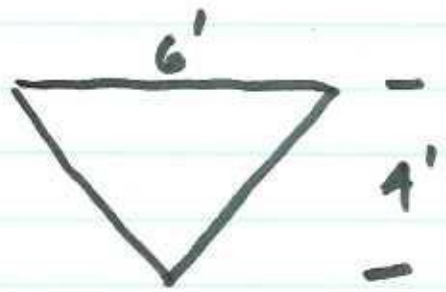
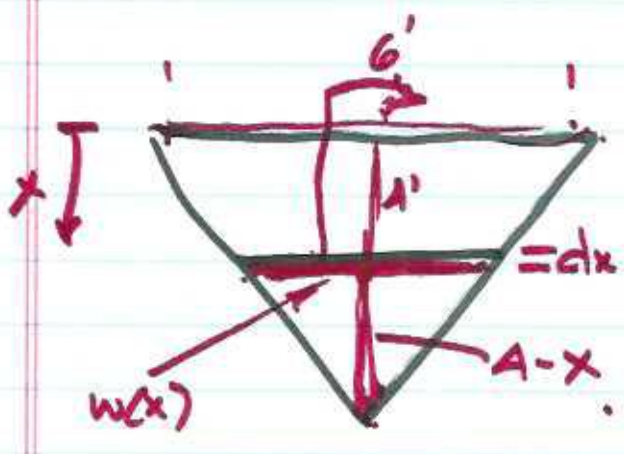
9/17
(1)

Problem #3





w(x) width

$$\left(125\,[w(x)]^2\,dx\right)x = dW(x)$$

$$\frac{w(x)}{4-x} = \frac{6}{4} = \frac{3}{2}$$

$$w(x) = \frac{3}{2}(4-x)$$

$$125\int \frac{9}{4}\left(16-8x+x^2\right)x\,dx = 6000 \text{ lbs}$$

$x'(t) = k\,x(t)$ exponential behavior

time

if $k>0$ growth if $k<0$ decay

$$\frac{dx}{dt} = kx \Rightarrow \frac{dx}{x} = k\,dt$$

this is "separated"

x's on left

t on right

$$\int \frac{dx}{x} = \ln|x| \qquad \int k\,dt = kt$$

$$\ln|x| = kt + C$$

$$e^{\ln x} = x = e^{kt+c} = \underbrace{e^{c}}_{\text{constant}}\underline{e^{kt}}$$

