J29A2

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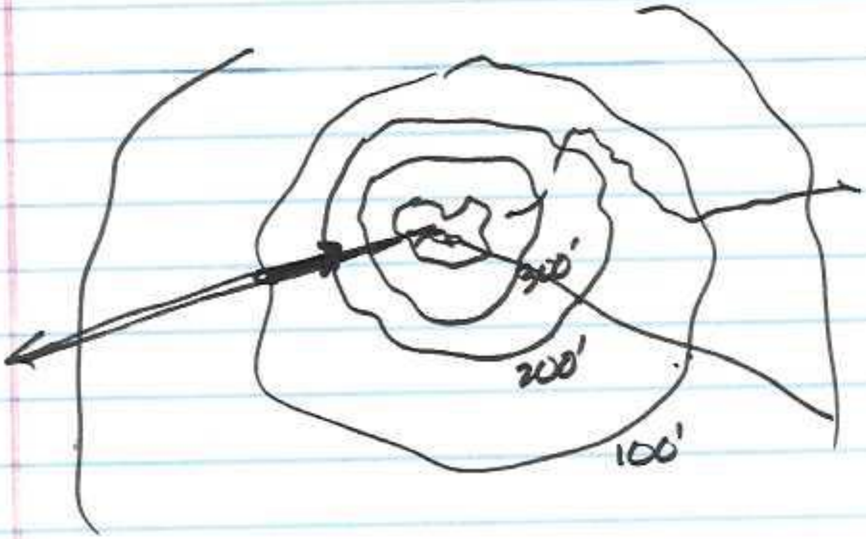
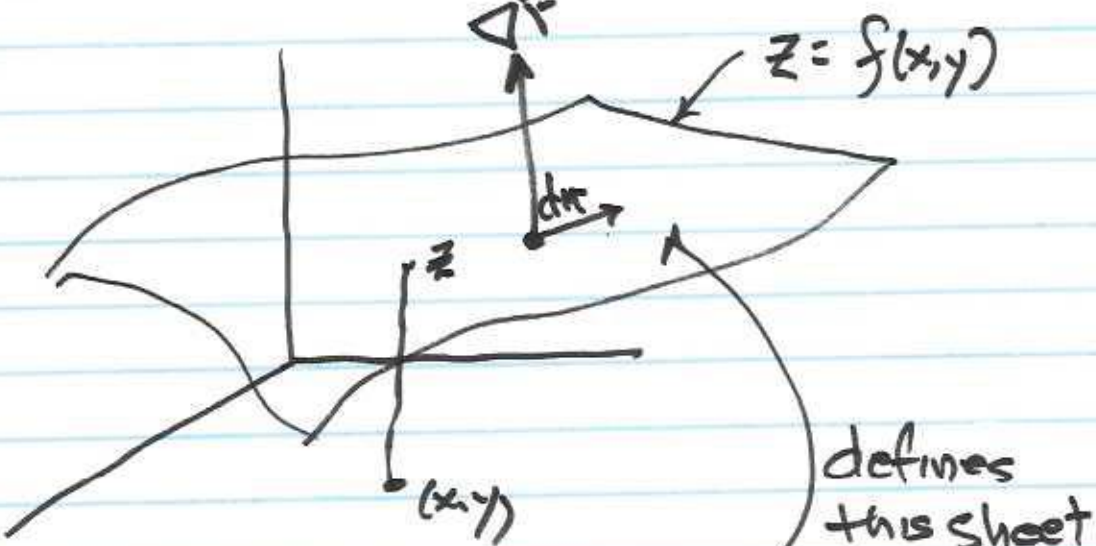
The Gradient (§ 14.5)





$$z - f(x,y) = 0 \Rightarrow F(x,y,z) = 0$$

$$\Delta F = \frac{\partial F}{\partial x}\Delta x + \cdots$$

$$dF = \frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz$$

$$r = x\hat{i} + y\hat{j} + z\hat{k}$$

$$dr = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$ ← differential motion in sheet

If we create the vector

$$\underbrace{\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)}_{\nabla}$$ ← gradient vector

