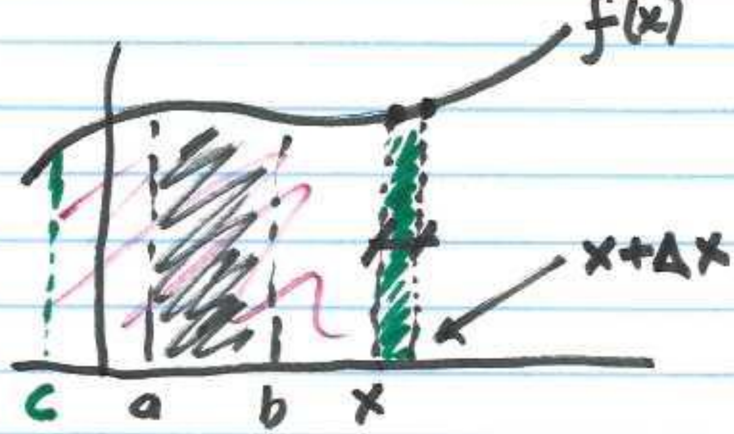
① 5:30 8·20



F(x) is the area under curve / above x-axis / right of c / stops @ x

(i) $\lim_{\Delta x \to 0} \dfrac{F(x+\Delta x) - F(x)}{\Delta x}$

(ii) $F(x+\Delta x) - F(x) = f(x) \cdot \Delta x$

Combine rate of chg of F(x) $= \lim_{\Delta x \to 0} \dfrac{f(x) \cdot \cancel{\Delta x}}{\cancel{\Delta x}}$

$$F'(x) = f(x)$$

Greek "x"

$$F(x) = \int_c^x f(x)\,dx \quad \left(\text{or } \int_c^x f(\xi)\,d\xi\right)$$

$$F(b) - F(a) = \int_a^b f(x)\,dx \quad \text{FTC}$$

Method of Substitution:

$$\int f(u)\,du \leftarrow \text{difficult}$$

$$u = g(x) \qquad \frac{du}{dx} = g'(x) \qquad du = g'(x)\,dx$$

$$\int f(g(x))\,g'(x)\,dx$$

Ex: $\int \sin^3 u \cos u \, du$

Let $u = \sin x$ then $\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

Ex: $\int x e^{x^2} dx$

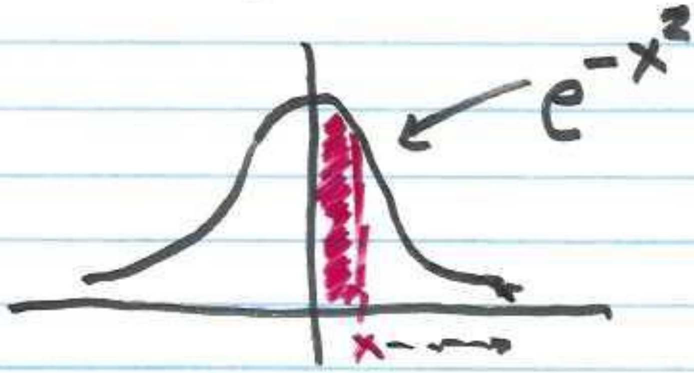
Let $y = x^2$ $\quad \frac{dy}{dx} = 2x \Rightarrow dy = 2x\,dx$

$\frac{dy}{2x} = dx$

$$\int \frac{\cancel{x} e^y dy}{2\cancel{x}} = \frac{1}{2}\int e^y dy = \frac{e^y}{2} + C$$

$$= \boxed{\frac{1}{2}e^{x^2} + C}$$

$$\int e^{x^2} dx, \quad \int \frac{dx}{\ln x}$$



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1) $\int 7(\sqrt{7x-1})\,dx$

