9GK72

① 2/20

In 1-D, given $f(x)$

can write $f(x)$: [redacted] $f(0) + f'(0)(x) + \underline{f''(0)\dfrac{x^2}{2}} + f'''(0)\cdot\dfrac{x^3}{6}$

$f^{(n)}(0)\cdot\dfrac{x^n}{n!} + \cdots$ start @ 0 means this is a Maclaurin series

$$\sin x = 0 + (1)(x) + 0 + -\frac{x^3}{6} + 0 + \frac{x^5}{120} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$(x-0)^n$$

$$\uparrow$$

$$\cdots \frac{(x-a)^n}{n!} \cdots$$

$$n(x,0,1) = \frac{1}{\sqrt{2\pi}} e^{\frac{-(x-0)^2}{2\cdot 1^2}}$$

$$\int e^{x^2}\, dx$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \cdots \frac{x^n}{n!}$$

Let $x = y^2$

$$e^{y^2} = \sum_{n=0}^{\infty} \frac{y^{2n}}{n!} = 1 + y^2 + \frac{y^4}{2!} + \cdots$$

$$\rightarrow \int e^{y^2} dy = \int 1 dy + \int y^2 dy + \frac{1}{2!}\int y^4 dy + \cdots \rightarrow$$

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§14.10 Partials with constraints:

$$F(x,y,u,v) = 0, \; G(x,y,u,v) = 0$$

From these relations, designate x, y as independent & u, v as dependent

(i.e. $u = u(x,y)$ & $v = v(x,y)$)

Suppose you need $\frac{\partial u}{\partial x}$

Method:

Find total differential of F & G w/r/t x, y

(1) $dF = F_x dx + F_y dy + F_u du + F_v dv$

(2) $dG = G_x dx + G_y dy + G_u du + G_v dv$

Note $\left\{ \begin{array}{l} du = u_x dx + u_y dy \\ dv = v_x dx + v_y dy \end{array} \right\}$

(1') $dF = F_x dx + F_y dy + \underbrace{F_u u_x dx + F_v v_x dx}_{w/r/t\ x} +$

$\underbrace{F_u u_y dy + F_v v_y dy}_{w/r/t\ y}$

$$\left\{ \begin{array}{l} 0 = \text{(1'')}\ dF = (F_x + F_u u_x + F_v v_x)dx + (F_y + F_u u_y + F_v v_y)dy \\ 0 = \text{(2'')}\ dG = (G_x + G_u u_x + G_v v_x)dx + (G_y + G_u u_y + G_v v_y)dy \end{array} \right.$$

By independence of x & y, coefficients vanish

$$\Rightarrow \left.\begin{aligned} &F_x + F_u u_x + F_v v_x = 0 \\ &F_y + F_u u_y + F_v v_y = 0 \\ &G_x + G_u u_x + G_v v_x = 0 \\ &G_y + G_u u_y + G_v v_y = 0 \end{aligned}\right\} \text{4 eqns.}$$

$$\boxed{\begin{aligned} ax + by &= c \\ gx + dy &= f \end{aligned}}$$

$$\left.\begin{aligned} F_u u_x + F_v v_x &= -F_x \\ G_u u_x + G_v v_x &= -G_x \end{aligned}\right\} \text{system \# 1}$$

$$\left.\begin{aligned} F_u u_y + F_v v_y &= -F_y \\ G_u u_y + G_v v_y &= -G_y \end{aligned}\right\} \text{system \# 2}$$

Aside: Cramer's Rule - (2-D)

Given $\left.\begin{array}{l} ax + by = c \\ dx + ey = f \end{array}\right\}$ 2 eqns, 2 unk

$$D = \begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{vmatrix} a & b \\ d & e \end{vmatrix} \end{array} = \underline{ae - bd}$$

$$D_x = \begin{vmatrix} c & b \\ f & e \end{vmatrix} = ce - bf$$

$$D_y = \begin{vmatrix} a & c \\ d & f \end{vmatrix} = af - cd$$

$$x = \frac{D_x}{D} \qquad y = \frac{D_y}{D}$$

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For system #1

$$\rightarrow D_1 = \begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}$$

$$D_{u_x} = \begin{vmatrix} -F_x & F_v \\ -G_x & G_v \end{vmatrix}$$

$$\rightarrow u_x = \frac{D_{u_x}}{D}$$

$$D_{v_x} = \begin{vmatrix} F_u & -F_x \\ G_u & -G_x \end{vmatrix} \Rightarrow v_x = \frac{D_{v_x}}{D}$$

For system #2

$$D_1 = D_2$$

$$D_2 = \begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}$$

$$D_{u_y} = \begin{array}{c} \begin{matrix} u & \quad v \end{matrix} \\ \begin{vmatrix} -F_y & F_v \\ -G_y & G_v \end{vmatrix} \end{array} \Rightarrow u_y$$

$$D_{v_y} = \begin{vmatrix} F_u & -F_y \\ G_u & -G_y \end{vmatrix} \Rightarrow v_y$$

Example: $u^2 - v = 3x + y$

$u - 2v^2 = x - 2y$

$$F(x,y,u,v) = u^2 - v - 3x - y = 0$$

$$G(x,y,u,v) = u - 2v^2 - x + 2y = 0$$

$$u_x = \frac{\begin{vmatrix} -F_x & F_v \\ -G_x & G_v \end{vmatrix}}{\begin{vmatrix} F_u & F_v \\ G_u & G_v \end{vmatrix}} \quad \leftarrow D_{u_x}$$

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$F_x = -3 \qquad F_v = -1 \qquad G_x = -1 \qquad G_v = -4v$

$F_u = 2u \qquad\qquad\qquad\qquad G_u = 1$

$$U_x = \frac{\begin{vmatrix} 3 & -1 \\ 1 & -4v \end{vmatrix}}{\begin{vmatrix} 2u & -1 \\ 1 & -4v \end{vmatrix}} = \boxed{\frac{1 - 12v}{1 - 8vu}} = U_x$$

$(1 - 8vu) \neq 0$

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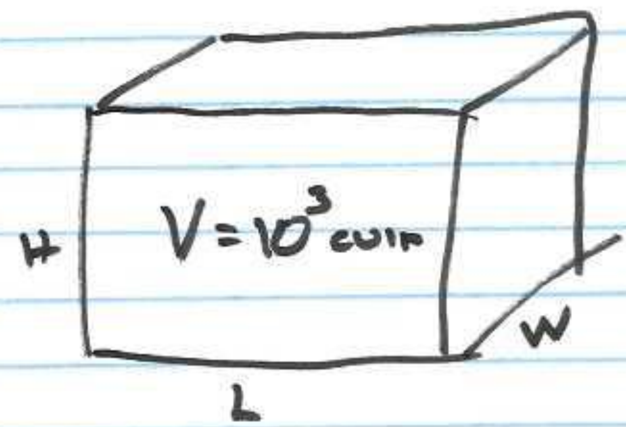

Minimize surface ?

Note: $L \cdot W \cdot H = 10^3$

$$H = \frac{10^3}{LW}$$

Obj. fcn: Surface area $= 2\left(HL + HW + LW\right)$

$$= 2\left(\frac{10^3 \cancel{L}}{\cancel{L}W} + \frac{10^3 \cancel{W}}{L\cancel{W}} + LW\right)$$

$$\rightarrow \quad = 2\left(\frac{10^3}{W} + \frac{10^3}{L} + LW\right)$$