UYZMD

8-25 ①

$$\int \frac{\sin\sqrt{\theta}}{\sqrt{\theta \cos^3\sqrt{\theta}}}\, d\theta = ?$$

Best substitution is $u = \cos\sqrt{\theta}$

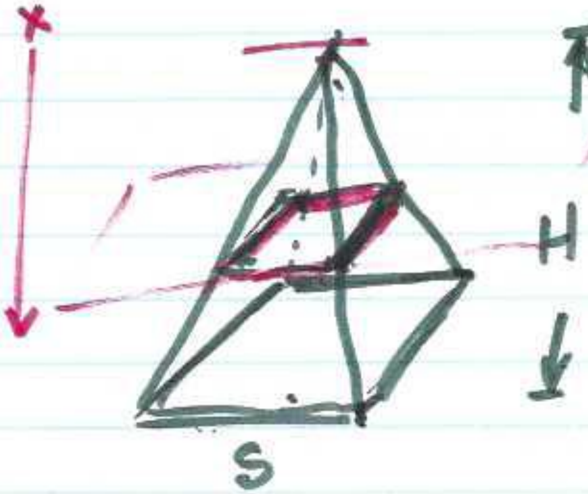
$$du = -\sin\sqrt{\theta} \cdot \frac{1}{2\sqrt{\theta}}\, d\theta$$

invert $d\theta = du \cdot \left[\frac{2\sqrt{\theta}}{-\sin\sqrt{\theta}}\right]$

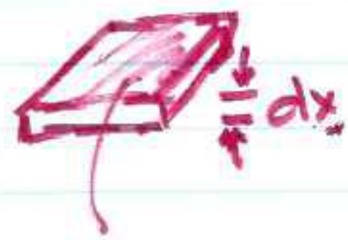
Plug in: $\displaystyle\int \frac{\cancel{\sin\sqrt{\theta}}}{\cancel{\sqrt{\theta}}\sqrt{\cos^3\sqrt{\theta}}}\, du \left[\frac{2\cancel{\sqrt{\theta}}}{-\cancel{\sin\sqrt{\theta}}}\right]$

$$2\int \frac{-du}{\sqrt{u^3}} = 2\int -u^{-3/2}\, du = -2\, u^{-\frac{1}{2}}(-2)$$

$$= \frac{4}{\sqrt{u}} = \frac{4}{\sqrt{\cos\sqrt{\theta}}} + C$$



$A(x)dx = dV(x)$



A(x) is area of section

Method of Sections

$$Vol = \int_0^H dV(x) = \int_0^H A(x)\,dx$$

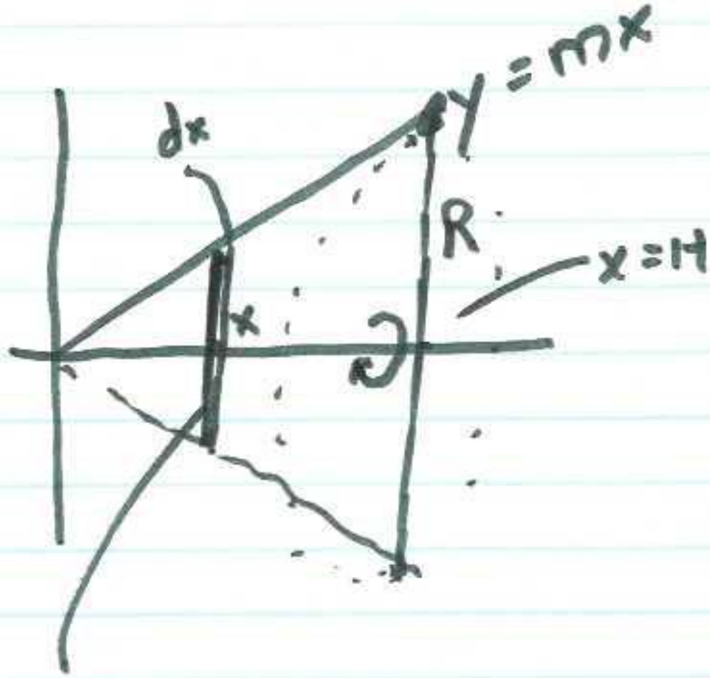
$$\frac{S(x)}{S} = \frac{x}{H} \quad \text{so} \quad S(x) = \frac{S}{H}x \checkmark$$