→ $$\nabla F = \hat{i}\frac{\partial F}{\partial x} + \hat{j}\frac{\partial F}{\partial y} + \hat{k}\frac{\partial F}{\partial z}$$

in plane

$$\nabla F \cdot dr = 0$$ ←

$$\frac{\partial F}{\partial x}dx + \frac{\partial F}{\partial y}dy + \frac{\partial F}{\partial z}dz = 0 = dF$$

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

Rules for gradients:

1) $\nabla(F \pm G) = \nabla F \pm \nabla G$

2) $\nabla(cF) = c\nabla F$

3) $\nabla(FG) = F\nabla G + G\nabla F$

4) $\nabla\left(\frac{F}{G}\right) = \frac{G\nabla F - F\nabla G}{G^2}$

Get ∇

7) $f(x,y,z) = x^2 + y^2 - 2z^2 + z\ln x$

@ $(1,1,1)$ $\leftarrow$

$$\hat{i}\left(2x + \frac{z}{x}\right) + \hat{j}\left(2y\right) + \hat{k}\left(-4z + \ln x\right)$$

@ $(1,1,1)$ $\nabla f_{(1,1,1)} = 3\hat{i} + 2\hat{j} + -4\hat{k}$

10 [redacted]) $f(x,y,z) = e^{x+y}\cos z + (y+1)\arcsin x$

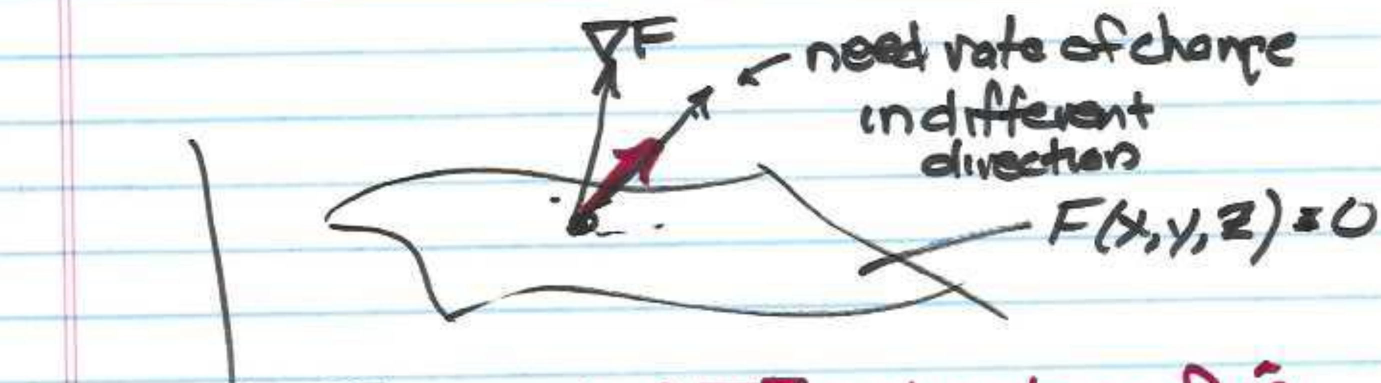
eval @ $(0,0,\frac{\pi}{6})$

$$\nabla f = \hat{\imath}\left[\left(e^{x+y}\cos z + (y+1)\left(\frac{1}{\sqrt{1-x^2}}\right)\right] +$$

