8WZ89

late 10/20 (1)

1) Type 1 : one or both LOIs are ∞

2) Type 2 : integrand undefined @ points in interval of integration

3) Type 3 : Type 1 & Type 2

Ex: $\int_1^{\infty} \frac{dx}{x} = \lim_{M\to\infty} \int_1^{M} \frac{dx}{x} = \left[\ln x\right]_1^{M} =$

$\lim_{M\to\infty} \left[\ln M - \underbrace{\ln 1}_{0}\right] = \infty$ does not converge

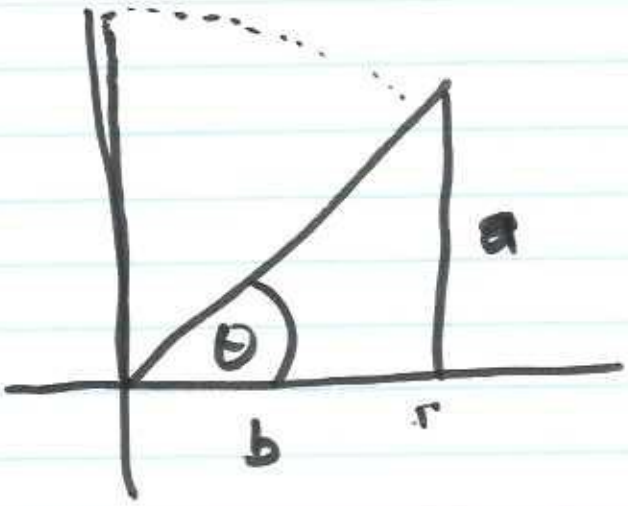
Ex: $\int_1^{\infty} \frac{dx}{\sqrt{x}} = \int_1^{M} \frac{dx}{\sqrt{x}} = \left[2\sqrt{x}\right]_1^{M} = 2\sqrt{M} - 2$

$\lim_{M\to\infty} \left(2\sqrt{M} - 2\right) = \infty$

(2)

$$\int_0^{\infty} \frac{dx}{x^2+1} = \int_0^{M} \frac{dx}{x^2+1} = \left[\arctan x\right]_0^{M}$$

$$\lim_{M\to\infty} \left[\arctan M - \arctan 0\right] = \frac{\pi}{2} - 0 = \frac{\pi}{2}$$



$$\int_1^{\infty} \frac{dx}{x^p} = \int_1^{M} \frac{dx}{x^p} = \frac{x^{1-p}}{1-p}\Big|_1^{M}$$

$$\frac{M^{1-p}}{1-p} - \frac{1}{1-p} = \frac{1}{p-1} - \frac{M^{1-p}}{p-1}$$

If $p > 1$ $(M^{1-p} \to 0)$ as $M \to \infty$

and $\int \ldots = \frac{1}{p-1}$

If $p < 1$ $\int \ldots \to \infty$

If $p = 1$ $\int \ldots \to \infty$

$$\int_1^\infty \frac{dx}{x^p} = \begin{cases} \frac{1}{p-1} & \text{if } p < 1 \\ \infty & \text{if } p \geq 1 \end{cases}$$

Type 2 case: $\int_0^3 \frac{dx}{(x-1)^{2/3}} =$

$$\int_0^{1-\varepsilon} \frac{dx}{(x-1)^{2/3}} + \int_{1+\varepsilon}^3 \frac{dx}{(x-1)^{2/3}}$$

④

part 1: $\int_0^{1-\varepsilon} \frac{dx}{(x-1)^{2/3}} \overset{u=x-1}{=} 3(x-1)^{1/3}\Big|_0^{1-\varepsilon}$

$= \sqrt[3]{-\varepsilon}^{\,3} - \sqrt[3]{-1}^{\,3} = 3$

($\varepsilon \to 0$, -1)

part 2: $\int_{1+\varepsilon}^{3} \frac{dx}{(x-1)^{2/3}} = 3(x-1)^{1/3}\Big|_{1+\varepsilon}^{3} = ?$

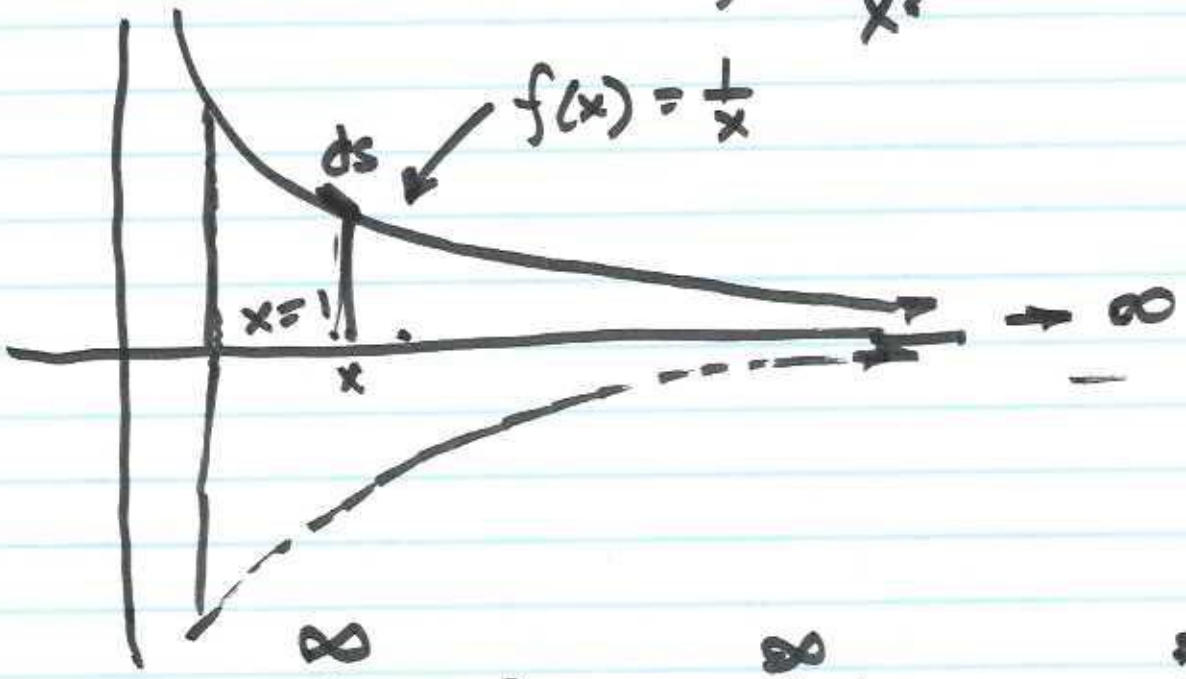
$3\sqrt[3]{2} - 3\varepsilon^{1/3} \to 3\sqrt[3]{2}$ so