$$A(x) = \left[S(x)\right]^2 = \frac{S^2}{H^2}\cdot x^2$$

$$V = \int_0^H \frac{S^2}{H^2}x^2\,dx = \frac{S^2}{H^2}\left[\frac{x^3}{3}\right]_0^H = \frac{S^2H^3}{3H^2}$$

$\frac{1}{3}S^2H$ $\quad$ $\frac{1}{3}\pi R^2H$

③

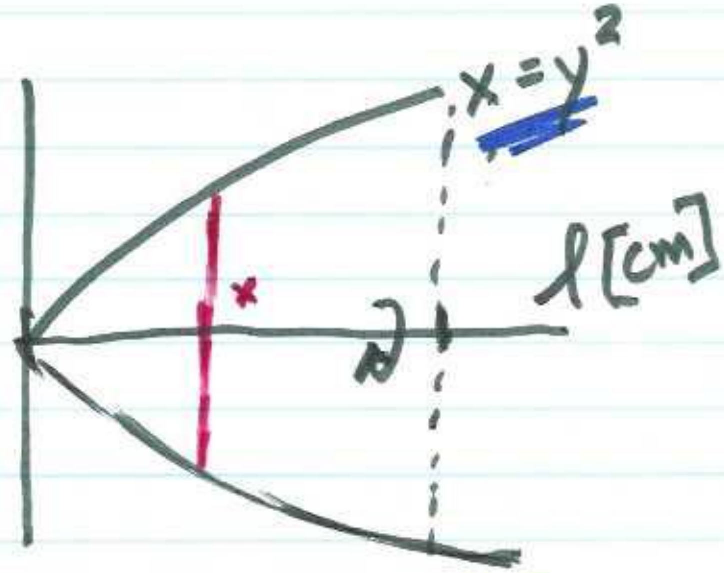


$$A(x) = \pi(mx)^2 = \pi m^2 x^2$$

$$A(x)dx = dV(x) = \pi m^2 x^2 dx$$

$$V = \int_0^H \pi m^2 x^2 dx = \pi m^2 \left[\frac{x^3}{3}\right]_0^H$$

$$= \frac{\pi m^2 H^3}{3} = \frac{\pi R^2 H}{3}$$



$$A(x) = (\overset{\text{radius}}{\sqrt{x}})^2 \cdot \pi = \pi x$$

$$dV(x) = \pi x dx$$

$$Vol = \int_0^l \pi x dx = \left[\frac{\pi x^2}{2}\right]_0^l = \frac{\pi}{2} l^2 \,[cm]^3$$

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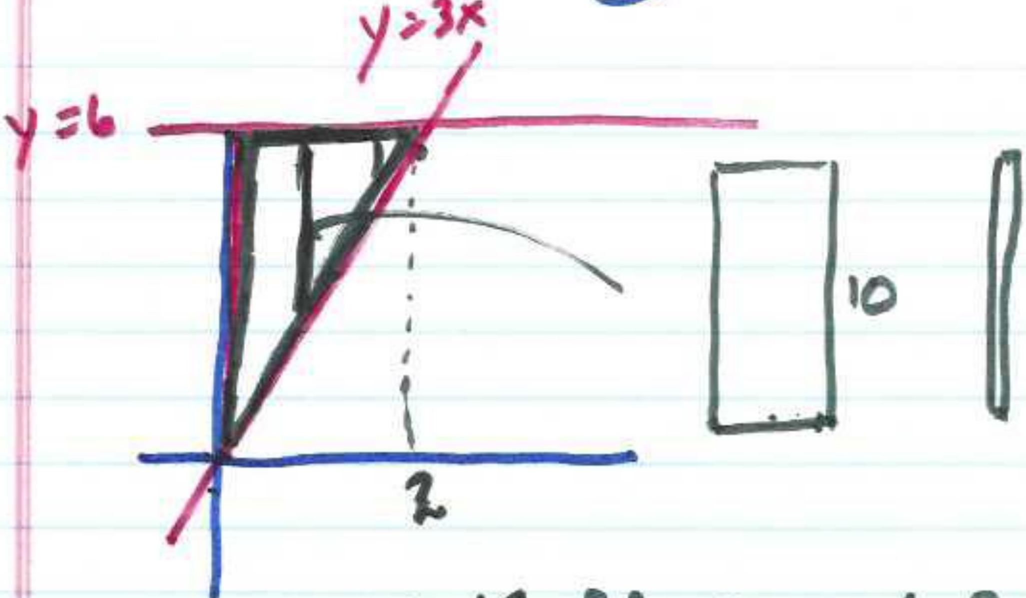