Let $y = 7x-1$ $\quad dy = 7dx$ or $dx = \frac{1}{7}dy$

$\int 7y^{1/2}\frac{1}{7}dy = \int y^{1/2}dy = \frac{2}{3}y^{3/2} + C$

$= \frac{2}{3}(7x-1)^{3/2} + C$

2) $\int \frac{(1+\sqrt{x})^{1/3}}{\sqrt{x}}\,dx$

Let $y = 1+\sqrt{x}$ $\quad dy = \frac{1}{2\sqrt{x}}dx$

$= \int \frac{y^{1/3}}{\cancel{\sqrt{x}}}(2\cancel{\sqrt{x}})dy$ $\quad dx = 2\sqrt{x}\,dy$

$= 2\int y^{1/3}dy = \cancel{2}\cdot\frac{3}{\cancel{4}_2}\cdot y^{4/3} + C$

$= \frac{3}{2}y^{4/3} + C$

$$= \boxed{\frac{3}{2}\left(1+\sqrt{x}\right)^{4/3}+C}$$

3) $\int \frac{1}{x^2}\cos^2\left(\frac{1}{x}\right)dx$

$y = \frac{1}{x} \quad dy = -\frac{1}{x^2}dx \Rightarrow dx = -x^2 dy$

$\int \frac{1}{\cancel{x^2}}\cos^2 y\left(-\cancel{x^2}dy\right) = -\int \cos^2 y\,dy$

$\cos^2\left(\frac{\theta}{2}\right) = \frac{1+\cos\theta}{2}$ (half-angle)

so... $\cos^2 y = \frac{1+\cos 2y}{2}$

$-\int\left(1+\frac{1}{2}\cos 2y\right)dy = -y - \frac{1}{2}\left(\frac{\sin 2y}{2}\right)$

$$\boxed{-\frac{1}{x} - \frac{1}{2}\sin\left(\frac{1}{x}\right) + C}$$

Trig Review

α, β

basic
$$\boxed{\begin{cases} \sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \\ \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta \end{cases}}$$

$$\sin(2\alpha) = \sin(\alpha+\alpha) = \sin\alpha\cos\alpha + \cos\alpha\sin\alpha$$
$$\boxed{= 2\sin\alpha\cos\alpha}$$

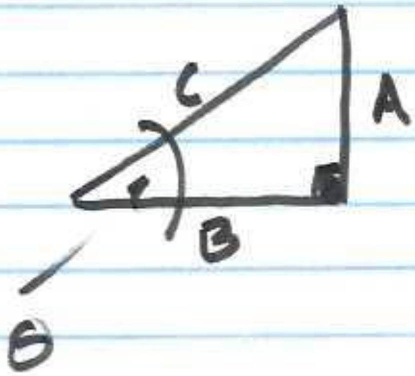
$$\cos(2\alpha) = \cos(\alpha+\alpha) =$$
$$\cos\alpha\cos\alpha - \sin\alpha\sin\alpha = \boxed{\cos^2\alpha - \sin^2\alpha}$$

1) $\cos^2\alpha + \sin^2\alpha = 1$

so... $\cos(2\alpha) = 1 - \sin^2\alpha - \sin^2\alpha = \boxed{1 - 2\sin^2\alpha}$

and $\cos(2\alpha) = \cos^2\alpha - (1 - \cos^2\alpha) =$

$\boxed{2\cos^2\alpha - 1}$



$$A^2 + B^2 = C^2$$

$$\frac{A^2}{A^2} + \frac{B^2}{A^2} = \frac{C^2}{A^2}$$

(i) $\boxed{1 + \cot^2\theta = \csc^2\theta}$

$$\frac{A^2}{B^2} + \frac{B^2}{B^2} = \frac{C^2}{B^2}$$

(ii) $\boxed{\tan^2\theta + 1 = \sec^2\theta}$

$$\frac{A^2}{C^2} + \frac{B^2}{C^2} = \frac{C^2}{C^2}$$

$\boxed{\sin^2\theta + \cos^2\theta = 1}$ ✓

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \tan\beta}$$

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(i) $\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$

