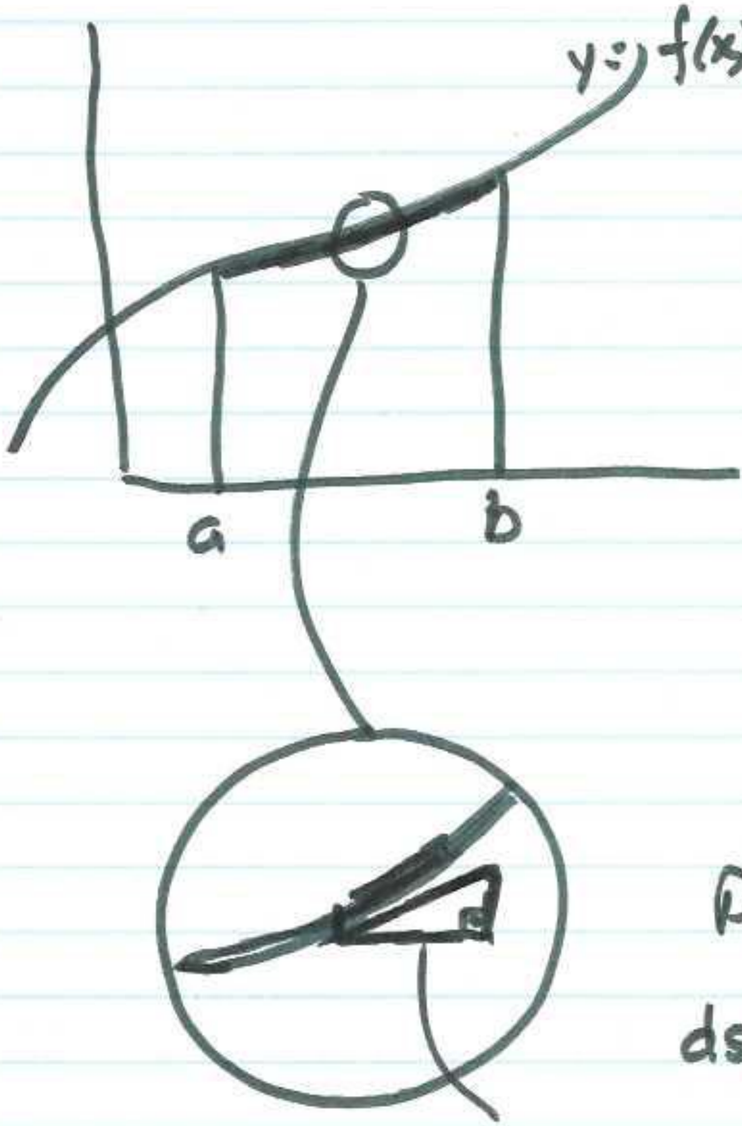
XZGLD

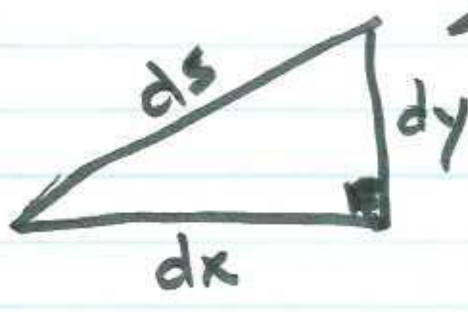
late 1

8/27



Pythagoras says

$ds^2 = dx^2 + dy^2$



s means length

$$ds^2 = \frac{(dx^2 + dy^2)}{dx^2} dx^2$$

$$\downarrow$$

$$ds^2 = \left(1 + \left(\frac{dy}{dx}\right)^2\right) dx^2$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

So length (on the curve) from a to b is:

