XLUQ9

late
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①

Hyperbolic functions:

$\sinh x : \dfrac{e^{x}-e^{-x}}{2}$ $\quad \frac{1}{2}$(diff)

$\cosh x : \dfrac{e^{x}+e^{-x}}{2}$ $\quad \frac{1}{2}$(avg.)

$\tanh x : \dfrac{\sinh x}{\cosh x} \quad x \in \mathbb{R}$

$\coth x : \dfrac{\cosh x}{\sinh x} = \dfrac{1}{\tanh x} \quad x \in \mathbb{R} - \{0\}$

$\operatorname{sech} x : \dfrac{1}{\cosh x} \quad x \in \mathbb{R}$

$\operatorname{csch} x : \dfrac{1}{\sinh x}$

$x \in \mathbb{R} - \{0\}$

$A \setminus B$

What is $\frac{d}{dx}(\sinh x)$?

$$\frac{d}{dx}\left(\frac{e^x - e^{-x}}{2}\right) = \frac{1}{2}\left(e^x + e^{-x}\right) = \cosh x$$

How about $\frac{d}{dx}(\cosh x)$?

$$\frac{d}{dx}(\cosh x) = \frac{1}{2}\left(e^x - e^{-x}\right) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \left(\frac{\sinh x}{\cosh x}\right)' = \frac{\cosh^2 x - \sinh^2 x}{\cosh^2 x}$$

Note: $1 + \sinh^2 x = \cosh^2 x$

$$\frac{1}{\cosh^2 x} = \operatorname{sech}^2 x$$

$$\frac{d}{dx}(\coth x) = \left(\frac{\cosh x}{\sinh x}\right)' = \frac{\sinh^2 x - \cosh^2 x}{\sinh^2 x}$$

$$= -\operatorname{csch}^2 x$$

$$\frac{d}{dx}(\operatorname{sech}x) = \frac{d}{dx}\left(\frac{1}{\cosh x}\right) = \frac{\cosh x\,(0)-(1)\sinh x}{\cosh^2 x}$$

$$= -\frac{\sinh x}{\cosh^2 x} = -\tanh x \cdot \operatorname{sech} x$$

Claim: $1+\sinh^2 x = \cosh^2 x$ Prove?

$$1+\left(\frac{e^x-e^{-x}}{2}\right)^2 = \left(\frac{e^x+e^{-x}}{2}\right)^2$$

$(a+b)^2 = a^2+2ab+b^2$

$$4+(e^x-e^{-x})^2 \stackrel{?}{=} (e^x+e^{-x})^2$$

$$\cancel{4}+\left[\cancel{e^{2x}}-\cancel{2}+\cancel{e^{-2x}}\right] = \left[\cancel{e^{2x}}+\cancel{2}+\cancel{e^{-2x}}\right]$$

Double angle in trig: $\underline{\sin 2\theta = 2\sin\theta\cos\theta}$

$$\sinh 2x \stackrel{?}{=} 2\sinh x\cosh x \quad ??$$

$$\frac{e^{2x}-e^{-2x}}{\cancel{2}} \stackrel{?}{=} \cancel{2}\left[\frac{e^{x}-e^{-x}}{\cancel{2}}\right]\cdot\left[\frac{e^{x}+e^{-x}}{\cancel{2}}\right]$$

$$e^{2x}-e^{-2x} \stackrel{?}{=} (e^{x}-e^{-x})(e^{x}+e^{-x})$$

$$= e^{2x}\cancel{+1}\cancel{+(-1)}-e^{-2x}$$

$$\sin(x+y) = \sin x\cos y + \cos x \sin y$$

$$\sinh(x+y) \stackrel{?}{=} \sinh x\cosh y + \cosh x \sinh y$$

$$\frac{e^{x+y}-e^{-(x+y)}}{2} \stackrel{?}{=} \underbrace{\left[\frac{e^{x}-e^{-x}}{2}\right]\left[\frac{e^{y}+e^{-y}}{2}\right]} + \left[\frac{e^{x}+e^{-x}}{2}\right]\left[\frac{e^{y}-e^{-y}}{2}\right]$$

$$\begin{cases} \frac{1}{4}\left[e^{x}\cancel{e^{y}}+e^{x}e^{-y}-e^{-x}e^{y}-e^{-x}e^{-y}\right] \; \boxed{+} \\ \frac{1}{4}\left[e^{x}\cancel{e^{y}}-e^{x}e^{-y}+e^{-x}e^{y}-e^{-x}e^{-y}\right] \end{cases}$$

$$= \frac{1}{4}\left[2e^{x+y} - 2e^{-x-y}\right] = \frac{1}{2}\left(e^{x+y} - e^{-(x+y)}\right)$$

$y = \sinh x$ want $x = f(y)$?

