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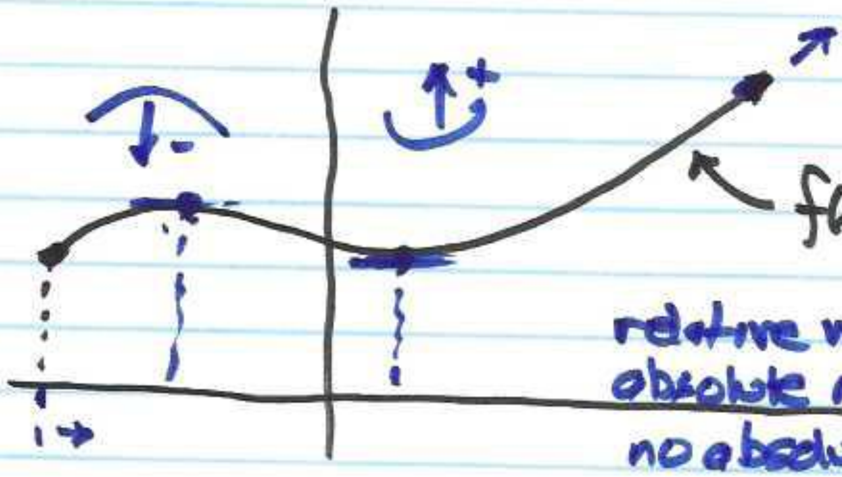
2/16

Optimization in 2 (or more) variables:

Recall procedure for 1-dimension



Calculate $f'(x)$

Look at where $f'(x) = 0$ or is undefined.

These are critical pts.

Need to check max/mins @ all

critical points & boundary pts.

To decide ("convexity") shape of curve

at critical points ($f'(x) = 0$)

Calculate $f''(x)$ @ each pt. where

$f'(x) = 0$. Say a is crit. pt.

If $f''(a) > 0 \Rightarrow$ min

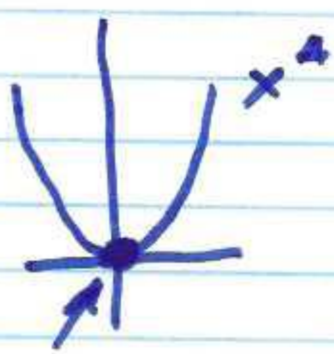
$f''(a) < 0 \Rightarrow$ max

$f''(a) = 0$ no conclusion –

based on convexity. Try testing

points in neighborhood of a.

Ex. $f(x) = x^2$



$f(x) = x^4$

$f'(x) = 4x^3 = 0$

$\Rightarrow x = 0$

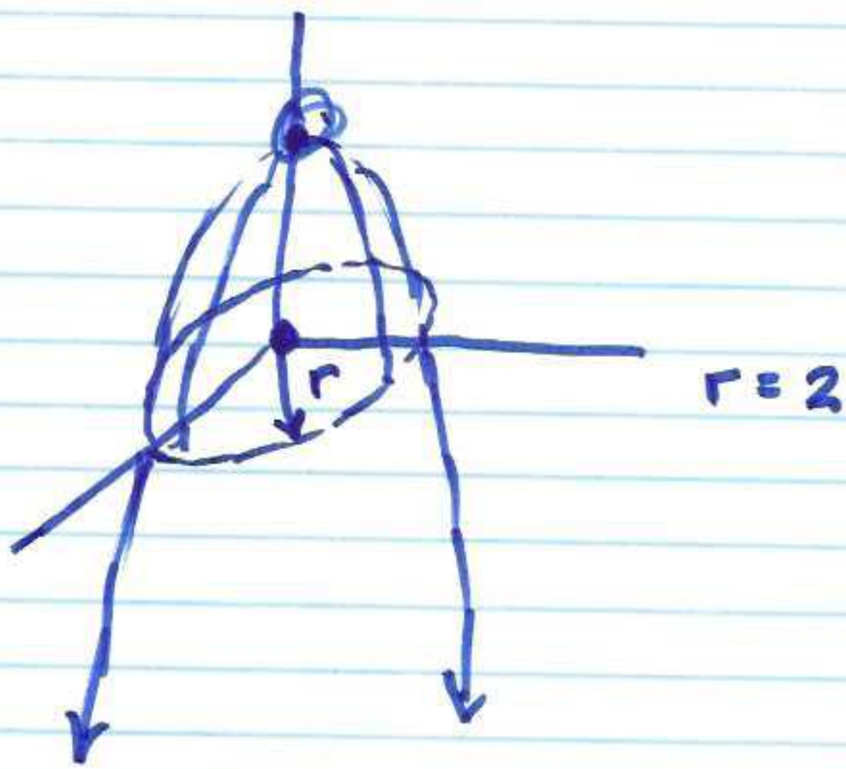
$f''(0) = 12x^2\Big|_{x=0} = 0$

move $+\varepsilon$ to right f inc

" $-\varepsilon$ to left "

Try 2-D:

$$z = f(x,y) = 4 - x^2 - y^2$$



Step 1: Calculate f_x & f_y

Step 2: Solve $f_x = 0$ $f_y = 0$ for candidate point(s)

Step 3: For each candidate point (x_0, y_0)

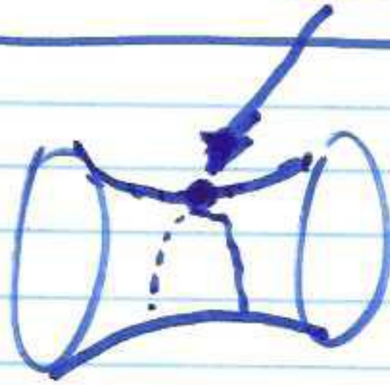
So we get f_{xx} & f_{yy} ... but also

$f_{xy} = f_{yx}$

Step 4: Calculate $\Delta := f_{xx} f_{yy} - f_{xy}^2$

Aside: Why do we need Δ?

