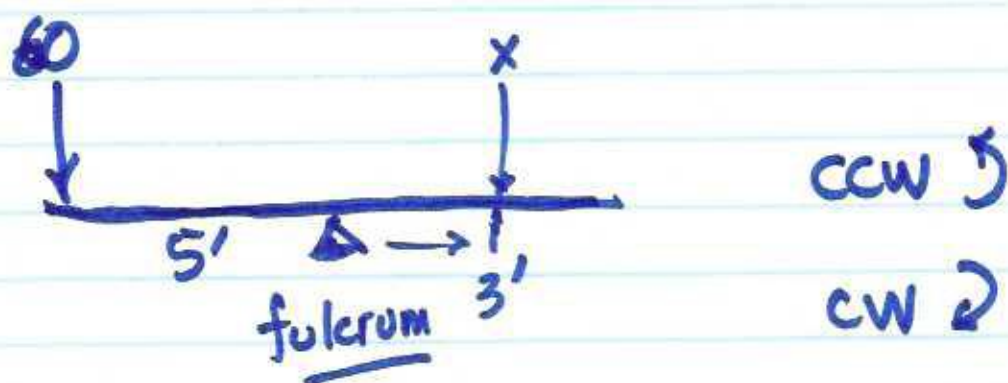


YUBRR

late



$$5 \cdot 60 = 300 \text{ ft} \cdot \text{lbs torque } \curvearrowright$$

$$3 \cdot x = \curvearrowleft \text{ ft} \cdot \text{lbs torque}$$

$$3x = 300 \Rightarrow x = 100 \text{ lbs}$$



torque about P by F is $F \cdot l$



$$m_1 g x_1 + m_2 g x_2 + m_3 g x_3 + m_4 g x_4 = 0$$

for stability

②

$$\sum_{i=1}^n m_i x_i = 0 \text{ for stability}$$

x_i is the moment arm of the i^{th} mass

Given system of masses $m_1, \dots, m_n$

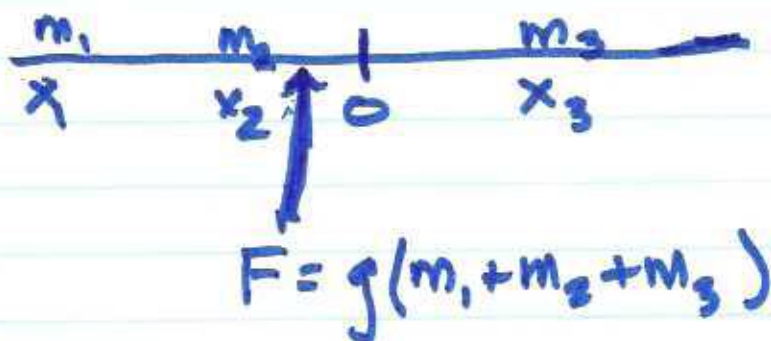
with locations $x_1, \dots, x_n$, on the x-axis,

where is the "balance point"? That is

the unique location of an upward force

equal to the weight of the system where no

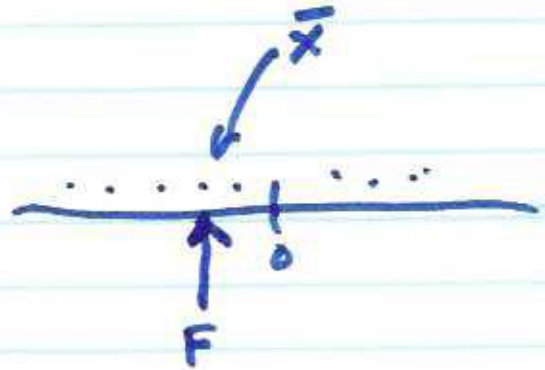
rotation (i.e. net torque) would be present.