$$y = \frac{e^x - e^{-x}}{2} \Rightarrow 2y = e^x - e^{-x}$$

$$e^x - e^{-x} - 2y = 0 \qquad \underline{ax^2 + bx + c = 0}$$

$$\cdot e^x \quad e^{2x} - 1 - 2ye^x = 0 \quad (e^x)^2 - 2y(e^x) - 1 = 0$$

$$e^x = \frac{2y \pm \sqrt{4y^2 + 4}}{2} = y \pm \sqrt{y^2+1}$$

"spurious"

$$e^x = y + \sqrt{y^2+1}$$

$$x = \ln\left(y + \sqrt{y^2+1}\right)$$

$$x = f\left(f^{-1}(x)\right)$$

$$1 = f'\left(f^{-1}(x)\right) \cdot \left[f^{-1}(x)\right]'$$

want this

$$\frac{1}{f'\left(f^{-1}(x)\right)} = \left[f^{-1}(x)\right]' \leftarrow$$

Ex: $x = e^{\ln x}$

$1 = e^{\ln x} \cdot [\ln x]'$

$$\frac{1}{e^{\ln x}} = \boxed{[\ln x]' = \frac{1}{x}}$$

⑦

$$\frac{d}{dx}(\arcsin x)$$

$$x = \sin(\arcsin x)$$

diff: $1 = \cos(\arcsin x) \cdot [\arcsin x]'$

$$\frac{1}{\cos(\arcsin x)} = [\arcsin x]'$$

$$\uparrow$$

$$\sqrt{1 - \sin^2(\arcsin x)}$$

$$\downarrow$$

$$\sqrt{1 - x^2}$$

$$\cos\alpha = \sqrt{1 - \sin^2\alpha}$$

$$\int_a^b \frac{dx}{\sqrt{1-x^2}} = \arcsin x \Big|_a^b$$

Now : $[\arcsin x]' = \frac{1}{\sqrt{1-x^2}}$ $\leftarrow$

We say $f(x)$ grows faster than $g(x)$ as $x \to \infty$ iff $\lim_{x\to\infty} \left| \frac{g(x)}{f(x)} \right| = 0$.

We say $f(x)$ and $g(x)$ have same growth rate if $\lim_{x\to\infty} \left| \frac{g(x)}{f(x)} \right| = K$, constant > 0

Compare growth of x^2 & 2^x $\qquad \frac{\infty}{\infty}$

Look @ $\lim_{x\to\infty} \left| \frac{x^2}{2^x} \right| = ?$

$$\lim_{x\to\infty} \left| \frac{2x}{2^x \ln 2} \right| = ?$$

$$\lim_{x\to\infty} \left| \frac{2}{2^x (\ln 2)^2} \right| \qquad \frac{2}{\infty} = 0$$

$\frac{x^n}{2^x}$

$\frac{nx^{n-1}}{\ln 2 \cdot 2^x}$

$\frac{n(n-1)x^{n-2}}{(\ln 2)^2 2^x} \to 0$

$\vdots$

Show that $\sqrt{x^2+5}$ & $(2\sqrt{x}-1)^2$ grow

@ same rate.

Compare w/ x

$$\lim_{x\to\infty}\left|\frac{\sqrt{x^2+5}}{x}\right| \stackrel{?}{=} \lim_{x\to\infty}\left|\sqrt{\frac{x^2+5}{x^2}}\right| =$$

$$\lim_{x\to\infty}\left|\sqrt{1+\frac{5}{x^2}}\right| = 1.$$

$$\lim_{x\to\infty}\left|\frac{(2\sqrt{x}-1)^2}{x}\right| = \lim_{x\to\infty}\left|\left(\frac{2\sqrt{x}-1}{\sqrt{x}}\right)^2\right|$$

$(\sqrt{x})^2$

$$\lim_{x\to\infty}\left|\left(2-\frac{1}{\sqrt{x}}\right)^2\right| = 4.$$

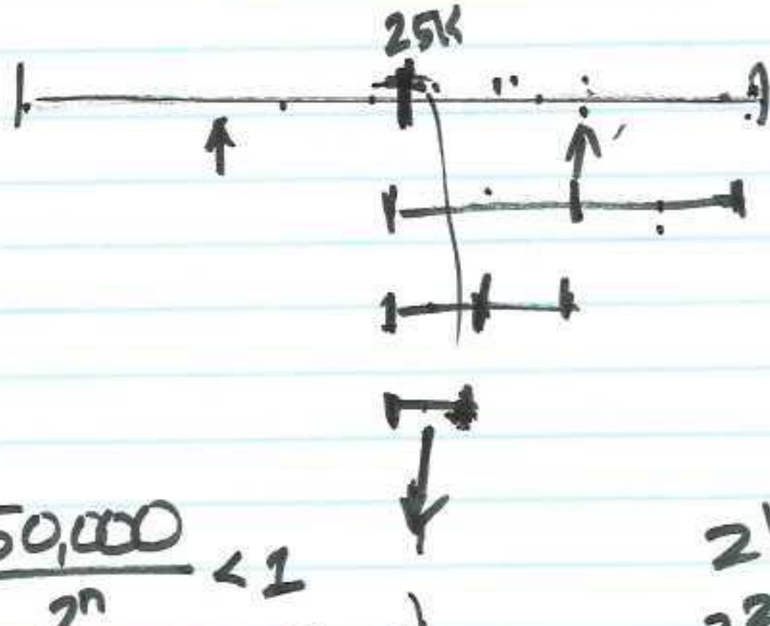
$\Theta(n)$ $o(n)$

Finding det(M_n) has order $n!$. $O(n!)$

But .. by Gaussian elimination $\sim O(n)$

Given word - need to find in dictionary

of 50K words.



$$\frac{50,000}{2^n} < 1$$

$2^{10} = 1024$

$2^{20} = 1,048,496$

$$2^n > 50,000$$