width ~~[illegible]~~ of fig @ x is $6-3x$

so.. $A(x) = 10(6-3x) = 60-30x$

$dV(x) = (60-30x)\,dx$

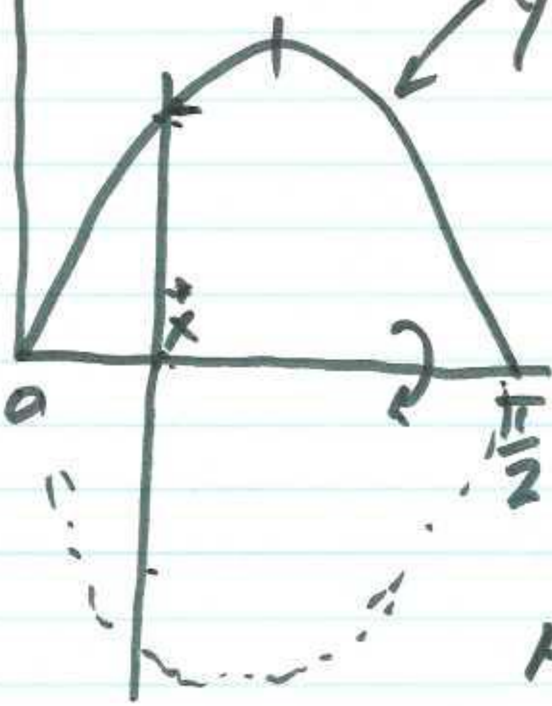
$$\text{Vol} = \int_0^2 (60-30x)\,dx = \left[60x-15x^2\right]_0^2$$

$120-60 = 60$ vol units

(6)

$y = \sin x \cos x$

$\sin 2x = 2\sin x \cos x$



$\frac{\sin 2x}{2} = \sin x \cos x$

$A(x) = \pi(\sin x \cos x)^2$

$dV(x) = \pi \sin^2 x \cos^2 x \, dx$

$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$

$$Vol = \pi\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx = ?$$

$\downarrow$

$$= \frac{\pi}{4}\int_0^{\pi/2} \sin^2 2x \, dx = \frac{\pi}{4}\int_0^{\pi/2} \left(\frac{1-\cos 4x}{2}\right) dx$$