$$\hat{\jmath}\left(e^{x+y}\cos z + \arcsin x\right) +$$

$$\hat{k}\left(e^{x+y}(-\sin z)\right)$$

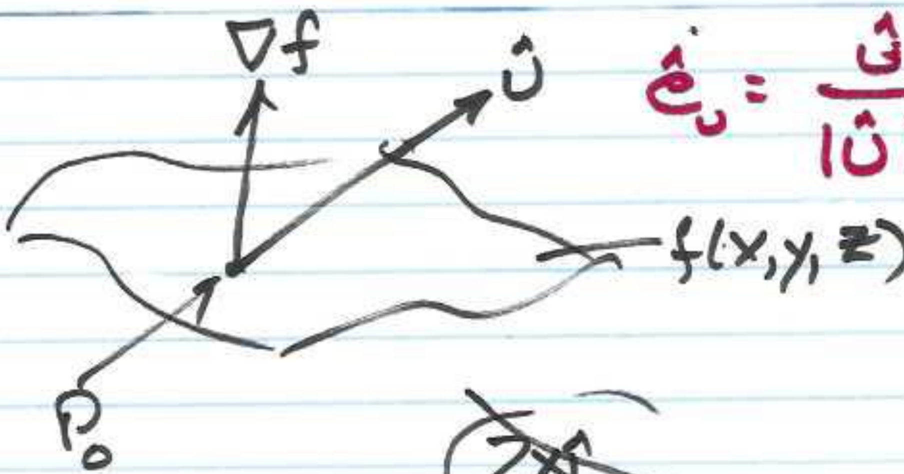
$$\nabla f_{(0,0,30^\circ)} = \hat{\imath}\left(\frac{\sqrt{3}}{2}+1\right) + \hat{\jmath}\left(\frac{\sqrt{3}}{2}\right) + \hat{k}\left(-\frac{1}{2}\right)$$



component of ∇F in direction of $\hat{u}$ is the "directional derivative"

16) $f(x,y,z) = x^2 + 2y^2 - 3z^2$

$P_o(1,1,1)$ $\quad \hat{u} = 3\hat{i} + 6\hat{j} - 2\hat{k}$



$\hat{e}_u = \dfrac{\hat{u}}{|\hat{u}|}$

$\cancel{2x}$

$\nabla f = \hat{i}(2x) + \hat{j}(4y) + \hat{k}(-6z)$

$\nabla f_{P_o} = 2\hat{i} + 4\hat{j} - 6\hat{k}$ $\quad |\nabla f_{P_o}| = \sqrt{56}$

$\hat{e}_u$ = unit vector in direction $\hat{u}$

$\hat{e}_u = \frac{1}{7}(3\hat{i} + 6\hat{j} - 2\hat{k})$

$D_u(f) = \nabla f \cdot \hat{e}_u = \frac{6}{7} + \frac{24}{7} + \frac{12}{7} = 6$

22) $g(x,y,z) = xe^{y} + z^{2} \quad \Big| \quad P_0 = (1, \ln 2, {}^{1}\!/_{2})$

$\nabla g = \hat{i}(e^{y}) + \hat{j}(xe^{y}) + \hat{k}(2z)$

$(\nabla g)_{P_0} = \hat{i}(2) + \hat{j}(2) + \hat{k}(1)$

direction of max rate of change is ∇g
" " min " " " is $-\nabla g$

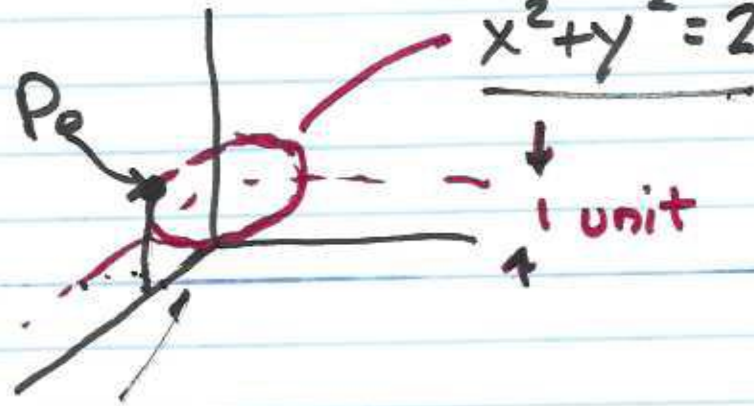
$|(\nabla g)_{P_0}| = 3$

$u_{\nabla g} = \hat{i}\cdot\frac{2}{3} + \hat{j}\cdot\frac{2}{3} + \hat{k}\cdot\frac{1}{3}$

34) $T(x,y,z) = 2xy - yz$

@ $P_0 = (1,-1,1)$ is -3 (C°/ft)

$T = 2xy - yz$ 7



$$\nabla T_{(1,-1,1)} = -2\hat{i} + \hat{j} + 1\hat{k}$$ rate of change of temp w/r/t dist.