To make sure we have max's or min's in both directions and not a "saddle point"



If $\Delta > 0$, then we have max or min

$\Delta = 0$, no conclusion / test nbhd

$\Delta < 0$, saddle point

Now... Step 5 (having confirmed $\Delta > 0$)

Conclude: If f_{xx} (or f_{yy}) > 0

candidate point is relative min.

If f_{xx} (or f_{yy}) < 0, rel. max. Done!

For given problem:

$f_x = -2x$; $f_y = -2y$

$-2x = 0 \Rightarrow x = 0$; $-2y = 0 \Rightarrow y = 0$

$f_{xx} = -2$; $f_{yy} = -2$ $f_{xy} = f_{yx} = 0$

$\Delta = (-2)(-2) - 0^2 = +4 > 0$

This implies $(0,0)$ is relative max

hence $f(0,0) = 4$.

What about $f(x,y,z)$

f_x, f_y, f_z set $= 0$ & solve

$f_{xx}, f_{yy}, f_{zz}, f_{xy}, f_{xz}, f_{yz}$

$$
\begin{array}{c|ccc|}
 & x & y & z \\
x & \boxed{f_{xx}} & f_{xy} & f_{xz} \\
y & f_{yx} & f_{yy} & f_{yz} \\
z & f_{zx} & f_{zy} & f_{zz}
\end{array}
\leftarrow
$$

f_{xx} $\qquad$ n

$f(x_1, \cdots, x_n)$

53) Find three numbers whose som is 9 and whose sum of squares is a minimum.

$x,\ y,\ 9-x-y \qquad [9-(x+y)]^2 \qquad (a-b)^2$

18

$$S(x,y) = \cancel{x^2} + \cancel{y^2} + \left(81 - \cancel{(x+y)} + (\cancel{x^2} + \cancel{2xy} + \cancel{y^2}\right)$$

$$= 2x^2 + 2y^2 - 2x - 2y + 2xy + 81$$

$$S_x = 4x - 2 + 2y = 0$$

$$S_y = 4y - 2 + 2x = 0$$

1) $4x + 2y = 2$	$2x + y = 1$
2) $4y + 2x = 2$	$2y + x = 1$

$y = 1 - 2x$ from (1)

$2(1-2x) + x = 1$

$2 - 4x + x = 1$ or $1 = 3x$ $\left\{ \begin{array}{l} x = \frac{1}{3} \\ y = \frac{1}{3} \end{array} \right\}$

$S_{xx} = 4$, $S_{yy} = 4$, $S_{xy} = 2 = S_{yx}$

$\Delta = (4)(4) - 2^2 = 12 > 0$

So $x = y = \frac{1}{3}$ gives min

Three #'s are $\frac{1}{3}, \frac{1}{3}, 8\frac{1}{3}$

$$\frac{1}{9} + \frac{1}{9} + \frac{625}{9} = \frac{627}{9}$$

NO FORGET THIS

$S(x,y) = x^2 + y^2 + (81 - 18(x+y) + (x^2 + 2xy + y^2)$

$= 2x^2 + 2y^2 - 18x - 18y + 2xy + 81$

1) $S_x = 4x - 18 + 2y = 0$

2) $S_y = 4y - 18 + 2x = 0$

1') $2x - 9 + y = 0$

2') $2y - 9 + x = 0$

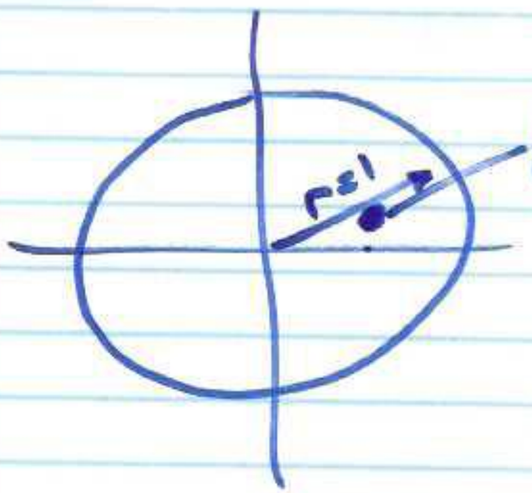
9

from 1') $y = 9-2x$

into 2') $2(9-2x)-9+x=0$

$18-4x-9+x=0$

$9-3x=0 \Rightarrow \underline{x=3}$



$T(x,y) = x^2+2y-x$

$\frac{1}{4}-\frac{1}{2} = -2y$

$-\frac{1}{2} = -2y$

$T_x = 2x-1 \quad T_y = 2$

$T_{xx} = 2 \quad T_{xy} = 0$

candidate pts. $\left(\frac{1}{2},\frac{1}{4}\right)$ ← test adjacent values

~~$T_{xx} = 2$~~

$\Delta = 0 - \underline{0 = 0}$