$$= \frac{\pi}{8}\int_0^{\pi/2} (1-\cos 4x)\, dx = \frac{\pi^2}{16}$$

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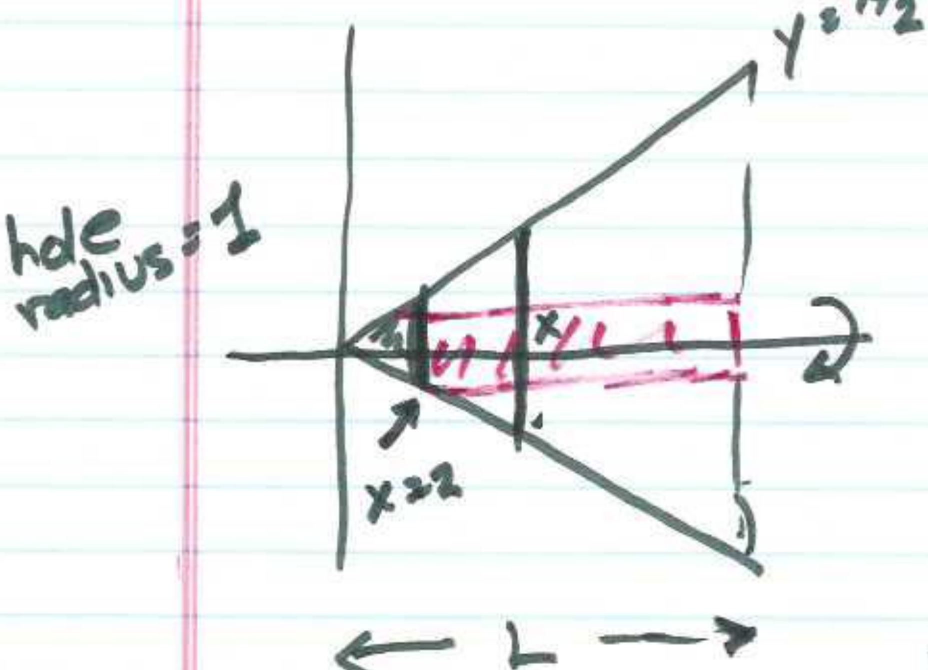

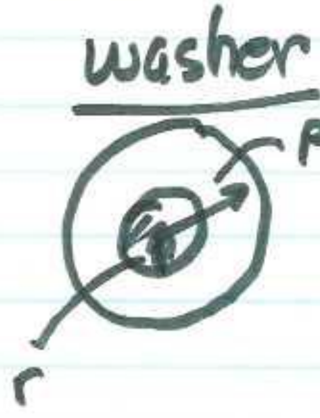

$$\pi\left(\frac{x}{2}\right)^2 - \pi(1^2)$$

R r

from 2 to L

$$A(x) = \left(\frac{\pi x^2}{4} - \pi\right)$$

$$dV(x) = \pi\left(\frac{x^2}{4} - 1\right)dx$$

$$\pi\int_2^L \left(\frac{x^2}{4} - 1\right)dx = \pi\left[\frac{x^3}{12} - x\right]_2^L \sim$$

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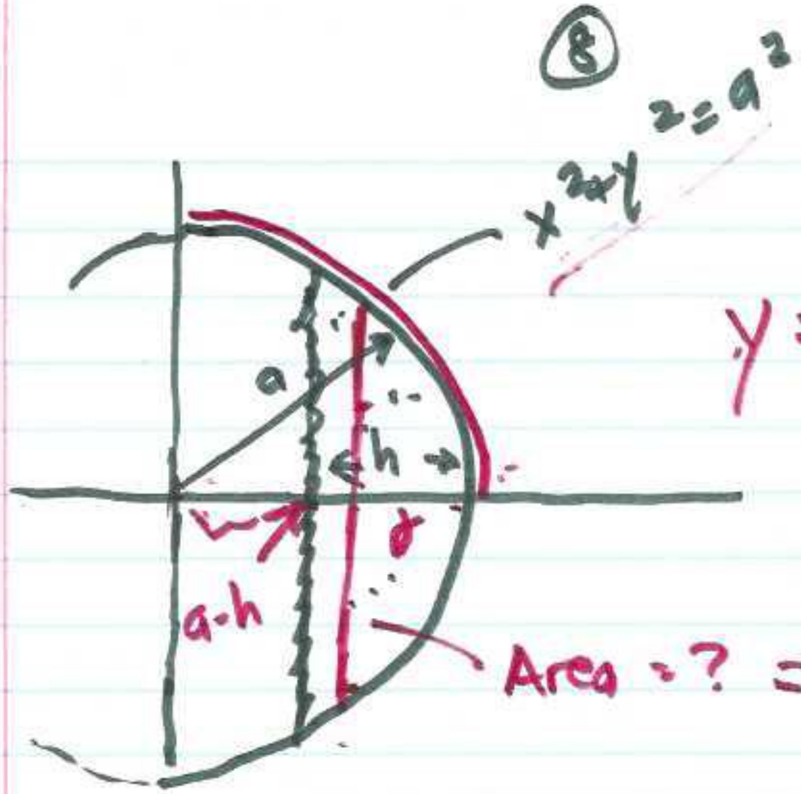
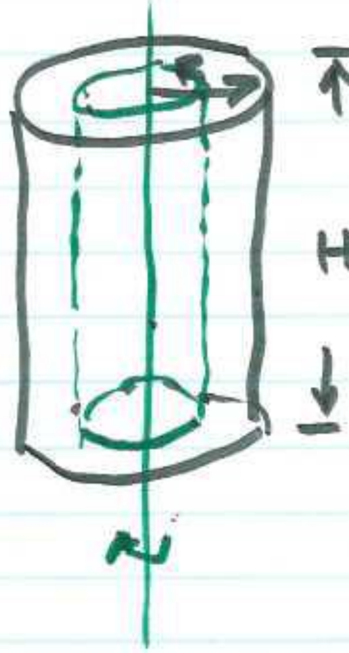