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, \underline{dx}$$

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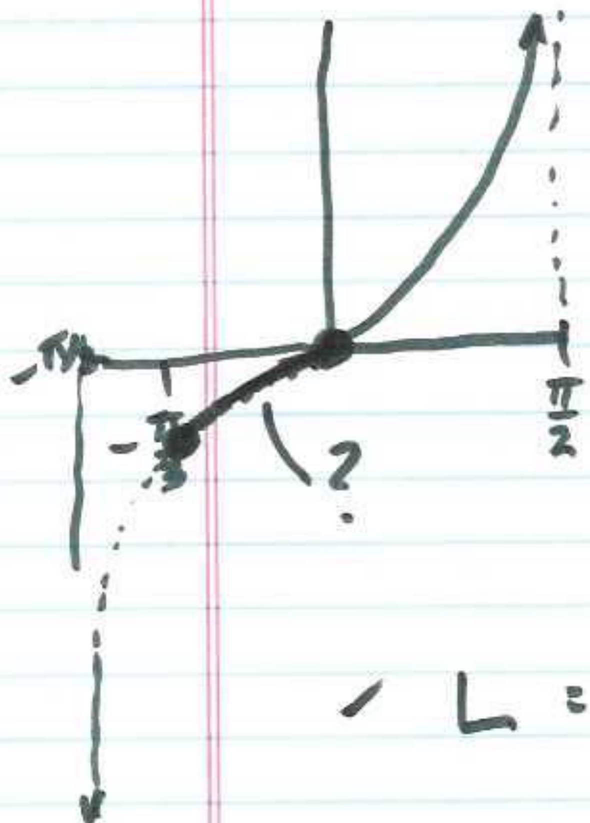
(14) $y = \tan x \qquad -\frac{\pi}{3} \le x \le 0$

③



$$\begin{array}{c|c} s & sct \\ \hline t & c \end{array}$$

$y' = \sec^2 x = \frac{dy}{dx}$

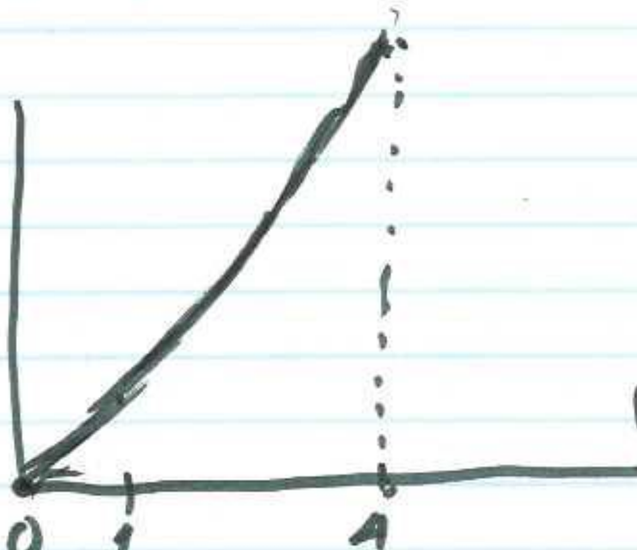
$L = \int_{-\frac{\pi}{3}}^{0} \sqrt{1 + \sec^4 x}\, dx$

Try: $u = 1 + \sec^4 x$

$du = 4\sec^3 x (\sec x \tan x)\, dx$

$$\int (u)^{1/2} \cdot \frac{du}{4\underbrace{\sec^4 x}_{u-1} \tan x}$$

$$y=(\sqrt{x})^3 = x^{3/2}$$



$$\sqrt{\frac{4+9x}{4}} = \frac{1}{2}\sqrt{4+9x}$$

$$y' = \frac{3}{2}x^{1/2} \Rightarrow ds = \sqrt{\frac{4}{4}+\frac{9x}{4}} = \frac{1}{2}\sqrt{9x+4}$$

$$L = \frac{1}{2}\int_0^4 \sqrt{9x+4}\,dx$$

$$u = 9x+4 \Rightarrow du = 9\,dx \Rightarrow dx = \frac{du}{9}$$

$$L = \frac{1}{2}\int_{x=0}^{x=4} u^{1/2}\,\frac{du}{9} = \frac{1}{18}\left(\frac{2}{3}u^{3/2}\right)\Big|$$

$$\frac{1}{27}(9x+4)^{3/2}\Big|_0^4 = \frac{1}{27}\left(40^{3/2} - 8\right) \approx 8.9$$