③

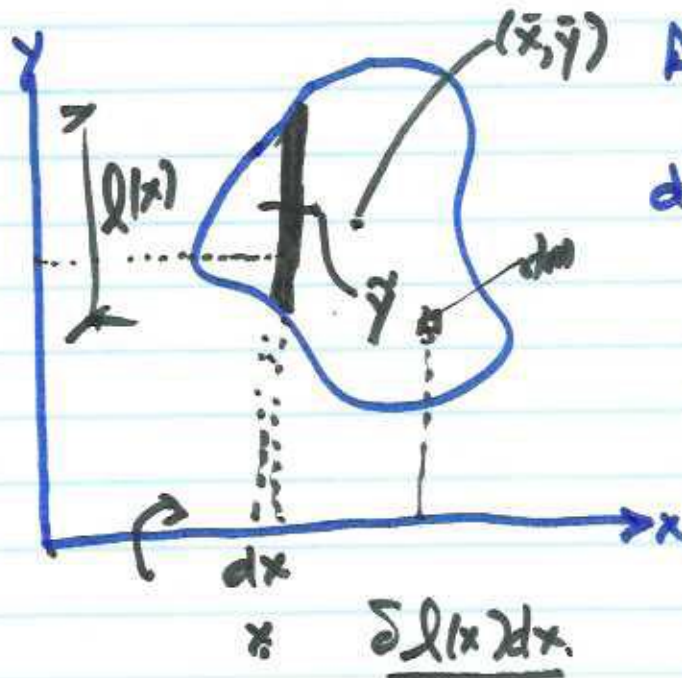
$$\frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \bar{x}$$

$$\frac{\sum_{i=1}^n m_i x_i}{\sum_{i=1}^n m_i} = \bar{x}$$



"~" is tide

$$F\bar{x} = \sum x_i m_i g$$



Assume constant
density
density = δ

CM (COM)

Moment about
the y-axis

④

M_x is moment about x-axis

M_y is " " y-axis

$$\text{So } \frac{M_x}{M} = \bar{y} \text{ and } \frac{M_y}{M} = \bar{x}$$

$$M_y = \sum \tilde{x} \Delta m$$

$$M_x = \sum \tilde{y} \Delta m$$

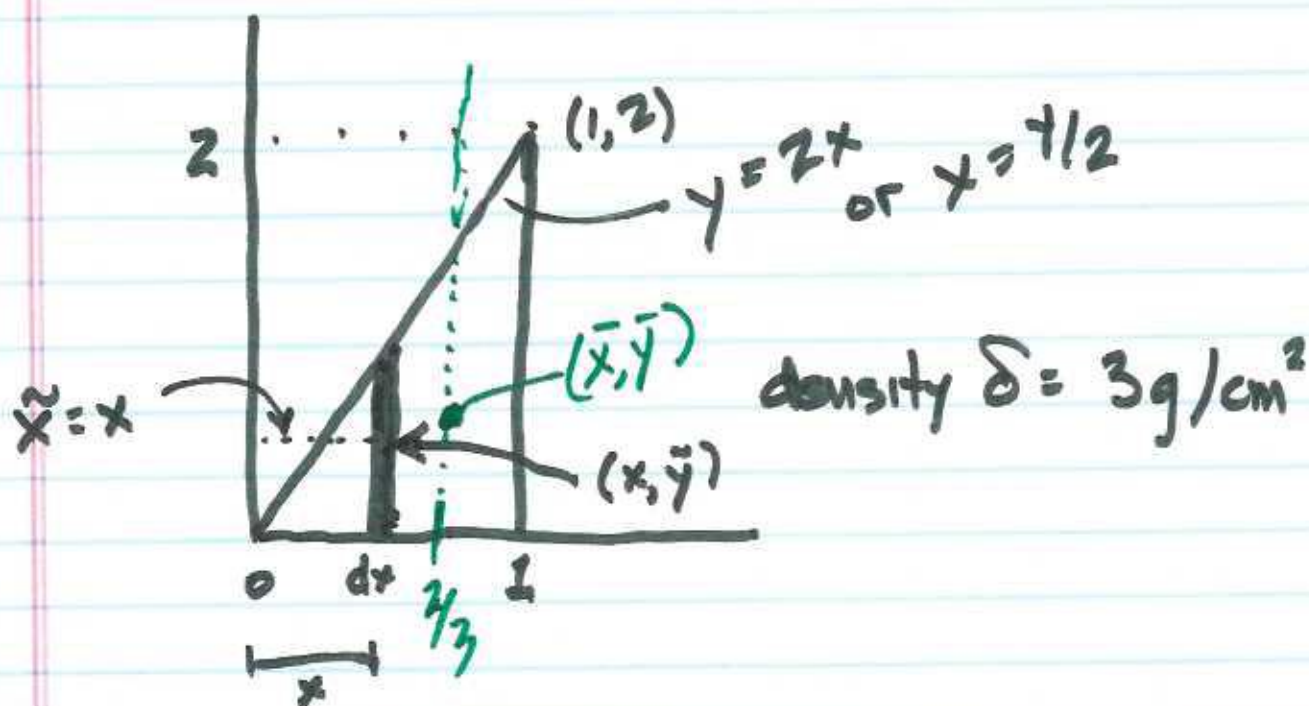
$\bar{y}$ = avg y position of entire shape