(ii) $\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$

(iii) $\tan^2\frac{\theta}{2} \quad \frac{1-\cos\theta}{1+\cos\theta}$

$s, -c, -t$	s, c, t
$-s, -c, t$	$-s, c, -t$

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$$\int \csc^2(2\theta)\cot(2\theta)\,d\theta$$

$$y = \csc(2\theta) \Rightarrow \cancel{dy =}$$ [redacted]

$$dy = \csc(2\theta)\cdot\cot(2\theta)(2)\,d\theta$$

$$\frac{1}{2}\int y^2 \cancel{\cot(2\theta)} \frac{dy}{\csc(2\theta)\cancel{\cot(2\theta)}}$$

$$\frac{1}{2}\int y^2 \cdot \cancel{\frac{\cos(2\theta)}{\sin(2\theta)} \cdot \frac{1}{\cos(2\theta)}}\, dy$$

$$\frac{1}{2}\int y^2 \cdot y\, dy = \frac{1}{2}\int y^3 dy = \frac{y^4}{8} + C$$

$$\boxed{\frac{\csc^4(2\theta)}{8} + C}$$

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$$\int (\cos x) e^{\sin x} dx$$

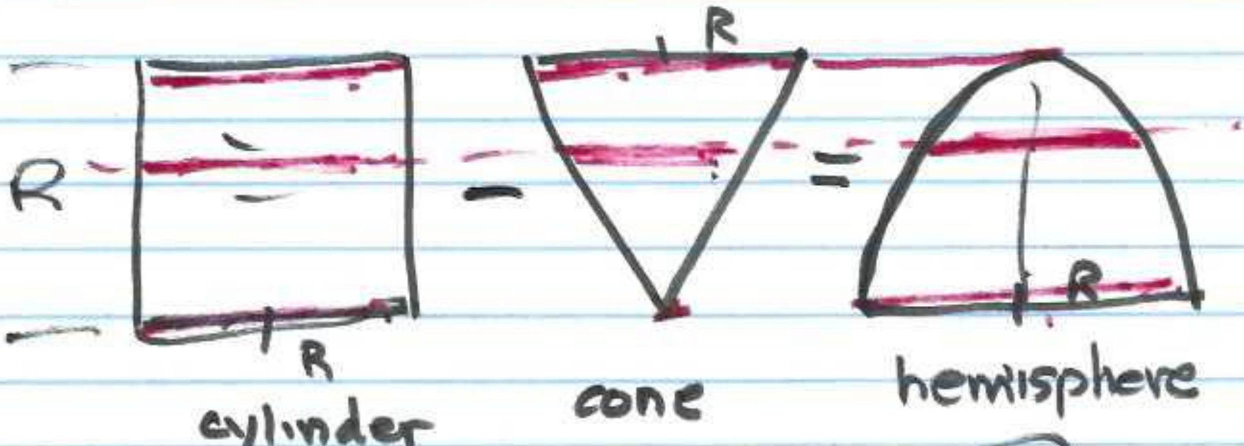
Try: $y = \sin x$ $dy = \cos x\, dx$

$dx = \dfrac{dy}{\cos x}$

$$\int \cancel{\frac{\cos x}{\cos x}} e^y dy = \int e^y dy = e^y + C,$$

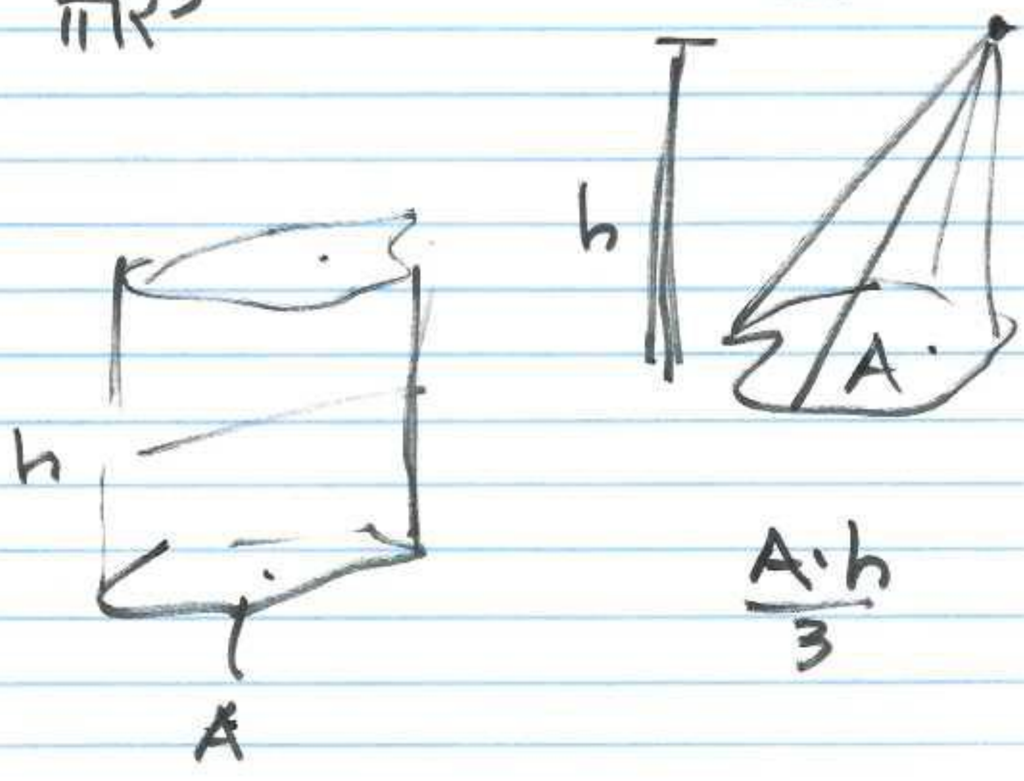
$$\boxed{e^{\sin x} + C}$$

6.1 - Volumes by "Method of Disks"



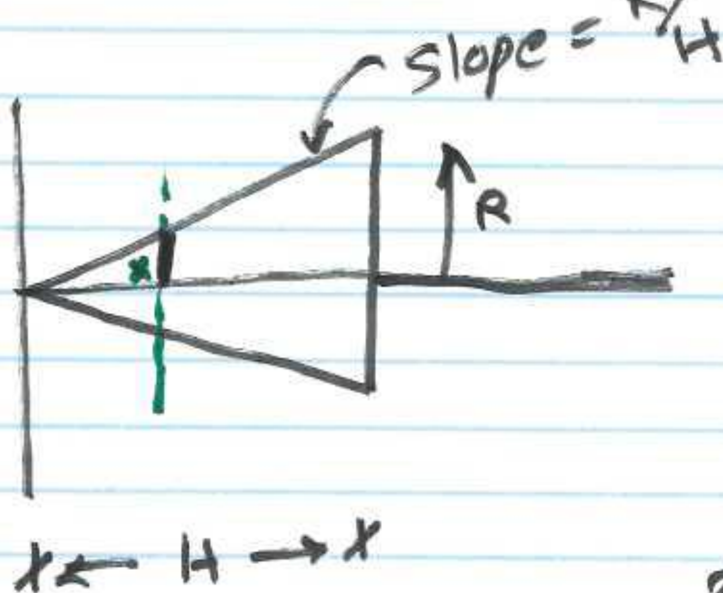
$$(\pi R^2 \cdot R) - \frac{1}{3}\pi R^3 = \frac{2}{3}\pi R^3$$

$$\pi R^3$$



$$\frac{A \cdot h}{3}$$

(12)



$$\pi\left(\frac{R}{H}\cdot x\right)^2 = A(x)$$

A(x)

$$dV(x) = \frac{\pi R^2 x^2}{H^2}\cdot dx$$

$$V = \int_0^H \frac{\pi R^2 x^2}{H^2}\,dx = \frac{\pi R^2}{H^2}\int_0^H x^2\,dx$$

$$= \frac{\pi R^2}{H^2}\left[\frac{x^3}{3}\right]_0^H = \frac{\pi R^2}{3H^2}\cdot H^3 = \boxed{\frac{\pi^2 R^2 H}{3}}$$