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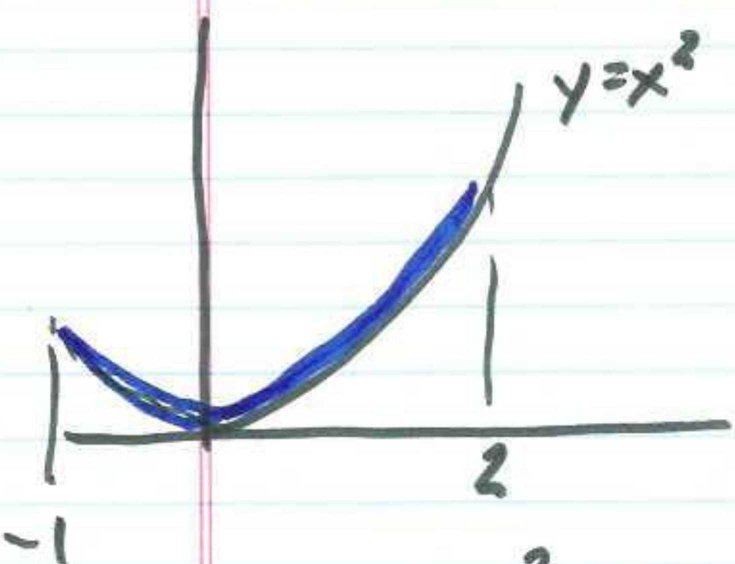

$$L = \int_{-1}^{2} \sqrt{1 + 4x^2}\, dx$$

$$\downarrow$$

$$2\sqrt{\frac{1}{4} + x^2} \qquad \left.\begin{matrix} a = 1/2 \\ x = x \end{matrix}\right\}$$

$$\int \sqrt{a^2+x^2} = \frac{x}{2}\sqrt{a^2+x^2} + \frac{a^2}{2}\ln\left|x + \sqrt{a^2+x^2}\right|$$

18) $y = \sin x - x\cos x \qquad 0 \le x \le \pi$

$$y' = \underbrace{\cos x - [(1)\cos x}_{0} - x\sin x]$$

$$y' = x\sin x$$

$$L = \int_0^{\pi} \sqrt{1 + x^2\sin^2 x}\, dx$$

$u = x\sin x \qquad du = [(1)\sin x + x\cos x]\, dx$

$$L = \int_{x=0}^{\pi} \sqrt{1+u^2}\ \frac{du}{\sin x + x\cos x}$$

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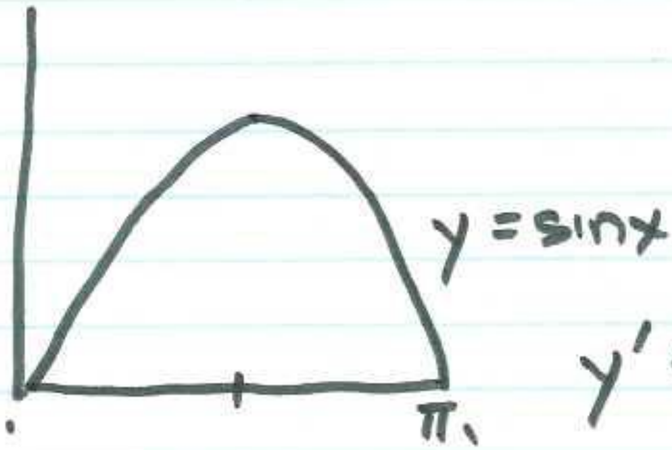

$y' = \cos x.$

$$L = \int_0^{\pi} \sqrt{1+\cos^2 x}\, dx$$

Try $u = \cos x$ $\quad du = -\sin x\, dx$

$$\int_0^{\pi} \sqrt{1+u^2}\, \frac{du}{-\sin x} \qquad \sin^2 x + \cos^2 x = 1$$

$\cosh^2 x = 1 + \sinh^2 x$

Try $\cos x = \sinh u$

$1 + \cos^2 x \Rightarrow 1 + \sinh^2 u = \cosh^2 u$

$$\sqrt{\cosh^2 u}$$

$$-\sin x = \cosh u \frac{du}{dx}$$

$$\frac{-\sin x \, dx}{\cosh u} = du \qquad dx = -\frac{\cosh u \, du}{\sin x}$$

$$\int \cosh u$$ OOPS!