$\tilde{y}$ = avg y position of vertical strip

$\bar{x}$ = avg x position of entire shape

$\tilde{x}$ = avg x position of horiz strip

(5)



length of strip $= 2x$

width of strip $= dx$

area of strip $= 2x dx$

mass of strip $= 6x dx = dm$

$\bar{x} = x$

moment of strip about y-axis is

$\bar{x} dm = 6x^2 dx$

diff moment
about y-axis

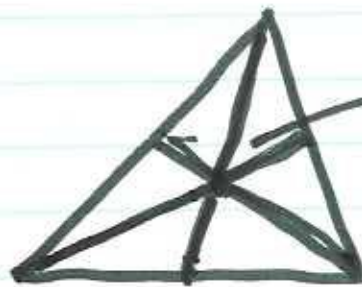
$M_y = \int_0^1 6x^2 dx = [2x^3]_0^1 = \boxed{2} \text{ g-cm}$

⑥

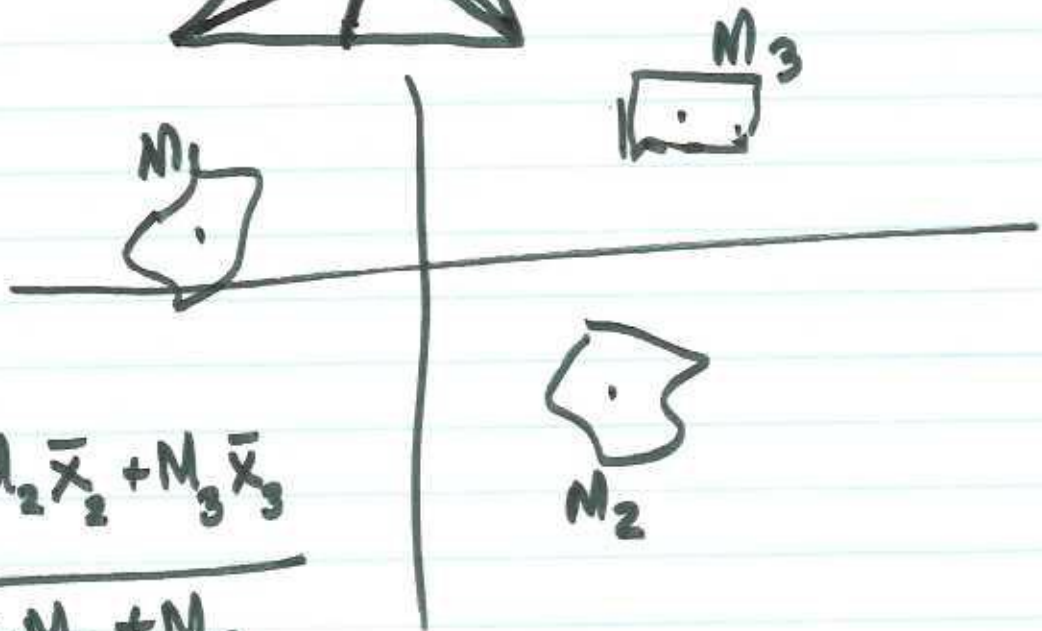
$$M = \int_0^1 6x dx = \left[3x^2 \right]_0^1 = \boxed{3}$$

So now $\frac{M_y}{M} = \bar{x} = \boxed{\frac{2}{3}}$

For triangles:

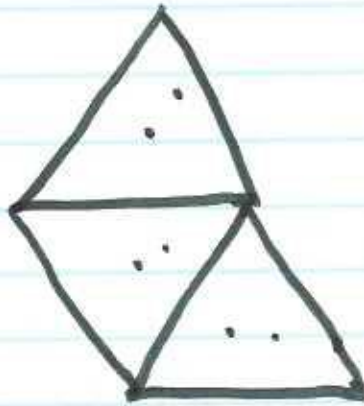


CM of triangle

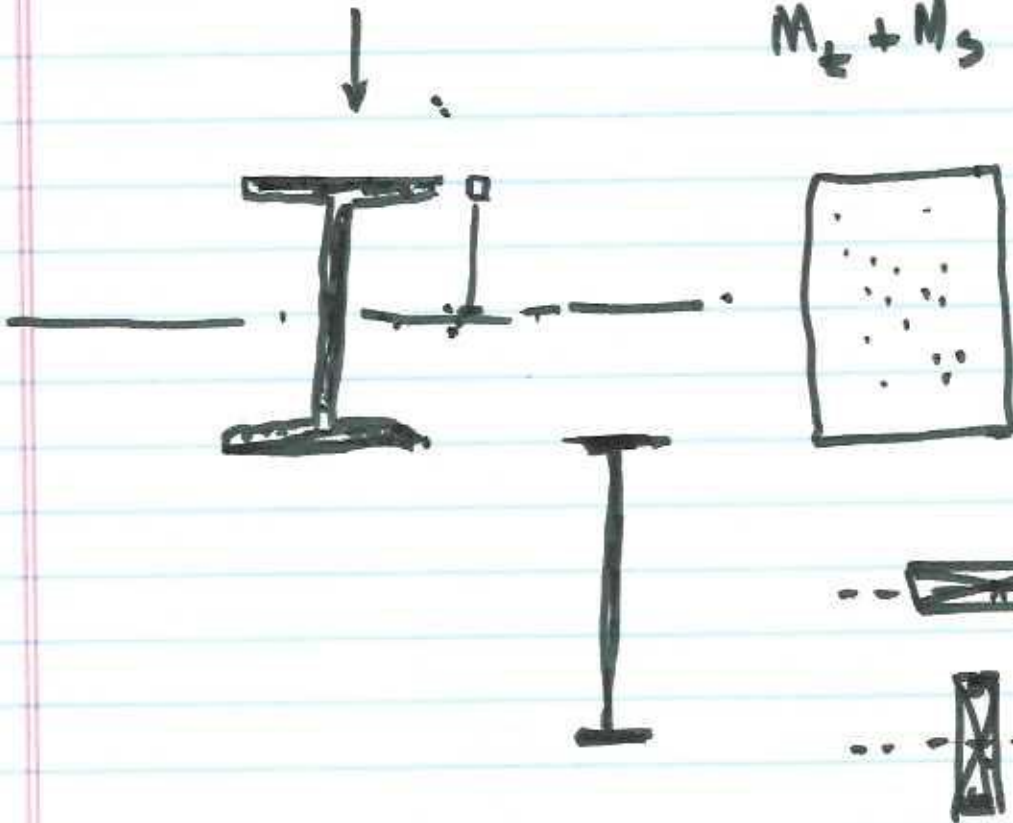


$$\bar{x}_{sys} = \frac{M_1 \bar{x}_1 + M_2 \bar{x}_2 + M_3 \bar{x}_3}{M_1 + M_2 + M_3}$$

