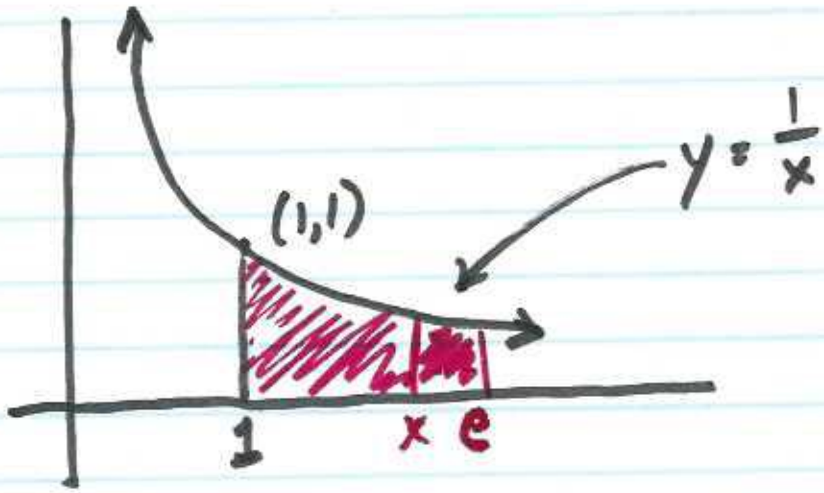
late 1/15

①

Ch. 7.1



$\ln(x)$ = area under $\frac{1}{x}$ from 1 to x

FTC $\rightarrow$ $\ln(x) = \int_1^x \frac{1}{t}\,dt$ $\qquad \int_1^e \frac{dt}{t} = ?$

$\ln(1) = 0$

$\ln(e) = 1$

$f(\ldots)$

$\left[\ln(x)\right]' = \frac{1}{x}$

$$\boxed{\left[e^{g(x)}\right]'} = e^{g(x)} \cdot g'(x)$$

$$x = e^{\ln x}$$

$$1 = e^{\ln x} \cdot [\ln x]'$$

$$\uparrow$$

$$1 = x \cdot [\ln x]' \Rightarrow [\ln x]' = \frac{1}{x}$$

(i) $a^x \cdot a^y = a^{x+y}$

(ii) $a^x / a^y = a^{x-y}$

(iii) $(a^x)^n = a^{nx}$

(iv) $\left[\sqrt[n]{a^x}\right] = a^{x/n}$

a here is called a base

What are legitimate bases

1) no negatives

2) $4 = \left(\frac{1}{2}\right)^x$

$2^2 = (2^{-1})^x = 2^{-x}$

$-x = 2 \Rightarrow x = -2$

3) base $\neq 1$

$$b^x = b^y$$

$$x = y$$

$\log_b A = x \Leftrightarrow b^x = A$

$\log A$ = common log base 10

$\ln A$ = logarithmus naturalis base 10

$\lg A$: base = 2

Log Rules: assume base b

Suppose $b^x = A$; $b^y = B$ | $\log_b A = x$ / $\log_b B = y$

(i) What is $\log_b(A \cdot B)$?

$$A \cdot B = b^x \cdot b^y = b^{x+y}$$

$$\log_b(A \cdot B) = x + y = \log_b A + \log_b B$$

(ii) What is $\log_b\left(A/B\right)$?

$$A/B = \frac{b^x}{b^y} = b^{x-y}$$

so... $\log_b\left(A/B\right) = x - y$

$$\log_b\left(\frac{A}{B}\right) = \log_b A - \log_b B$$

(iii) What is $\log_b A^m$?

$$A^m = b^{x \cdot m} \rightarrow \log_b A^m = m \cdot x = m \cdot \log_b A$$

(iv) What is $\log \sqrt[m]{A}$

$$A^{1/m} = b^{x \cdot \frac{1}{m}} = b^{x/m}$$

$$\log \sqrt[m]{A} = \frac{\log_b A}{m}$$

Change of base formula

aka Euler's Golden Rule

$$A = b^x = c^y$$

$$\log_b A = x \quad \log_c A = y$$

Write c as $b^{\log_b c}$

$$b^x = \big(\underbrace{b^{\log_b c}}_{c}\big)^y$$

$$x = \log_b c \cdot y$$

$\log 3 = .47712$
$\log 2 = .30103$
$\ln 2 = .69315$

$$\log_b A = \log_b c \cdot \log_c A$$

$$\frac{\log_b A}{\log_c A} = \log_b c$$

$b = 10$
$c = e$

$$\frac{\log A}{\ln A} = \log e \Rightarrow \ln A = \frac{\log A}{\log e}$$

⑥

$$.69315 = \frac{.30103}{\log e}$$

$$\log e = \frac{.30103}{.69315} \sim \frac{1}{2.30258}$$

$$\frac{d}{dx}(2^x) = \frac{d}{dx}\left(e^{\ln 2}\right)^x = \frac{d}{dx}\left(e^{x\ln 2}\right)$$

$$\underbrace{e^{x\ln 2}}_{2^x}\cdot\ln 2 = \ln 2\cdot 2^x$$

$$\boxed{r \in \mathbb{R}_{0+} \setminus \{1\}}$$

$$\frac{d}{dx}(r^x) = \frac{d}{dx}\left(e^{\ln r}\right)^x = \frac{d}{dx}\left(e^{x\ln r}\right) ?$$

$$e^{x\ln r}\cdot\ln r = \boxed{\ln r\cdot r^x}$$

$$\int \frac{dx}{\cos x} = \int \sec x \, dx \cdot \frac{\sec x + \tan x}{\sec x + \tan x}$$

$$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx = \int \frac{du}{u} = \ln|u| + C$$

Let $u = \sec x + \tan x$

$$du = (\sec^2 x + \sec x \tan x)\, dx$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| + C$$

$$\underline{\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C}$$