

10/5
late
①

8.1

$$\int \frac{dx}{\sqrt{8x-x^2}}$$

$$x^2+ax+b$$
$$\left(\frac{a}{2}\right)^2$$

$$\frac{1}{\sqrt{8x-x^2}} = \frac{1}{\sqrt{-(x^2-8x+16)+16}}$$
$$-(x-4)^2+16$$

$$\text{Let } x-4 = u \quad dx = du$$

$$\int \frac{du}{\sqrt{16-u^2}} = \arcsin\left(\frac{u}{4}\right)$$

$$\boxed{\arcsin\left(\frac{x-4}{4}\right) + C}$$

②

Ex 2 $\int (\cos x \sin 2x + \sin x \cos 2x) dx$

$$\sin(\alpha + \beta) = \underbrace{\sin \alpha}_{2x} \underbrace{\cos \beta}_x + \underbrace{\cos \alpha}_{2x} \underbrace{\sin \beta}_x$$

$$\sin(2x + x) = \underline{\sin 3x}$$

$$= \int \underline{\sin 3x} dx$$

$$= \underline{-\frac{1}{3} \cos 3x}$$

$$\left(\frac{-\cos 3x}{3} \right)$$

$$(B) - \frac{(-\sin 3x)}{3}$$

$$+ \frac{\sin 3x}{3}$$

(3)

$$\frac{1}{2+\sqrt{3}} \cdot \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\int_0^{\pi/4} \frac{dx}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} \rightarrow$$

$$\int_0^{\pi/4} \frac{(1+\sin x)dx}{\underbrace{1-\sin^2 x}_{\cos^2 x}} = 2$$

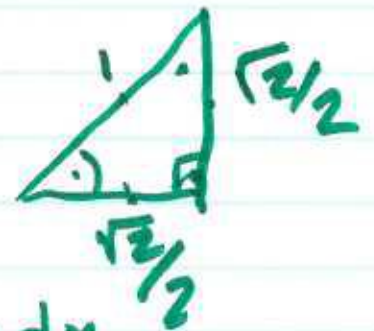
$$\int_0^{\pi/4} \frac{dx}{\cos^2 x} + \int_0^{\pi/4} \frac{\sin x}{\cos^2 x} dx$$

$$\downarrow$$

$$\int_0^{\pi/4} \sec^2 x dx + \int_0^{\pi/4} \tan x \sec x dx$$

$$\left[\tan x \right]_0^{\pi/4} + \left[\sec x \right]_0^{\pi/4}$$

$$(1-0) + \left(\frac{2}{\sqrt{2}} - 1 \right) = \sqrt{2}$$



(4)

$$\textcircled{2} \int \frac{x^2}{x^2+1} dx$$

$$\frac{dx}{dy} = \cosh y \uparrow$$

$$x = \sinh y$$

Try ~~$\frac{dx}{dy} = \sinh y$~~

$$\text{dx} = \cosh x \text{ dx}$$

~~$\frac{dx}{dy} = \sinh y$~~

$$\underline{dx = \cosh y dy}$$

$$\int \frac{\sinh^2 y}{\sinh^2 y + 1} \cdot \cosh y dy$$

$$\cosh^2 y$$

$$\int \frac{\sinh^2 y}{\cosh^2 y} dy = \int \tanh^2 y dy$$

$$\left| \int \frac{\sinh^2 y}{\cosh y} dy \right| = \int \underline{\sinh y \tanh y dy}$$

(5)

2^d try, let $x = \tan \theta$ $\left| \frac{dx}{d\theta} = \sec^2 \theta \right.$

$$\int \frac{\tan^2 \theta}{\underbrace{\tan^2 \theta + 1}_{\cancel{\sec^2 \theta}}} \sec^2 \theta d\theta$$

$$= \int \tan^2 \theta d\theta = \int (\sec^2 \theta - 1) d\theta$$

$$\int \sec^2 \theta d\theta - \int d\theta$$

↓

$\tan \theta - \theta$

$x = \arctan x$

(6)

(14) ~~$\int$~~ $\int \csc t \cdot \sin 3t \, dt$

$$\sin 3t = \sin(t+2t) = \underline{\sin t \cos 2t} + \cos t \sin 2t$$

$$= \sin t (\cos^2 t - \sin^2 t) + \cos t (2 \sin t \cos t)$$

$$\frac{1}{\sin t} (\sin t \cos^2 t - \sin^3 t + 2 \cos^2 t \sin t)$$

$$= \cos^2 t - \sin^2 t + 2 \cos^2 t$$

$$= \int 3 \cos^2 t - \sin^2 t \, dt$$

Use $\sin^2 t + \cos^2 t = 1$ | i.e. $\sin^2 t = 1 - \cos^2 t$

$$= \int (4 \cos^2 t - 1) \, dt$$

(7)

$$\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}}$$

$$\star \cos \frac{\theta}{2} = \sqrt{\frac{1 + \cos \theta}{2}}$$

$$\tan \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{1 + \cos \theta}}$$

$$\text{Let } \theta = 2t$$

$$\cos^2 t = \frac{1 + \cos 2t}{2}$$

$$2 \int \left[\frac{1 + \cos 2t}{2} - 1 \right] dt$$

$$\int \cos 2t dt = \frac{\sin 2t}{2} + C$$

⑧

$$(34) \int e^{z+e^z} dz = \int \underline{e^z e^{e^z}} dz$$

$$\text{Let } u = e^z$$

$$du = e^z dz$$

$$\int \cancel{e^z} e^u \cdot \frac{du}{\cancel{e^z}}$$

$$dz = \frac{du}{u}$$

$$\int e^u du = e^u = \boxed{e^{e^z} + C}$$

$$(16) \int \frac{\sqrt{x} dx}{1+x^3}$$

$$\text{Let } u = x^{3/2}$$

$$\int \frac{\cancel{\sqrt{x}} \left(\frac{2}{3} \frac{1}{\cancel{\sqrt{x}}} \right) du}{1+u^2}$$

$$\frac{du}{dx} = \frac{3}{2} x^{1/2}$$

$$du = \frac{3}{2} \sqrt{x} dx$$

$$= \frac{2}{3} \int \frac{du}{1+u^2}$$

$$dx = \frac{2}{3} \frac{1}{\sqrt{x}} du$$

(9)

$$\frac{2}{3} \int \frac{du}{1+u^2} = \frac{2}{3} \arctan u = \quad \quad \quad$$

$$\frac{2}{3} \arctan((\sqrt{x})^3)$$