⑦

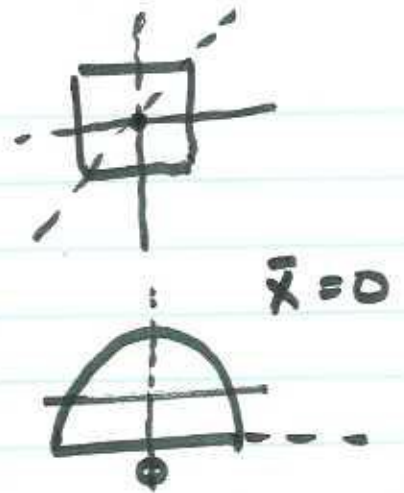
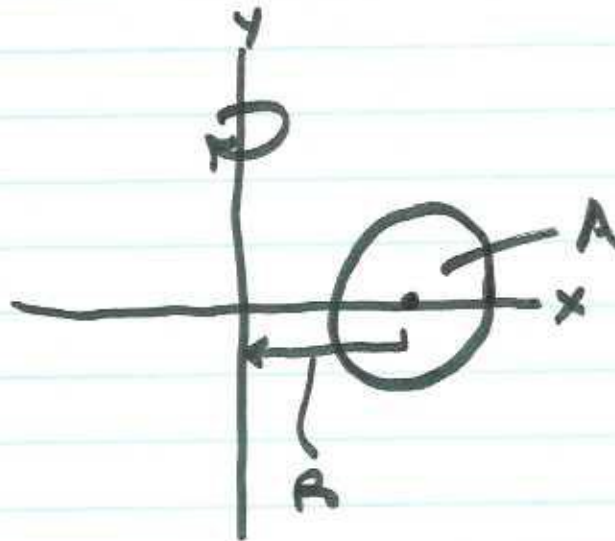


$$\frac{\bar{x}_t M_t + \bar{x}_s M_s}{M_t + M_s} = \bar{x}_{sys}$$



⑧

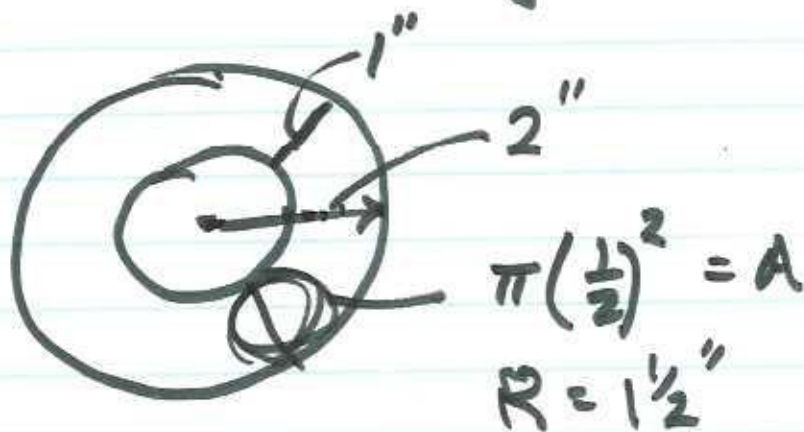
Thm of Pappus



Volume of rotated area is:

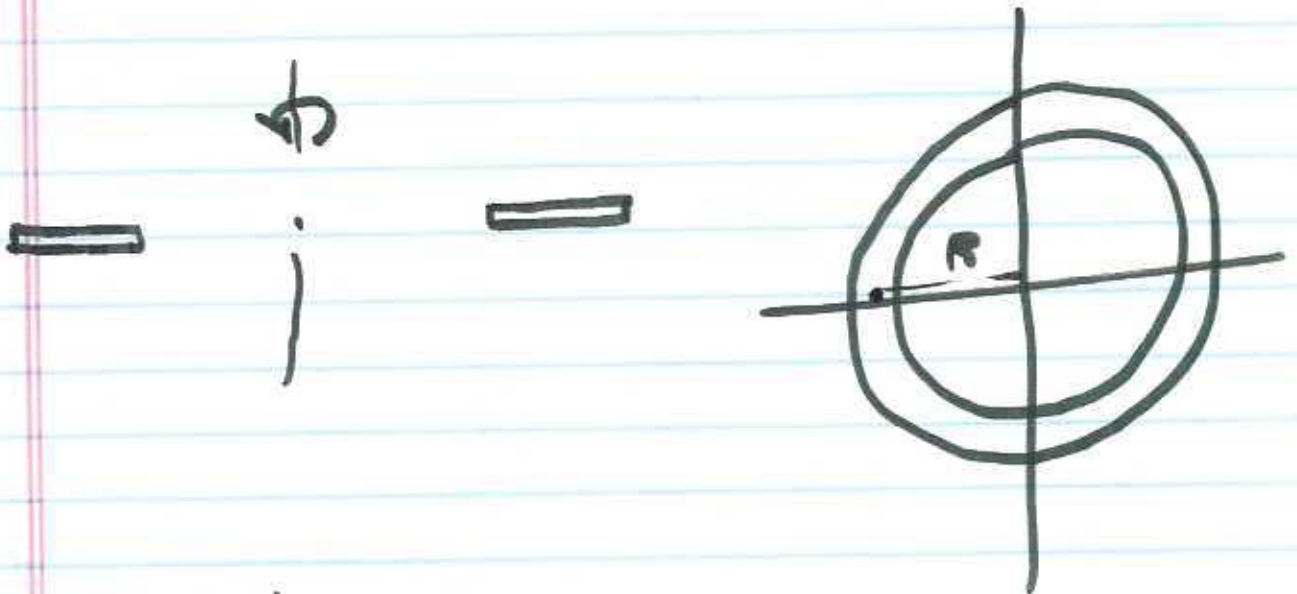
$$2\pi R \cdot A = V$$

What is volume of a doughnut

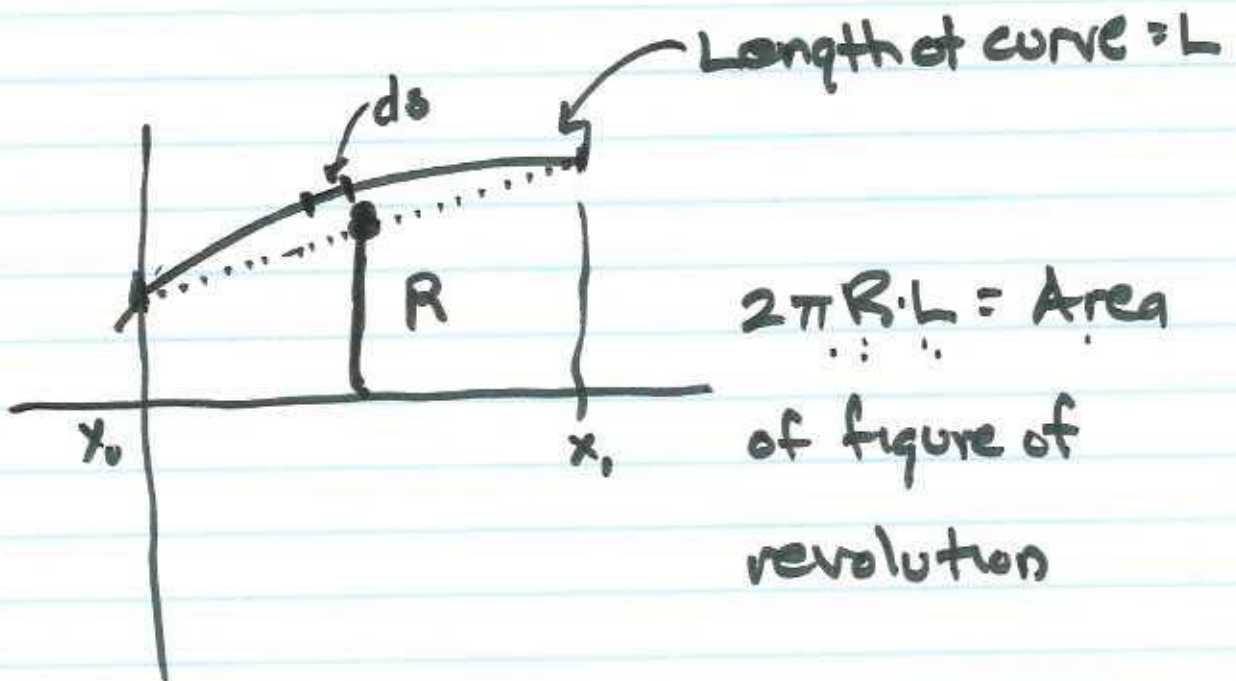


$$Vol = 2\pi(1.5)(.25)\pi$$

⑨

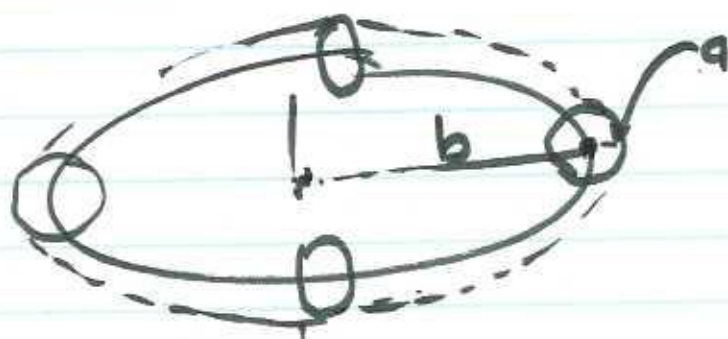


$$Vol = 2\pi RWH$$



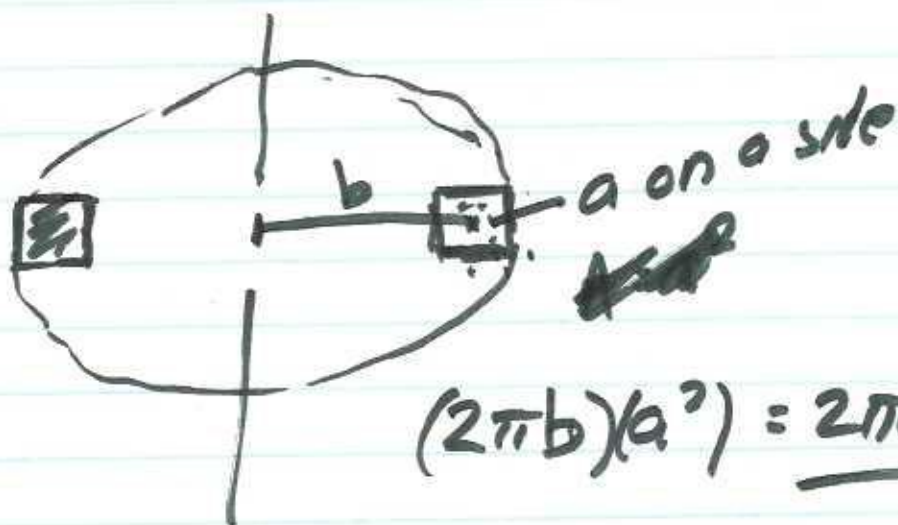
$$2\pi R \cdot L = \text{Area of figure of revolution}$$

