$ and $y = y(t)$

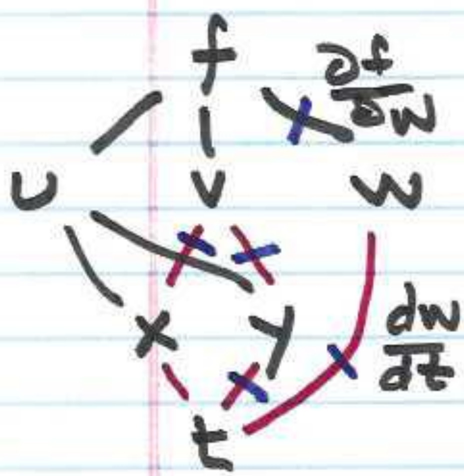


$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt}$$

Prob. [redacted] $f = f(u, v, w)$

$u = u(x, y)$ $v(x, y)$ $w(t)$

$x = x(t)$ $y(t)$



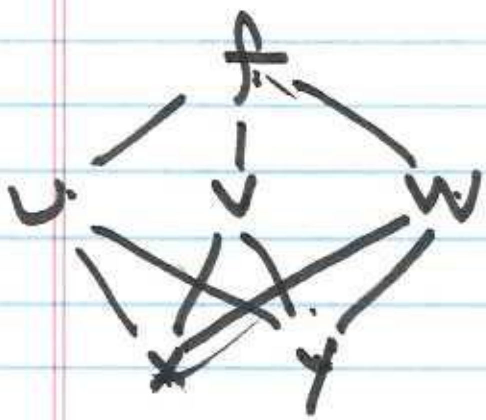
$$\frac{df}{dt} = \frac{\partial f}{\partial w} \cdot \frac{dw}{dt} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x} \cdot \frac{dx}{dt} +$$

$$+ \frac{\partial f}{\partial u} \frac{\partial u}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial u} \frac{\partial u}{\partial x} \frac{dx}{dt} \quad *$$

Case where single ultimate variable appears t.

Prob: $f(u,v,w)$

$u(x,y)$, $v(x,y)$, $w(x,y)$



x $$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial x} + \frac{\partial f}{\partial v}\cdot\frac{\partial v}{\partial x} + \frac{\partial f}{\partial w}\frac{\partial w}{\partial x}$$

y $$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}\cdot\frac{\partial u}{\partial y} + \frac{\partial f}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial f}{\partial w}\cdot\frac{\partial w}{\partial y}$$

$$\Delta f \approx f'(x)\cdot \Delta x \leftarrow$$

Note: $f'(x) = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x} \neq \frac{\partial f}{\partial y}$

$$\pi r^2 h = V(r,h)$$

$$\underline{\frac{\partial V}{\partial r}, \frac{\partial V}{\partial h}} \;\Big|\; \frac{dV}{dr} \quad \frac{dV}{dh}$$

$$\underline{\Delta V = \frac{\partial V}{\partial r}\Delta r + \frac{\partial V}{h}\Delta h}$$

$f(x,y,z)$

$$\underline{\Delta f = \frac{\partial f}{\partial x}\Delta x + \frac{\partial f}{\partial y}\Delta y + \frac{\partial f}{\partial z}\Delta z}$$

5

Suppose can has radius 2" & ht 5"

$V = 4\pi \cdot 5 = 20\pi$ cubic units

Quickly estimate volume if radius increases 10% and height decreases 5%

$$V = \pi r^2 h$$

$$\frac{\partial V}{\partial r} = 2\pi r h \qquad \frac{\partial V}{\partial h} = \pi r^2$$

$$\hookrightarrow 2\cdot\pi\cdot 2\cdot 5 = 20\pi \qquad \hookrightarrow 4\pi$$

$$\Delta V = (20\pi)(.2) - (4\pi)(4.75)$$

$$= \pi(4 - 19) = \text{[illegible] cubic units}$$

$$\Delta V \cong \frac{\partial V}{\partial r}\Delta r + \frac{\partial V}{\partial h}\Delta h$$