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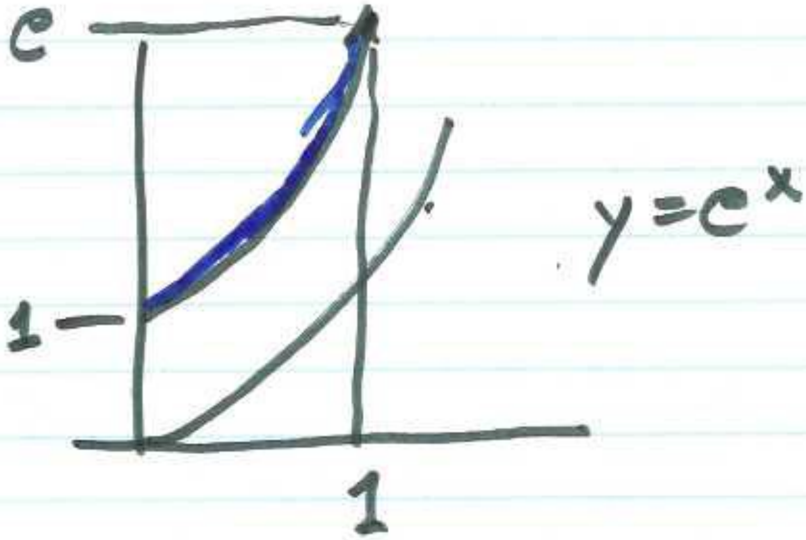