(10)



Pappus' Surface Area Th^m

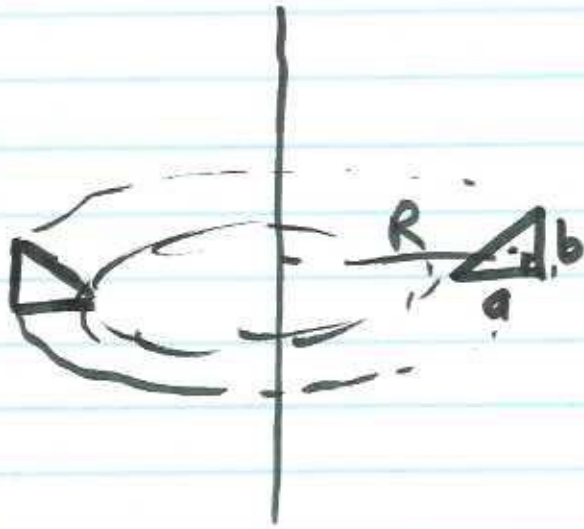
$$S = (2\pi b)(2\pi a) = 4\pi^2 ab$$



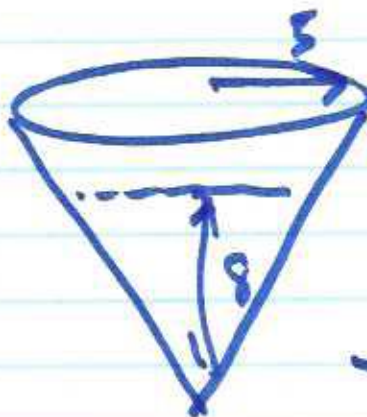
$$(2\pi b)(a^2) = \underline{2\pi a^2 b}$$

$$(2\pi b)(4a) = \underline{8\pi ab}$$

(11)



$$2\pi R \left(\frac{1}{2} ab \right) = \left[\frac{\pi}{2} ab R \right]$$



10

