B8M7F

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2/13

Fun with gradients: (14.5/6)

$$\nabla f(x,y,z) := \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right\rangle$$

Suppose $f(x,y)$ is constant on the curve $\mathbf{r}' : \underline{g'(t)\,\hat{\imath} + h'(t)}\,\hat{\jmath}$ (i.e.) it is a level curve - think a path on a hill that is always at the same height.

← red is velocity

$$\frac{d}{dt}\left(f[(g(t)), h(t)]\right) = (\text{by rules}) =$$

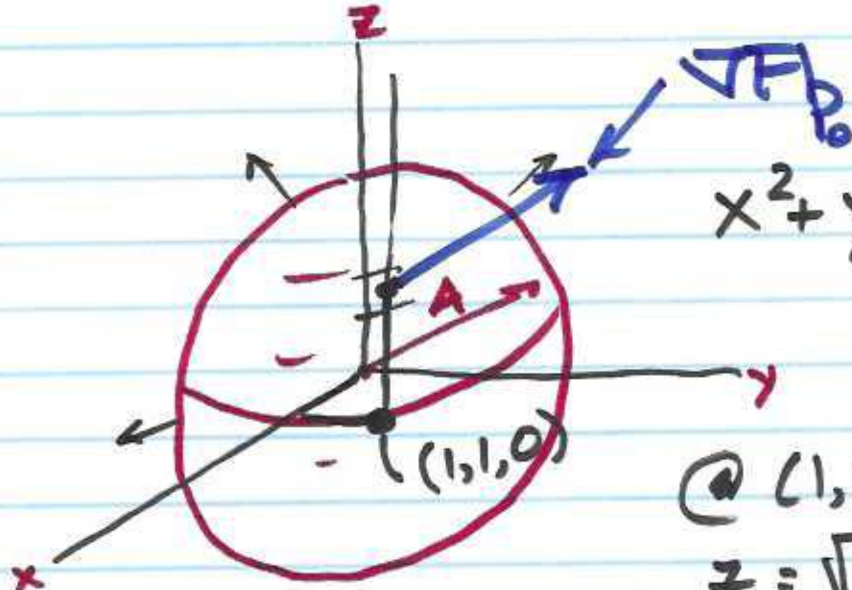
$$\star \quad \frac{\partial f}{\partial g}\frac{dg}{dt} + \frac{\partial f}{\partial y}\frac{dh}{dt} = \frac{d(c)}{dt} = 0.$$

Note: this can be written

$$\left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle \cdot \left\langle \frac{dg}{dt}, \frac{dh}{dt} \right\rangle = 0$$

↑ gradient oper. ↑ velocity

Tangent Plane to Surface



$$x^2+y^2+z^2=16$$

@ (1,1,0)

$$z=\sqrt{14}$$

Point on sphere is $\left(1,1,\sqrt{14}\right)=P_0$

Find equation of plane tangent to sphere @ P_0.

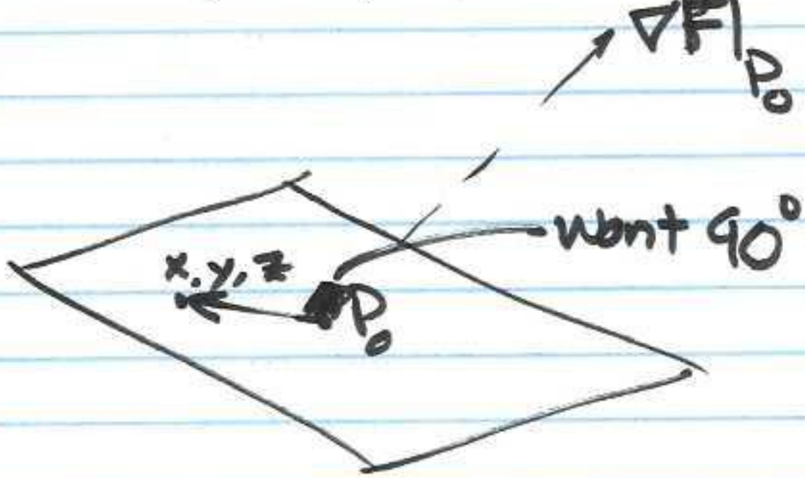
Let $F(x,y,z)=x^2+y^2+z^2-16=0$

Claim: ∇F is $\perp$ to surface of sphere

$$\nabla F=\langle 2x,2y,2z\rangle=2\langle x,y,z\rangle$$

$$\nabla F\Big|_{P_0}=\langle 2,2,2\sqrt{14}\rangle$$

Generic vector in tangent plane can be written as $\langle x-1, y-1, z-\sqrt{14}\rangle = V_0$



Set $\nabla F|_{P_0} \cdot V_0 = 0$ means

$$\langle 2, 2, 2\sqrt{14}\rangle \cdot \langle x-1, y-1, z-\sqrt{14}\rangle = 0$$

$$= 2(x-1) + 2(y-1) + 2\sqrt{14}\,(z-\sqrt{14}) = 0$$

$$= 2x + 2y + 2\sqrt{14}\,z = 2 + 2 + 28$$

$$= x + y + \sqrt{14}\,z = 16$$ ← Eqn of tangent plane

5 $\cos \pi x - x^2 y + e^{xz} + yz = 4$

$P_0 = (0,1,2)$

$1 - 0 + 1 + 2 = 4$ ✓

So we have verified pt is on surface

$F(x,y,z) = \cos(\pi x) - x^2 y + e^{xz} + yz - 4 = 0$

$\nabla F\Big|_{P_0} = \Big\langle -\pi \sin \pi x - 2xy + ze^{xz}, -x^2 + z,$
$xe^{xz} + y \Big\rangle \Big|_{P_0}$

