XN9ZT

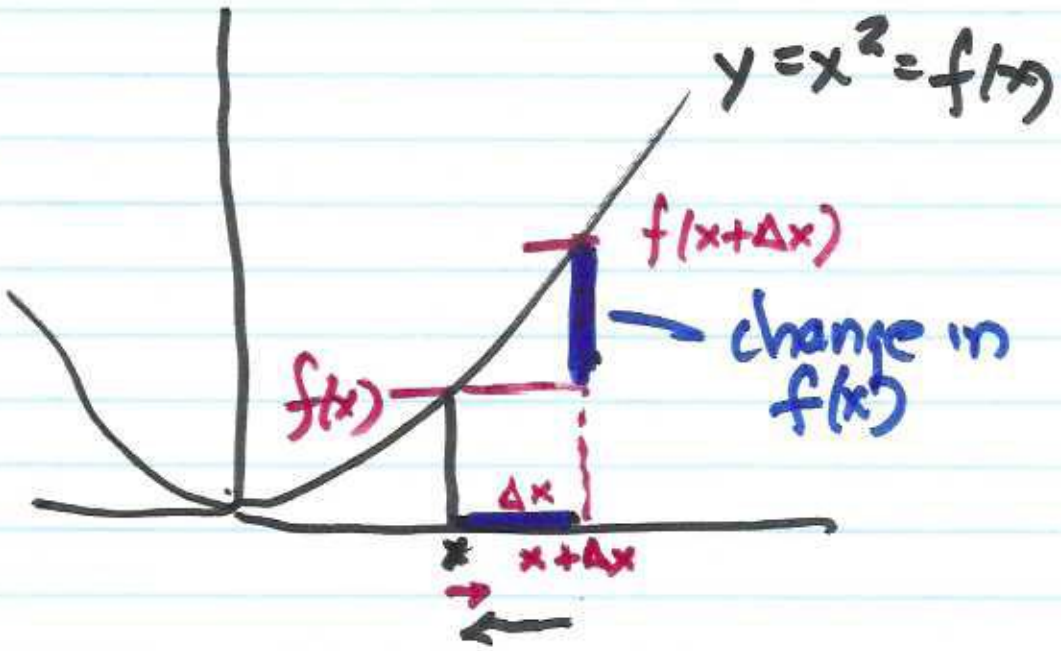
8/11/16

①

$f(x)$ $\quad f'(x), \frac{df}{dx}, D_x f$

Def: $f'(x) = \lim_{\Delta x \to 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} \right)$

Δ always implies an increment



$f(x) = x^2$

$f(x+\Delta x) = (x+\Delta x)^2 = x^2 + 2x\Delta x + (\Delta x)^2$

(2)

$$f'(x) = \lim_{\Delta x \to 0} \left(\frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \right)$$

$$= \lim_{\Delta x \to 0} \left(2x + \underbrace{\Delta x}_{0} \right) = 2x$$

$f(x)$	$f'(x)$	
c (constant)	0	
x	1	
cx	c	
x^2	$2x$	
x^3	$3x^2$	
$\vdots$		
x^n	nx^{n-1}	$\frac{1}{x} = x^{(-1)}$
e^x	e^x	
$e^{g(x)}$	$e^{g(x)} \cdot g'(x)$	
$\ln x$	$\frac{1}{x}$	
2^x	$(\ln 2)2^x$	

$g(x)$ is diff function

$e^{\ln 2 \cdot x}$

$\tanh x = \frac{\sinh x}{\cosh x}$

④ $\sinh x = \frac{e^x - e^{-x}}{2}$

S C S C S

$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\csc x$	$-\csc x \cot x$
$\sec x$	$\sec x \tan x$
$\tan x$	$\sec^2 x$
$\cot x$	$-\csc^2 x$
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	
$\coth x$	
$\operatorname{csch} x$	
$\operatorname{sech} x$	

otn x · cotx

⑤

$\sin^{-1}x \stackrel{?}{=} \frac{1}{\sin x}$ False

$\sin^{-1}x = \frac{1}{\sin^2 x}$ True

$f \quad f^{-1}$

Call $\sin^{-1}x$ | arcsin x

arccos x

arctan x

arccot x

arcsec x

arccsc x

Use these

f	f'
$\ln(g(x))$	$\frac{g'(x)}{g(x)}$ } relative change (%) in g

Some rules

1) Product Rule $f(x) = g(x) \cdot h(x)$

$$f'(x) = g'(x)h(x) + g(x)h'(x)$$

2) Quotient Rule $f(x) = \frac{g(x)}{h(x)}$

$$f'(x) = \frac{h(x)g'(x) - h'(x)g(x)}{[h(x)]^2}$$

Workaround $\frac{g(x)}{h(x)} = g(x) \cdot \frac{1}{h(x)}$

3) Chain Rule :