$$y' = e^x \,, \quad \sqrt{1+(y')^2} = \sqrt{1+e^{2x}}$$

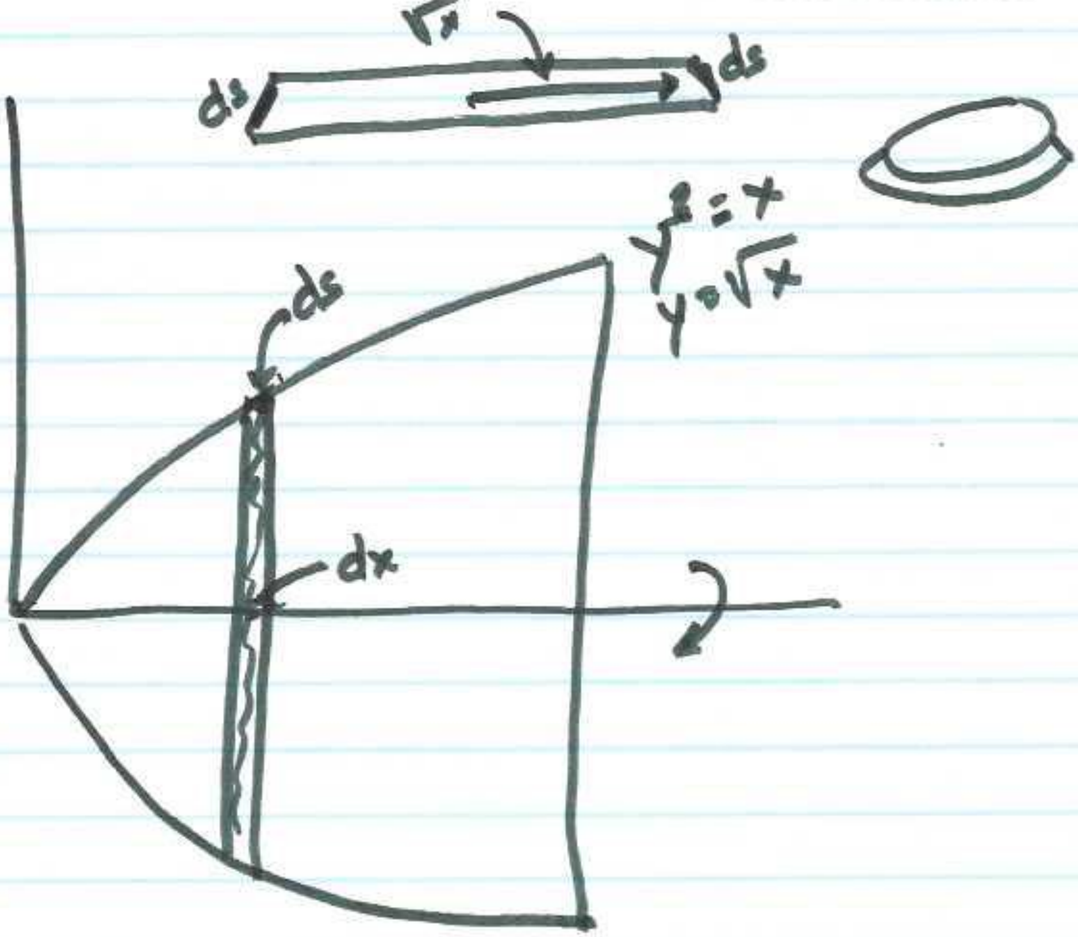
$$L = \int_0^1 \sqrt{1+e^{2x}}\,dx$$

Let $u = e^x$ $\quad du = e^x dx \quad dx = \frac{du}{e^x} \leftarrow u$

$$L = \int_{x=0}^{x=1} \sqrt{1+u^2}\,\frac{du}{u} = \int_{x=0}^{x=1} \sqrt{\frac{1}{u^2}+\frac{u^2}{u^2}}\,du$$

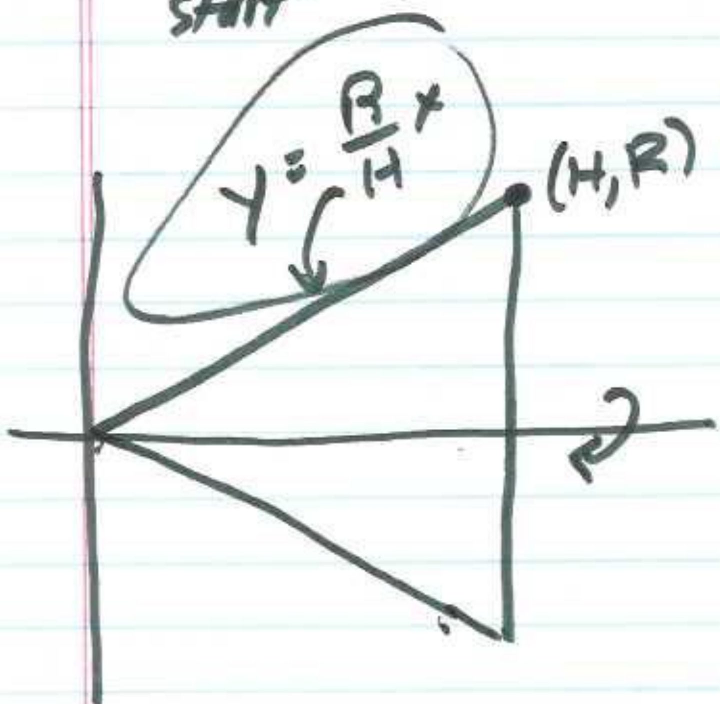
$$L = \int_{x=0}^{1} \sqrt{1+\frac{1}{u^2}}\,du = \int_{0}^{1} \frac{\sqrt{u^2+1}}{u}\,du$$

$$\star \int \frac{\sqrt{a^2+x^2}}{x}\,dx = \sqrt{a^2+x^2} - a\ln\left|\frac{a+\sqrt{a^2+x^2}}{x}\right|$$



$dA(x) = 2\pi\sqrt{x}\cdot ds$ Gen'l $dA(x) = \underline{2\pi y\cdot ds}$

$$S = \int_{\text{start}}^{\text{stop}} 2\pi y \sqrt{1+(y')^2}\, dx$$



$$\sqrt{\frac{H^2+R^2}{H^2}} = \frac{\sqrt{H^2+R^2}}{H}$$

$$S = \int_0^H 2\pi\left(\frac{Rx}{H}\right)\sqrt{1+\left(\frac{R}{H}\right)^2}\, dx$$

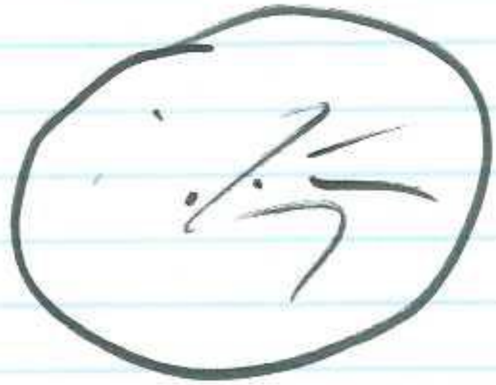
$$= 2\pi\, \frac{R\sqrt{1+R^2/H^2}}{H} \cdot \int_0^H x\, dx$$