@ $t=0$, say $x=x_0$

$$x_0 = e^{c}\cdot\underbrace{e^{0}}_{1} \Rightarrow x_0 = e^{c}$$

$$\boxed{x(t) = x_0 e^{kt}}$$

Finance: Let $x(t)$ be amount on deposit in cont. interest acc't. t in yrs

$$F(t) = F(0)e^{rt}$$

$\frac{\text{mil}}{000}$ $\frac{\%}{100}$

$$\underline{Pe^{rt}} \quad \underline{PERT}$$

$$? = 1000e^{(.05)(10)}$$

$$= 1000\underbrace{e^{.5}}_{\text{growth}} = \underline{\$1648}$$

$$\frac{dQ}{dt} = -\lambda Q(t) \qquad \lambda > 0$$

radiation strength

$$\frac{dQ}{Q(t)} = -\lambda dt$$

$$\ln|Q(t)| = -\lambda t + c$$

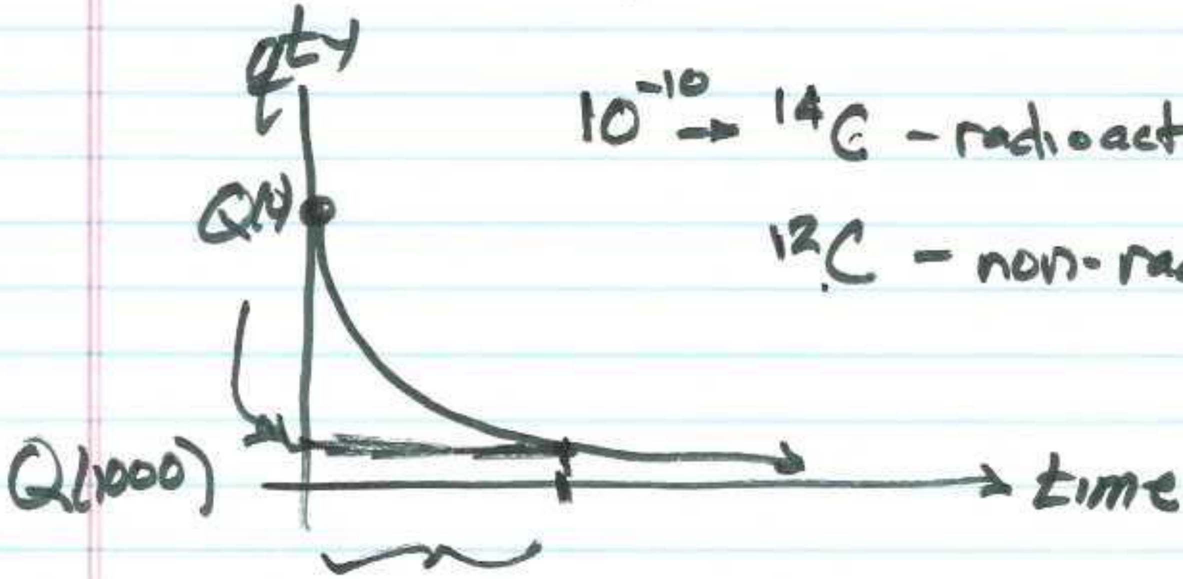
$$Q(t) = e^{-\lambda t + c} \rightarrow e^{c} e^{-\lambda t}$$