composition $(f \circ g)(x) = f(g(x))$ — $f \circ g$

$f \circ g = g \circ f$

g goes first

g is precomposed with f

f is postcomposed with g

C.R. $f(g(x))' = f'(g(x)) \cdot g'(x)$

$\frac{d}{dx}(x^x)$ $x = e^{\ln x}$ ~~$x \cdot x^{x-1} = x^x$~~

$\frac{d}{dx}\left(e^{\ln x}\right)^x = \frac{d}{dx}\left(e^{x \ln x}\right)$

$= \underbrace{e^{x \ln x}}_{x^x} \cdot \frac{d}{dx}(x \ln x)$

$= x^x \left[(1)\ln x + x\left(\frac{1}{x}\right)\right]$

$= x^x \left[\ln x + 1\right]$ ✓

$$\frac{d}{dx}(\sin x)^{\cos x} = ?$$

$$e^{g(x)}{}'$$

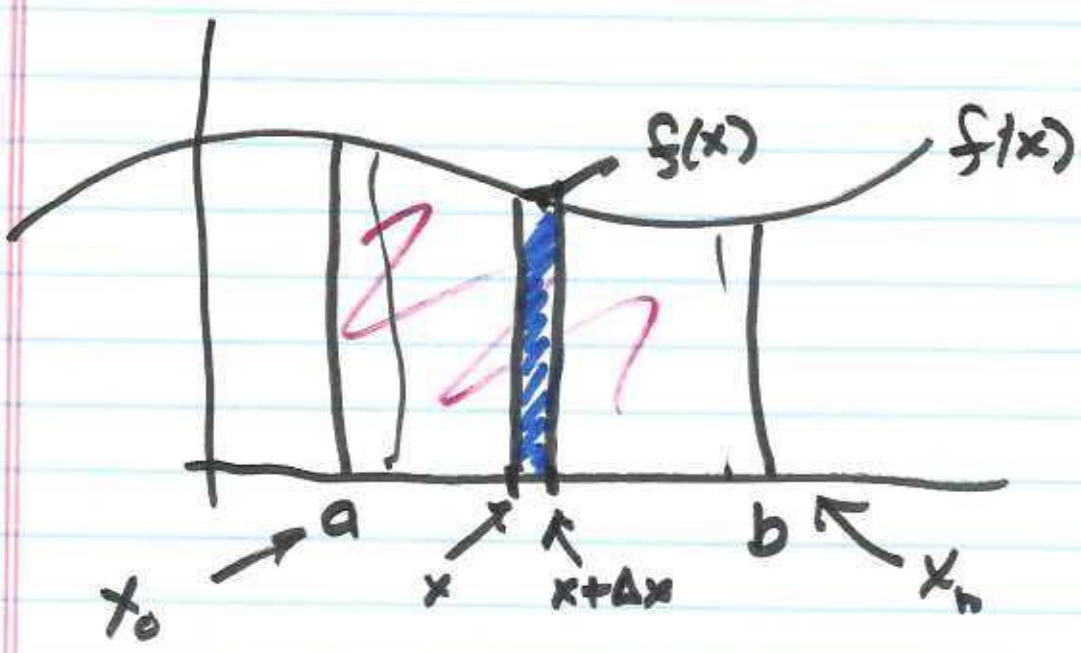
$$\left[e^{\ln \sin x}\right]^{\cos x} = e^{\cos x \cdot \ln \sin x}{}'$$

$$e^{\cos x \cdot \ln \sin x} \cdot \left[-\sin x \cdot \ln \sin x + \right.$$

$$\cos x \cdot$$

What is $(\ln \sin x)' = \frac{\cos x}{\sin x} = \cot x$

★ $(\sin x)^{\cos x}(-\sin x \ln \sin x + \cot x)$



area of strip $\Delta x \cdot f(x)$

$$\sum_{x=x_0}^{x_n} f(x)\Delta x \approx \text{Total Area}$$

$$\text{Area} = \int_a^b f(x)dx \quad \text{Riemann integral}$$