$$\nabla T = \hat{i}(2y) + \hat{j}(2x-z) + \hat{k}(-y)$$

$$y = \sqrt{2-x^2} \quad z = 1$$

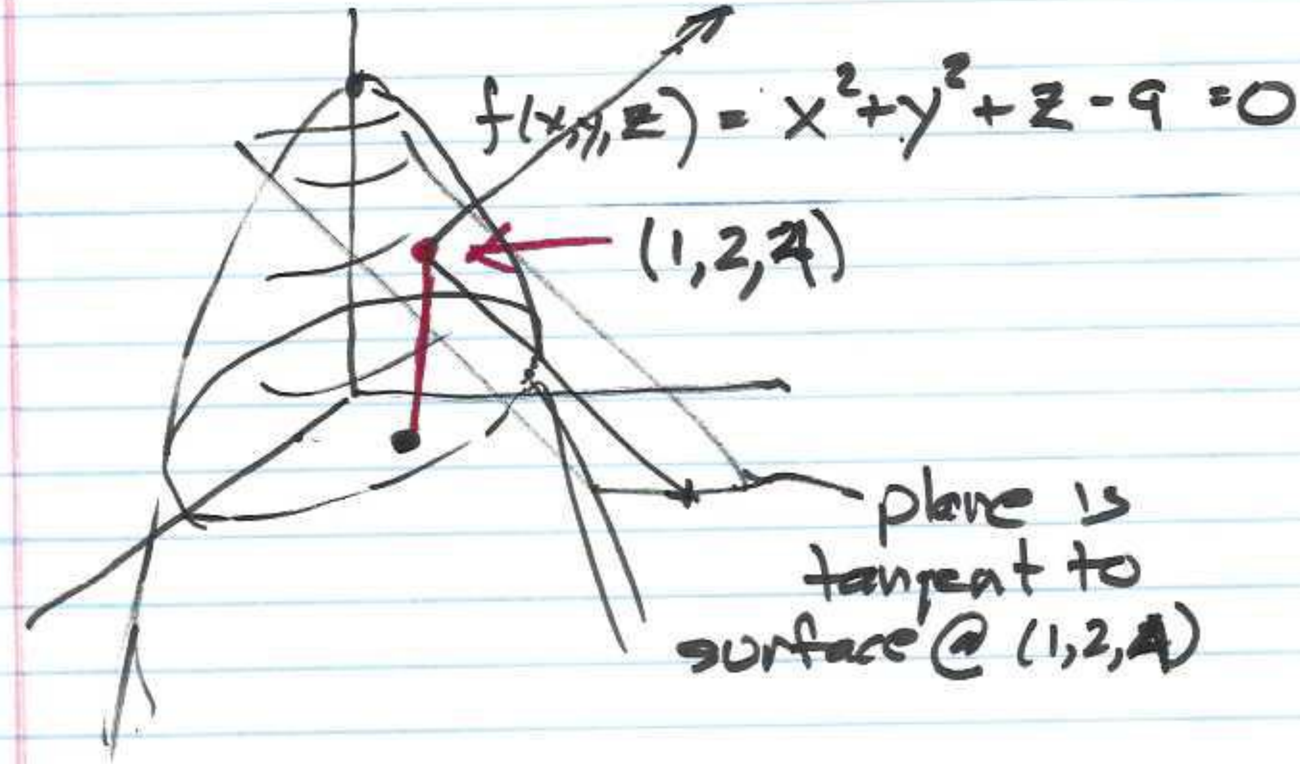
$$(\nabla T)_{ar} =$$

$$-3 = \left|\hat{i}\left(2\sqrt{2-x^2}\right) + \hat{j}(2x-1) + \hat{k}\left(-\sqrt{2-x^2}\right)\right|$$

$$9 = 4(2-x^2) + (4y^2-4x+1) + (2-x^2)$$

$$= 5(2-x^2) + (4x^2-4x+1)$$

$$9 = 10 - 5x^2 + 4x^2 - 4x + 1$$


f(x,y,z) = x^2+y^2+z-9 = 0
(1,2,4)
plane is tangent to surface @ (1,2,4)