@ $t=0$ $\quad Q(0) = e^{c} e^{0} \Rightarrow e^{c} = Q(0)$

$$Q(t) = Q(0) e^{-\lambda t}$$

$10^{-10} \rightarrow {}^{14}C$ - radioactive

${}^{12}C$ - non-radioactive



$\lambda \approx .00012$

$Q(t) = Q(0)\, e^{-.00012 t}$ $\quad$ ^{14}C dating

$[0, 25K]$ $\quad$ $\pm$ 50 yrs

Turin $\quad$ Shroud of Turin

92% C^{14}

$$\frac{Q(t)}{Q(0)} = .92 = e^{-.00012 t},$$

$$\ln(.92) = -.00012 t$$

$$t = \frac{\ln(.92)}{-.00012} \approx 694$$

$$P'(t) = k\,P(t)$$

$$\frac{dP}{dt} = kP \Rightarrow \int \frac{dP}{P} = \int k\,dt$$

$$\ln P = kt + c \Rightarrow$$

$$P(t) = e^{kt} \cdot e^{c}$$

$$P(0) = e^{c} \qquad .02$$

$$P(t) = P(0)\,e^{kt}$$

$$\underline{F = Pe^{rt}}$$

$$F(t) = P(0)\,e^{(r-i)t}$$

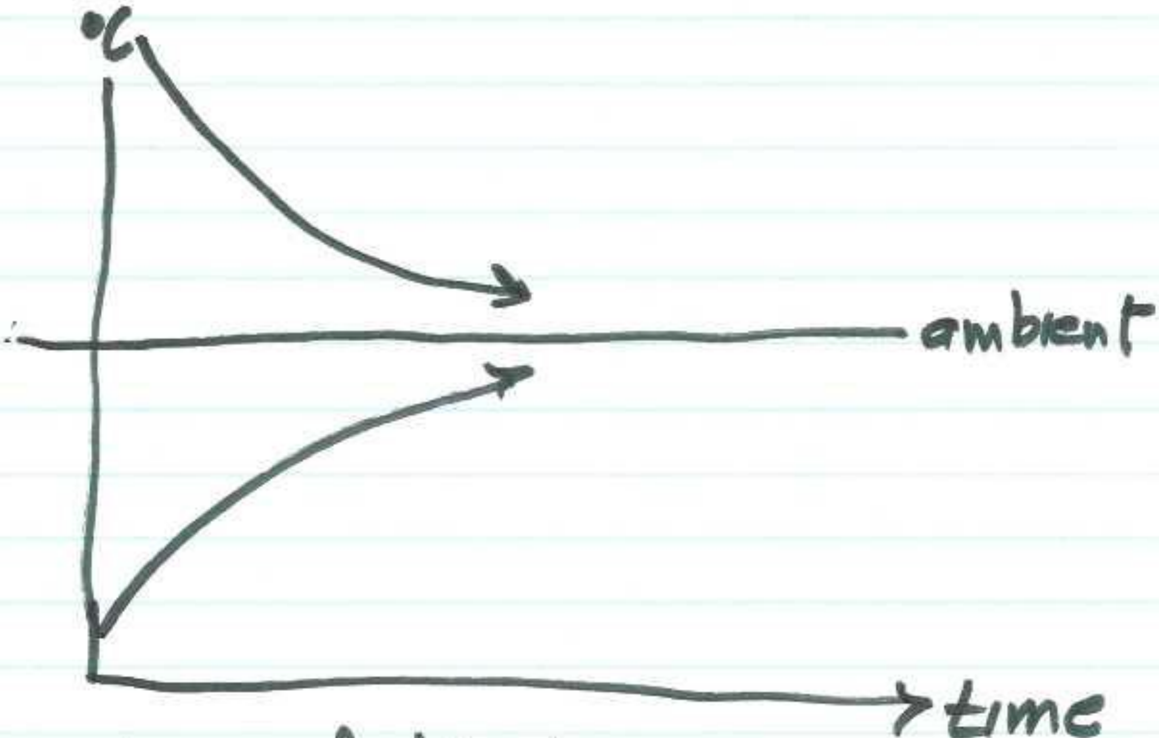
Hardboiled Egg @ 98°C

Put in water bath @ 18°C

Let sit for 5 min. - now egg @ 38°C

When will it be @ 20°C

Newton's Law (of Cooling)



H is temp of object

H_s is ambient temp

t is time (min)

K is thermal coefficient

$$\frac{dH}{dt} = -K(H-H_s)$$

Let $y = H - H_s$

What is $\frac{dy}{dt} = \frac{dH}{dt} \cancel{- \frac{dH_s}{dt}}$

So.. $\frac{dy}{dt} = -ky$ $\qquad \frac{dy}{y} = -k\,dt$

$\ln|y| = -kt + c$

$y = e^{-kt} \cdot e^{c}$

initial temp of egg

* $H - H_s = (H_0 - H_s)e^{-kt}$

$\sinh x = \dfrac{e^x - e^{-x}}{2}$	$\operatorname{csch} x = \dfrac{1}{\sinh x}$
$\cosh x = \dfrac{e^x + e^{-x}}{2}$	$\operatorname{sech} x = \dfrac{1}{\cosh x}$
$\tanh x = \dfrac{\sinh x}{\cosh x}$	$\coth x = \dfrac{1}{\tanh x}$

$\sin^2\theta + \cos^2\theta = 1$

$\sinh^2 x + 1 = \cosh^2 x$

$y = \sinh x \qquad x = \underline{\sinh^{-1}(y)}$

$y = \dfrac{e^x - e^{-x}}{2} \Rightarrow 2y = e^x - e^{-x}$

$e^x - e^{-x} - 2y = 0$

e^x: $(e^x)^2 - 1 - 2y(e^x) = 0$

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$$(e^x)^2 - 2y(e^x) - 1 = 0$$

$$e^x = \frac{2y \pm \sqrt{4y^2+4}}{2}$$

$$e^x = y \pm \sqrt{y^2+1} = y + \sqrt{y^2+1}$$

$$\boxed{x = \ln\left(y + \sqrt{y^2+1}\right)}$$