$y = \sqrt{a^2 - x^2}$

Area = ? $= \pi(a^2 - x^2) = A(x)$

$dV(x) = A(x)dx = \pi(a^2-x^2)dx$

$\pi\int_{a-h}^{a}(a^2-x^2)dx \quad \pi\left[a^2x - \frac{x^3}{3}\right]_{a-h}^{a}$

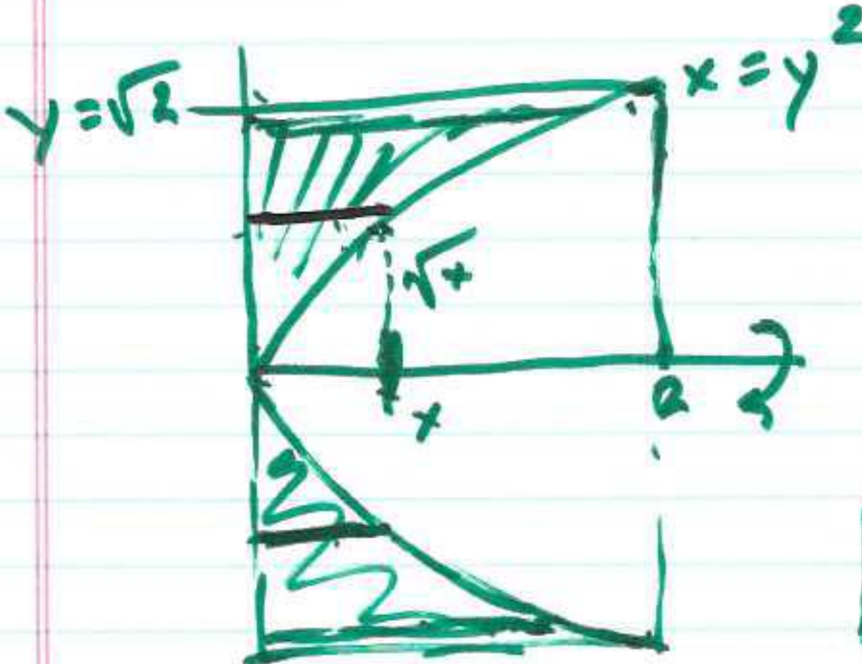


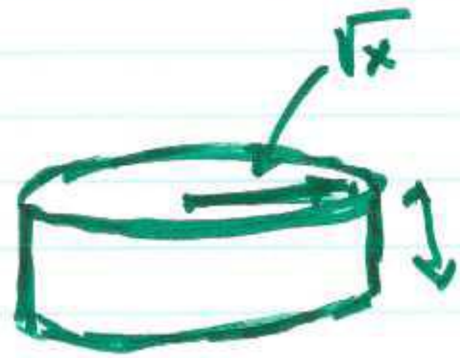
$\underline{\underline{\pi R^2 H}}$

$$\int_0^R 2\pi r\,dr \cdot H = V$$

$$H \cdot 2\pi \int_0^R r\,dr = 2\pi H\left(\frac{r^2}{2}\right)\Big|_0^R$$

$$= \cancel{2}\pi H \frac{R^2}{\cancel{2}} = \underline{\underline{\pi R^2 H}}$$





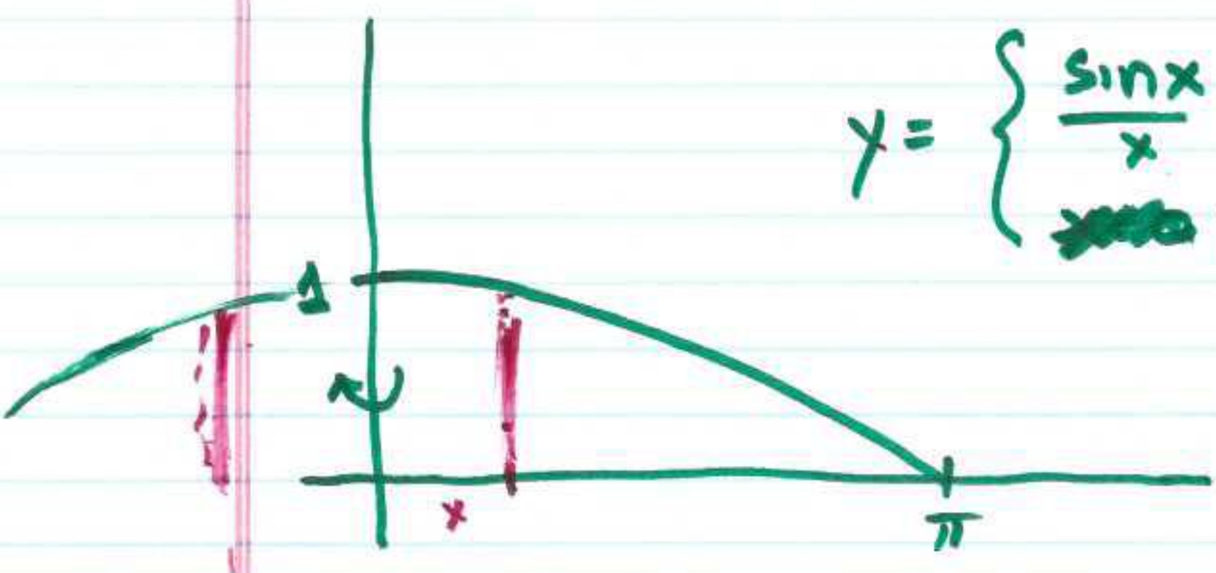
$$dV(x) = 2\pi\sqrt{x}\,dx \cdot x$$

$$\text{Vol} = \int_0^2 2\pi x^{3/2}\,dx = 2\pi\left(\frac{2}{5}\right)x^{5/2}\Big|_0^2$$

$$= \frac{4\pi}{5}\left(4\sqrt{2}\right)$$

$$= \frac{16\pi\sqrt{2}}{5}$$

$$y = \begin{cases} \dfrac{\sin x}{x} & x > 0 \\ 1 & \text{if } x = 0 \end{cases}$$



$$2\pi \cancel{x}\,dx \cdot \underbrace{\frac{\sin x}{\cancel{x}}}^{y}$$

$$\text{Vol} = \int_0^{\pi} 2\pi \sin x\,dx = -2\pi\big[\cos x\big]_0^{\pi}$$

$$2\pi + 2\pi = 4\pi$$

$$-2\pi\big[\cos\pi - \cos 0\big]$$

$$(-1 - 1)$$

$$(-2\pi)(-2)$$