$$= \frac{2\pi R}{H} \cdot \frac{\sqrt{R^2+H^2}}{H} \int_0^H x\, dx$$

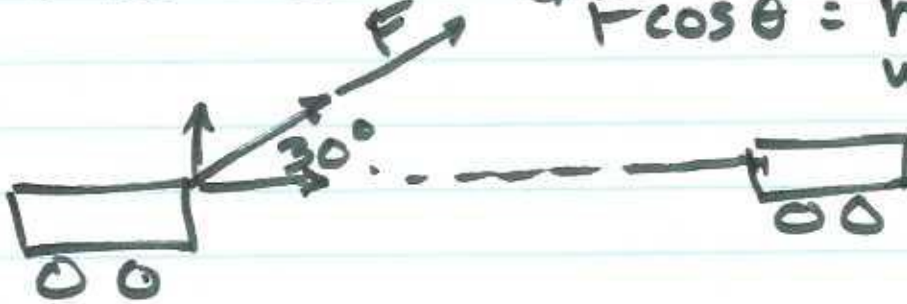
$$= \frac{2\pi R\sqrt{R^2+H^2}}{H^2}\left[\frac{x^2}{2}\right]_0^H$$

$$= \pi R\sqrt{R^2+H^2}$$

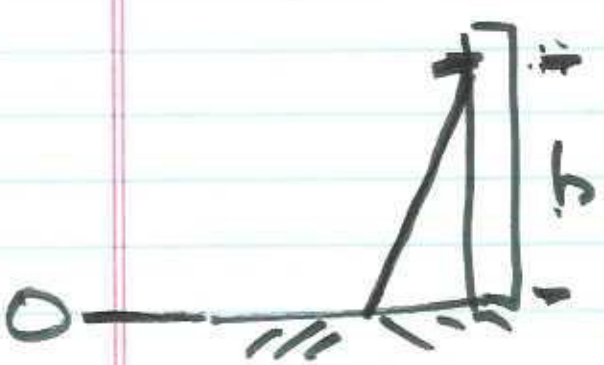
$$= \pi R\sqrt{R^2}$$



$F \cdot L = W$ = energy



$F\cos\theta$ = horiz work



M, g $W = Mg$

$g = 32.16 \text{ fps}^2$
$= 9.81 \text{ m/s}^2$

$Mg = F$

dist $= h$

So work to get to top $= Mgh$

This is also your potential energy

(relative to datum)

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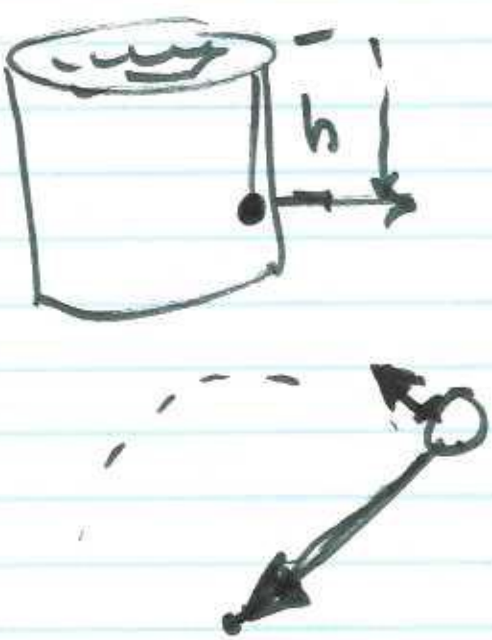
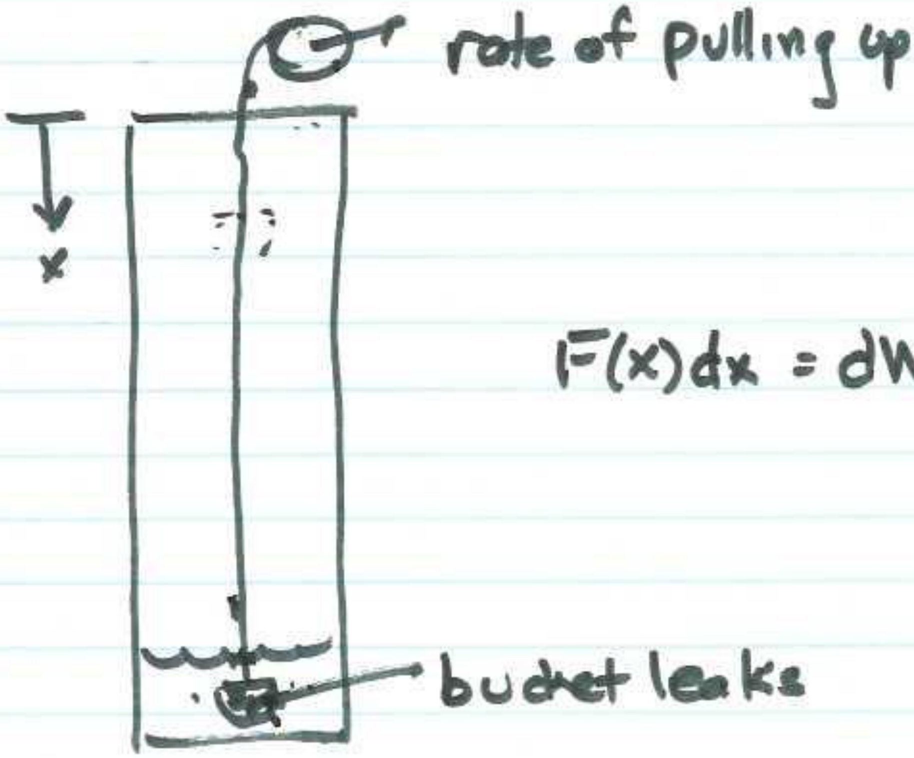
KE (energy of motion) $= \frac{Mv^2}{2}$

