7MYNT

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What is gradient?

$\nabla(\ )$ $\quad \text{grad}\,\phi = \nabla\phi$

$$\nabla f(x,y,z) = \hat{i}\, f_x + \hat{j}\, f_y + \hat{k}\, f_z$$

scalar

$$\nabla \cdot \nabla f = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$f_{xx} + f_{yy} + f_{zz}$$ ← scalar

∇ nabla

harp

$f(x,y,z) = \ln(xy)\sin z$ $\quad \hat{i}\, f_x = \frac{1}{(xy)}\, y\cdot\sin z$

$\nabla f =$ →

$$+ \hat{j}\, f_y = \frac{1}{(xy)}\, x\cdot\sin z$$

$$+ \hat{k}\, f_z = \ln(xy)\cos z$$

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$$\nabla\left(e^{xy}\cos yz\right) = ?$$

$\hat{\imath}$ $f_x = y e^{xy} \cos yz$

$+ \hat{\jmath}$ $f_y = x e^{xy} \cos yz + e^{xy}(-\sin yz) z$

$+ \hat{k}$ $f_z = e^{xy}(-\sin yz) y$

$$e^{xy}\langle y\cos yz,\ x\cos yz - z\sin yz,\ y\sin yz\rangle$$

$$\nabla\left(e^{xyz}\sin x + \ln(xy)\cos z\right) \quad ?$$

$$= \nabla\left(e^{xyz}\sin x\right) + \nabla \ln(xy)\cos z$$

$$\nabla(fg) \overset{?}{=} f\nabla g + g\nabla f$$

To find tangent plane :

③

Sphere $= x^2+y^2+z^2=9$

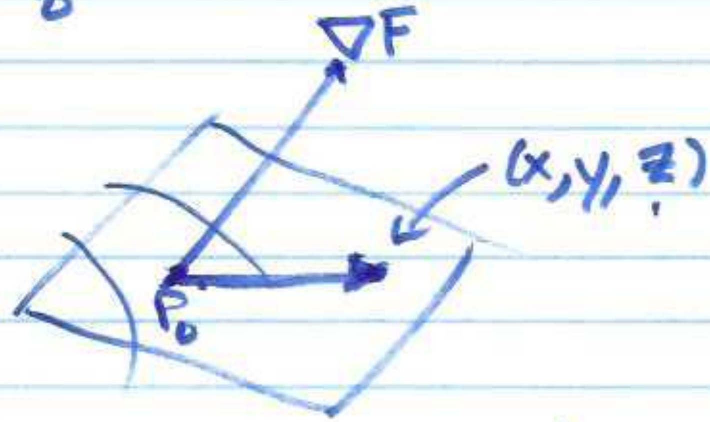
find point on sphere $x=y=1 \Rightarrow z=\sqrt{7}$

point on sphere is $(1,1,\sqrt{7}) = P_0$

$F(x,y,z) = x^2+y^2+z^2-9=0$

$\nabla F = \langle 2x, 2y, 2z \rangle$

$\nabla F \Big|_{P_0} = \langle 2, 2, 2\sqrt{7} \rangle$



$\langle x-1, y-1, z-\sqrt{7} \rangle$

So... $\langle 2,2,2\sqrt{7} \rangle \cdot \langle x-1, y-1, z-\sqrt{7} \rangle = 0$

Good enough $2(x-1)+2(y-1)+2\sqrt{7}(z-\sqrt{7}) = 0$

$2x+2y+2\sqrt{7}z = 2+2+14 = 18$

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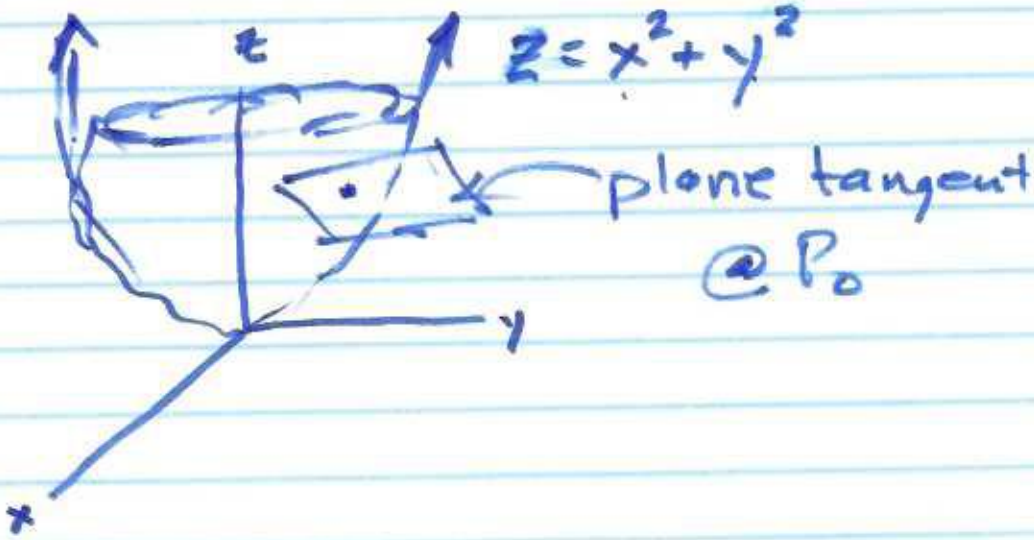
Plane eqn is: $x+y+\sqrt{7}\,z=9$



