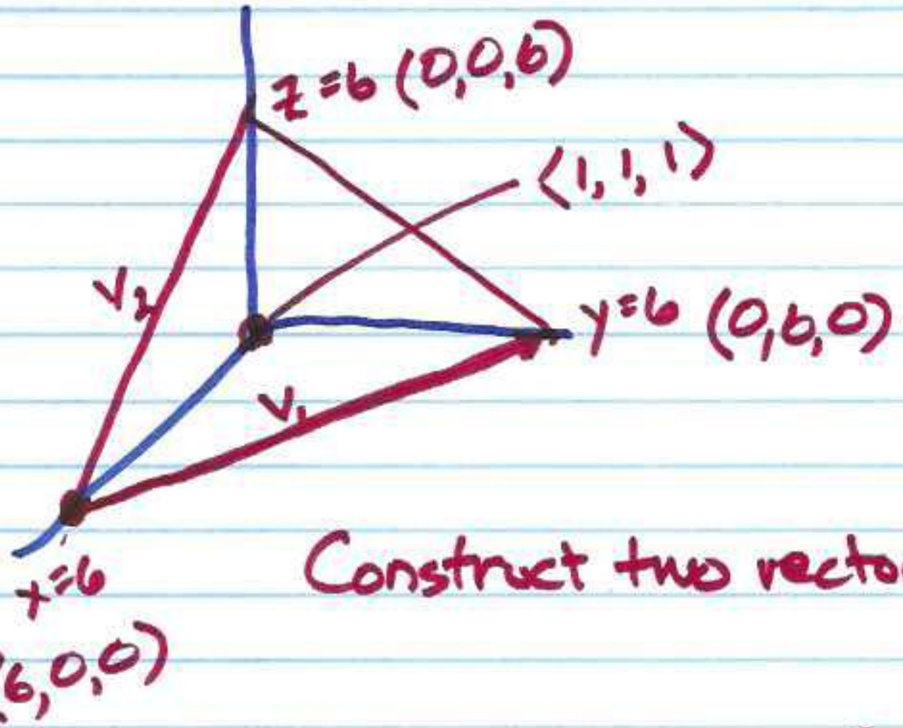
LRJ5W

1

2/18



Construct two vectors

$Ax+By+Cz = D$ (std plane)

Shortcut: $Ax=D \Rightarrow A\cdot 6 = D$

$By = D \Rightarrow B\cdot 6 = D$

$Cz = D \Rightarrow C\cdot 6 = D$

$Ax + \cancel{B}y + Az = D$

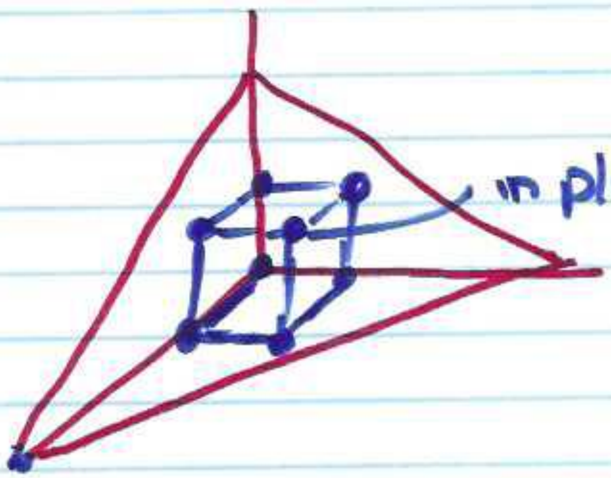
$A(x+y+z) = D$

$x+y+z = D$

$x=6;\ y=0,\ z=0 \Rightarrow D = 6$

2

Eqn of plane is $x+y+z=6$



Note: $f(x,y) = xyz$ where $z = 6-x-y$

$$= xy(6-x-y)$$

$$f(x,y) = 6xy - x^2y - xy^2$$

$f_x = 6y - 2xy - y^2$ | $f_y = 6x - x^2 - 2xy$

$$6y - 2\cancel{x}y - y^2 = 6x - x^2 - 2\cancel{x}y$$

(*) $6y - y^2 = 6x - x^2$

$\{0\}$ gives $V=0$

3

$$y^2-6y+9 = x^2-6x+9$$

$$(y-3)^2 = (x-3)^2$$

any pt (a,a) works

If we acknowledge (x,x) $\leftarrow y=x$ is always a solution, then $z = 6-2x$

$(2,2,2)$ being the candidate

$f(x,y,z) = xyz = xy(6-x-y)$

$f_{xx} = -2y \quad f_{yy} = -2x$

$f_{xy} = 6-2x-2y$

$f_{xx}(2,2) = -4 \; ; \; f_{yy}(2,2) = -4$

$f_{xy}(2,2) = -2$

$\Delta(2,2) = (-4)(-4)-(-2)^2 = 12 > 0$

so we have a max volume @ $(2,2,2)$

Lagrange Multipliers

Given an objective function $f(x,y)$

and the constraint function $g(x,y) = 0$
$g(x,y,z)$

Form the auxiliary function:

$\phi = \lambda f + g$ or $\boxed{\phi = f + \lambda g}$

then calculate

$\phi_x, \phi_y, \phi_\lambda = 0$

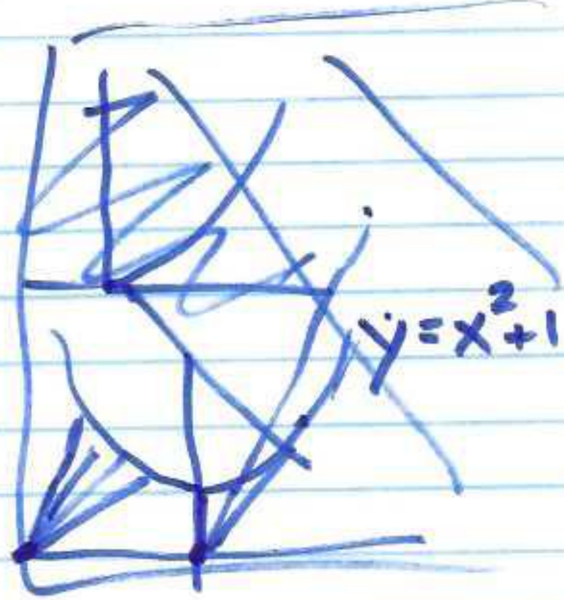
Solve simultaneously for candidate(s)

that are extrema. Done w/ math but

Check solution for realism.

⑤

Problem: Given the equation of the hyperbola $\underline{x^2 + 8xy + 7y^2 = 225}$, find the shortest distance from the origin to the curve.



$f(x,y) = \underline{x^2 + y^2}$

this is to be minimized

So objective fcn is $f(x,y) = x^2 + y^2$

constraint fcn is $g(x,y) = x^2 + 8xy + 7y^2 - 225$

auxilary function is $\phi(x,y) = \underline{\lambda f(x,y) + g(x,y)}$

⑥

$$\phi(x,y,\lambda) = \lambda(x^2+y^2) + x^2 + 8xy + 7y^2 - 225$$

$$\left.\begin{aligned} \phi_x &= 2x\lambda + 2x + 8y = 0 \\ \phi_y &= 2y\lambda + 8x + 14y = 0 \end{aligned}\right\} \text{solve simultaneously}$$

$$\phi_\lambda = x^2 + y^2$$

simplified

$$\phi_x : 2x(\lambda+1) + 8y = 0 \quad \Big| \quad x(\lambda+1) + 4y = 0$$

$$\phi_y : 2y(\lambda+7) + 8x = 0 \quad \Big| \quad y(\lambda+7) + 4x = 0$$

$$\underbrace{\begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \begin{bmatrix} \lambda+1 & 4 \\ 4 & \lambda+7 \end{bmatrix} \end{array}}_{\text{must be } 0} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$(\lambda+1)(\lambda+7) - 16 = 0$$

$$\lambda^2 + 8\lambda + 7 - 16 = \lambda^2 + 8\lambda - 9 = 0$$

$$(\lambda+9)(\lambda-1) = 0 \Rightarrow$$

$$\lambda \in \{-9, +1\}$$

Case I ($\lambda = +1$)

$\phi_x : 2x(2) + 8y = 0$

$= 4x = -8y$ or $x = -2y$

$g(x,y) = x^2 + 8xy + 7y^2 - 225 = 0$

Set $x = -2y$: $4y^2 - 16y^2 + 7y^2 = 225$

$y^2(4 - 16 + 7) = 225$

$-5y^2 = 225$

$\lambda = 1$ does not give realistic solution.

Case II ($\lambda = -9$)

$\phi_x = -16x + 8y = 0 \Rightarrow y = 2x$

$g(x,y)$ with $y = 2x$ gives

$x^2 + 16x^2 + 28x^2 = 225$

$45x^2 = 225$

$x^2 = 5$ so $x = \sqrt{5}$

$\Rightarrow y^2 = 20$

then $f(x,y) = x^2 + y^2 = 25$

and we conclude min. dist. is 5