$\nabla F\Big|_{(0,1,2)} = \langle 2, 2, 1 \rangle$

plane $\langle x-0, y-1, z-2 \rangle$

Then: $2x + 2(y-1) + (z-2) = 0$

$2x + 2y - 2 + z - 2 = 0 \Rightarrow 2x + 2y + z = 4$

Linearization:

Def^n The function $f(x,y)$ @ point $P_0 = (x_0, y_0)$

when $f(x,y)$ is differentiable @ P_0

can be (linearly) approximation

$$f(x,y) \approx L(x,y) = f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

this is the standard linear approx. of f.

Prob: Linearize $f(x,y) = x^2 - xy + \frac{y^2}{2} + 3$

$P_0 = (3,2)$

$f(3,2) = 3^2 - 3\cdot 2 + \frac{2^2}{2} + 3 = 8$

$f_x(x,y) = 2x - y \mid f_x(3,2) = 4$

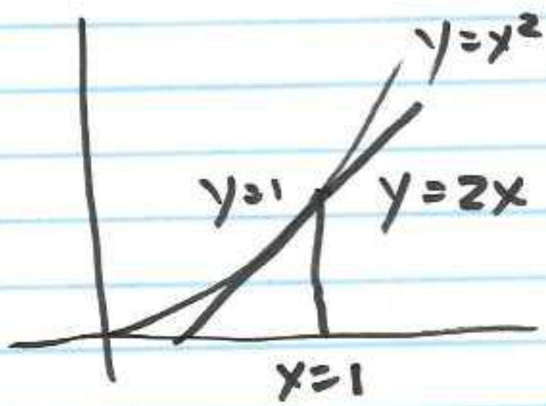
$f_y(x,y) = -x + y \mid f_y(3,2) = -1$

$$L_f(x,y) = 8 + 4(x-3) - (y-2)$$
$$= 8 + 4x - 12 - y + 2$$
$$\cancel{= 4x - y} \quad 4x - y - 2$$

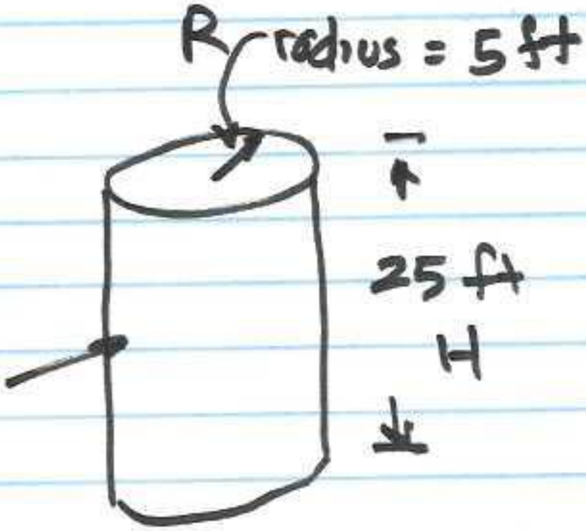
Differentials

The total differential of $f(x,y,z)$

$$df := f_x\,dx + f_y\,dy + f_z\,dz$$



$$y = (1+h)^2$$
$$= 1 + \underline{\underline{2h}} + h^2$$



$$V = \pi R^2 H$$

$$dV = \pi 2RH \cdot dR + \pi R^2 dH$$

$$= 250\pi\, dR + 25\pi\, dH$$

$$= 25\pi(10\,dR + dH)$$

Wilson Lot Size Formula

$$Q = \sqrt{2KM/h} = \left(\frac{2KM}{h}\right)^{1/2}$$

K : cost of placing order
M : weekly sales
h = " inventory (holding) cost

$$P_0 = (K_0, M_0, h_0) = (2, 20, .05)$$

$$dQ = Q_K\,dK + Q_M\,dM + Q_h\,dh$$

$$Q_K = \frac{1}{2}\left(\frac{2KM}{h}\right)^{-\frac{1}{2}} \cdot \left(\frac{2M}{h}\right) \sim \frac{1}{80}(800) = \cancel{[illegible]} \quad \boxed{=10}$$

$$Q_M = \frac{1}{2}\left(\frac{2KM}{h}\right)^{-1/2} \left(\frac{2K}{h}\right) \sim \frac{1}{80}(8)$$

$$Q_h = \frac{1}{2}\left(\frac{2KM}{h}\right)^{-1/2} \cdot \left(\frac{-2KM}{h^2}\right) \sim \frac{1}{80}\,\overset{32{,}000}{\cancel{([illegible])}}$$

$$dQ\Big|_{P_0} = \left(\frac{1}{2}\left(\frac{2 \cdot 2 \cdot 20}{.05}\right)^{-1/2} \cdot \right.$$

$$= \left(\frac{1}{2}\,\frac{1}{40}\right)$$

$$\left.\begin{array}{l} Q_K = 10 \\ Q_M = .1 \\ Q_h = 400 \end{array}\right\} ?!$$