$\cancel{M}gh = \frac{\cancel{M}v^2}{2}$ $\quad$ $\underline{v = \sqrt{2gh}}$



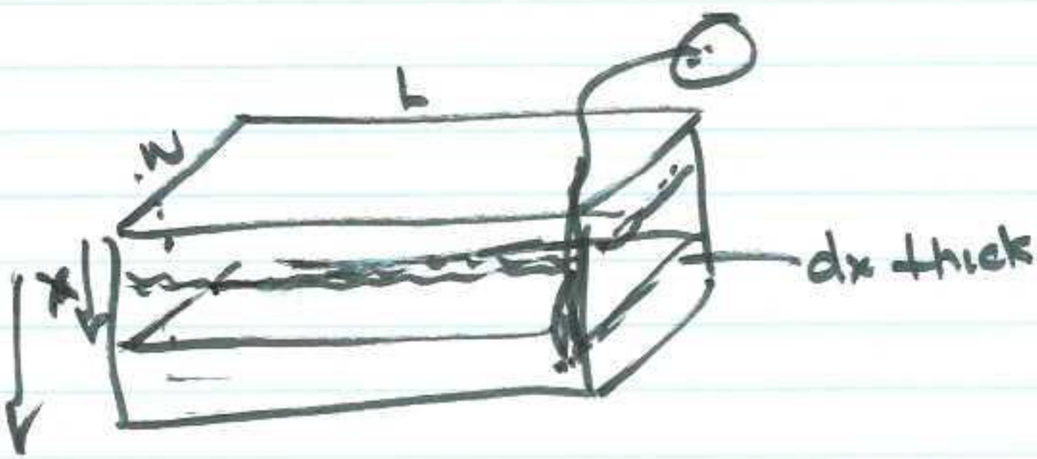
$F \cdot d \cos\theta = 0$

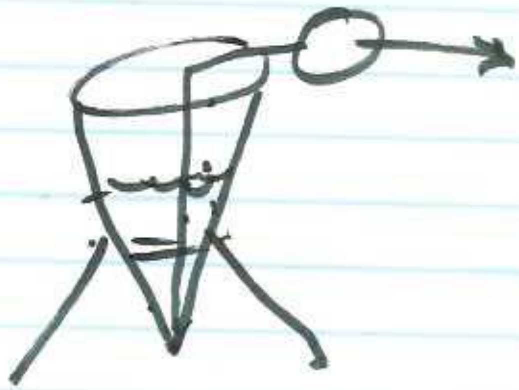
$\theta = 90^\circ$



$$F(x)dx = dW(x)$$

$$W = \int F(x)dx$$





$F = -kx$

$W = \frac{1}{2} F x^2$