$$(20\pi)(.2) + (4\pi)(-.25)$$

$$4\pi - \pi = 3\pi \text{ cu. in.}$$

Guiness Brewery

$$S(A^{1.5}E^{1.2}W^{1.7})$$

A 5% to 7%

E −10%

W −2%

$S = A^2$

$2A$

$S = A^{1.5}$ ___

$= (1.5)A^{.5}$

$$\Delta S = \frac{\partial S}{\partial A}\Delta A + \underbrace{\frac{\partial S}{\partial E}\Delta E + \frac{\partial S}{\partial W}\Delta W}$$

$\cancel{(1.5)A^{.5}\Delta A + (1.2)}$

$$\underbrace{(1.5)A^{1/2}E^{1.2}W^{1.7}}\Delta A$$

(7)

$$\underline{\Delta S} = (1.5)A^{.5}E^{1.2}W^{1.7}\Delta A +$$
$$(1.2)A^{1.5}E^{0.2}W^{1.7}\Delta E +$$
$$(1.7)A^{1.5}E^{1.2}W^{0.7}\Delta W$$

$\Delta A = .02$

$\Delta E = -.10$

$\Delta W = -.02$

$y(x)$

$\cancel{2y^3 + y - x = 0}$

$2y^3 + xy - x^2 = 0$ Want $\frac{dy}{dx}$

$$6y^2 \cdot \frac{dy}{dx} + \left(y + x\frac{dy}{dx}\right) - 2x = 0$$

$$\frac{dy}{dx}\left(6y^2 + x\right) = 2x - y$$

So... $\frac{dy}{dx} = \frac{2x-y}{6y^2+x}$

In 2-D:

Given $F(x,y)$ & note setting $F(x,y)=0$ defines y as a function of x implicitly.

Set $w = F(x,y) = 0$

$$\frac{dw}{dx} = 0 = \frac{\partial F}{\partial x}\cdot\underbrace{\frac{dx}{dx}}_{1} + \frac{\partial F}{\partial y}\cdot\frac{dy}{dx}$$

$$0 = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\cdot\frac{dy}{dx} \Rightarrow$$

$$\frac{dy}{dx} = -\frac{\partial F}{\partial x}\Big/\frac{\partial F}{\partial y} \Rightarrow y'(x) = \frac{-F_x}{F_y}$$

Prob: Given $y^2 - x^2 = \sin xy$, find $\frac{dy}{dx}$

Set $F(x,y) = y^2 - x^2 - \sin xy = 0$

$$\frac{dy}{dx} = \frac{-F_x}{F_y}$$

$$F_x = -2x - (\cos xy)\cdot y$$

$$F_y = 2y - (\cos xy)\cdot x$$

So... $$\frac{dy}{dx} = \frac{2x + y\cos xy}{2y - x\cos xy}$$

$$F(x,y,z) = 0 = F(x,y,f(x,y))$$

$$0 = F_x \underbrace{\frac{\partial x}{\partial x}}_{1} + F_y \underbrace{\frac{\partial y}{\partial x}}_{0} + F_z \frac{\partial z}{\partial x}$$

10

$$0 = F_x + F_z \frac{\partial z}{\partial x}$$

so $\frac{\partial z}{\partial x} = \frac{-F_x}{F_z}$

Likewise $\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$

Recall $z = f(x,y)$

So. $\frac{\partial f}{\partial x} = \frac{-F_x}{F_z}$ $\quad \frac{\partial f}{\partial y} = \frac{-F_y}{F_z}$

$F(x,y,z) = x^3 + z^2 + ye^{xz} + z\cos y = 0$

Want $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}$ @ origin $(0,0,0)$

$F_x = 3x^2 + yze^{xz}$

$F_y = e^{xz} - z \sin y$

$F_z = 2z + xye^{xz} + \cos y$

$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$ $\frac{\partial z}{\partial y} = \frac{-F_y}{F_z}$