$$\phi(x,y) = x^2y^3 \qquad \phi(x,y) = \boxed{\begin{matrix}\sin xy\\ \ln xy\end{matrix}}$$

$$\phi_x = 2xy^3$$

$$\phi_y = 3x^2y^2$$

$$\phi_{xy} = (\phi_x)_y = 6xy^2$$

$$\rightarrow \parallel$$

$$\phi_{yx} = 6xy^2$$

$$\phi_{xy} = \frac{\partial^2\phi}{\partial y\,\partial x} \quad \underline{\text{True}}$$

$$\frac{\partial^2\phi}{\partial x\,\partial y}$$

$f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$

Find max/min/saddle points

Script: 1) Calc f_x, f_y

2) Set $f_x = f_y = 0$ & solve for candidate pts. (a,b)

~~3) Eval Δ~~

3) Find f_{xx}, f_{yy}, & f_{xy}

4) Eval $\Delta := f_{xx} f_{yy} - f_{xy}^2$

5) Conclude:

a) If $\Delta(a,b) > 0$ then proceed
b) " " < 0 then saddle
c) " " $= 0$ then quit

6) Look @ f_{xx}, f_{yy}

if $f_{xx} > 0 \Rightarrow$ min
$f_{xx} < 0 \Rightarrow$ max

(a,b) (c,d) 6

x y

$f_x = y - 2x - 2 \quad f_y = x - 2y - 2$

$$\begin{array}{l|l} f_x = y - 2x - 2 = 0 & -2x + y = 2 \quad (1) \\ f_y = x - 2y - 2 = 0 & x - 2y = 2 \quad (2) \end{array}$$

solve simultaneously

$(1) + 2(2)$

$$\begin{array}{r} -2x + y = 2 \\ 2x - 4y = 4 \\ \hline -3y = 6 \end{array} \Rightarrow 3y = 6$$

so $y = \cancel{2}\ -2$ correct

$y = -2$

subs $\cancel{y=2}$ in (2) and we get ~~$x - 4 = 2 \Rightarrow x = 6$~~

~~Candidate is $(6, -2)$~~

$x - 2(-2) = 2, \quad x = -2$

Candidate is $(-2, -2)$

$\Delta(-2,-2) = f_{xx} f_{yy} - f_{xy}^2 = (-2)(-2) - 1^2 = 3 > 0$

$f_{xx} = -2, \quad f_{yy} = -2 \quad f_{xy} = 1$

Conclude there is a max @ $(-2,-2)$

$$f(-2,-2) = (-2)(-2) - (-2)^2 - (-2)^2 - 2(-2) - 2(-2) + 4$$

$$= 4 - 4 - 4 + 4 + 4 + 4 = 8$$

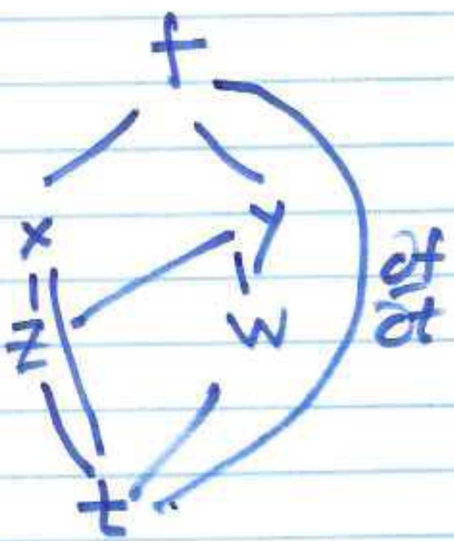
$f(x, y, z, w, t)$

$x = (z, t)$

$y = (z, w)$

$z = (w, t)$

$w = w(t)$



$df = \underline{\qquad\qquad\qquad}$

$$\frac{df}{dt} = \frac{\quad \cdot \quad \cdot \quad \cdot \quad}{\qquad} = \frac{\partial f}{\partial t}$$

$$f(x,t) = 2x + 3t$$

$$x = t^2+1$$

$$\begin{tikzcd} f \arrow[d, "f_x"'] \arrow[dd, bend left, "f_t"] \\ x \arrow[d, "x'(t)"'] \\ t \end{tikzcd}$$

$$df = f_x\, x'(t)\,dt + f_t\, dt$$

$$\frac{df}{dt} = f_x \cdot \frac{dx}{dt} + \boxed{f_t}$$

direct $f(t) = 2(t^2+1) + 3t = 2t^2+3t+2$

$$f'(t) = 4t + 3$$

$$f_x = 2 \quad x'(t) = 2t \quad f_t = 3$$

$$\frac{df}{dt} = 2(2t) + 3 = 4t+3$$