$\int_0^3 \frac{dx}{(x-1)^{2/3}} = 3 + 3\sqrt[3]{2}$

⑤ $A = \int_a^b 2\pi f(x)\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$

Gabriel's Horn

$f'(x) = -\frac{1}{x^2}$



$$\text{Volume} = \int_1^\infty \pi\left(\frac{1}{x}\right)^2 dx = \pi\int_1^\infty \frac{dx}{x^2} = \pi\left(-\frac{1}{x}\right)\Big|_1^\infty$$

$$= 0 - \left(-\frac{\pi}{1}\right) = \boxed{\pi}$$

$$\text{Area} = \int \frac{2\pi}{x}\sqrt{1+\left(\frac{1}{x^2}\right)^2}\,dx$$

$$\int f(x)\,dx < \int g(x)\,dx \quad g(x) > f(x) > 0$$

Claim: $\int_1^\infty \frac{2\pi}{x}\sqrt{1+\frac{1}{x^4}}\,dx > \int_1^\infty \frac{2\pi}{x}(1)\,dx$

$$2\pi\int_1^\infty \frac{dx}{x} = 2\pi \ln x\Big|_1^\infty = \underline{2\pi\ln(\infty) = \infty}$$

Surface Area $= \infty$
Volume $= \pi$

We have used one of these theorems:

Direct Comparison Test for Con/Divergence

If $f(x)$ & $g(x)$ are continuous on $[a,\infty]$ with $0 \le f(x) \le g(x)$ then:

(i) If $\int_a^\infty g(x)\,dx < \infty$ then $\int_a^\infty f(x)\,dx < \infty$

(ii) If $\int_a^\infty f(x)\,dx = \infty$ so does $\int_a^\infty g(x)\,dx = \infty$

Limit Comparson Test

If $f(x), g(x)$ are positive functions on $[a, \infty)$ then:

If $\lim_{x \to \infty} \frac{f(x)}{g(x)} = L \quad 0 < L < \infty$

then both $\int_a^\infty f(x)dx$ & $\int_a^\infty g(x)dx$ either converge or diverge together

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Consider $f(x) = \frac{1}{x^2}$, $g(x) = \frac{1}{1+x^2}$

on $[a, \infty]$ $a > 0$, what can you say about $\int_a^\infty \frac{dx}{1+x^2}$? First note $g(x) < f(x)$

$$\int_a^\infty \frac{dx}{x^2} = \left(-\frac{1}{x}\right)\Big|_a^\infty = -\frac{1}{\infty} - \left(-\frac{1}{a}\right) = \frac{1}{a}$$

(8)

(3) $$\int_0^1 \frac{dx}{\sqrt{x}} = \lim_{c\to 0} \int_c^1 \frac{dx}{\sqrt{x}} = \lim_{c\to 0} (2\sqrt{x})\Big|_c^1$$

$$= \lim_{c\to 0} \left[2\sqrt{1} - 2\sqrt{c}\right] = 2 - 0 = 2$$

(15) $$\int_0^1 \frac{x+1}{\sqrt{x^2+2x}}\,dx = \int_0^1 \frac{(x+1)\,dx}{\sqrt{(x+1)^2-1}}$$

let $u = x+1$ $$= \int_{x=0}^{1} \frac{u\,du}{\sqrt{u^2-1}}$$

Let $u = \sec\theta \Rightarrow du = \sec\theta\tan\theta\,d\theta$

$$\int_{x=0}^{1} \frac{(\sec\theta)(\sec\theta\tan\theta)\,d\theta}{\tan\theta}$$

$$= \int_{x=0}^{1} \sec^2\theta\,d\theta = \tan\theta\Big|_{x=0}^{1}$$

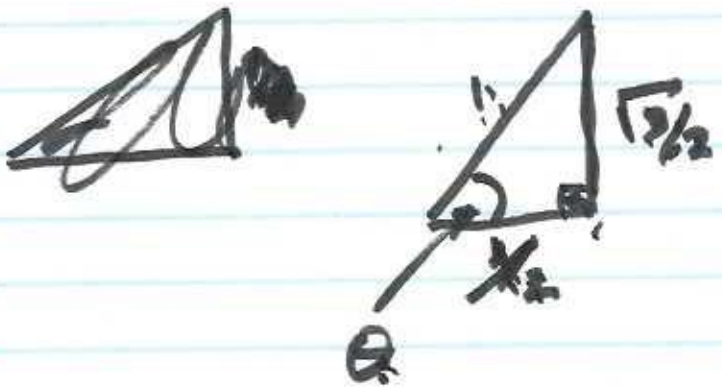
(9)

$\theta = \operatorname{arcsec}(x+1)$

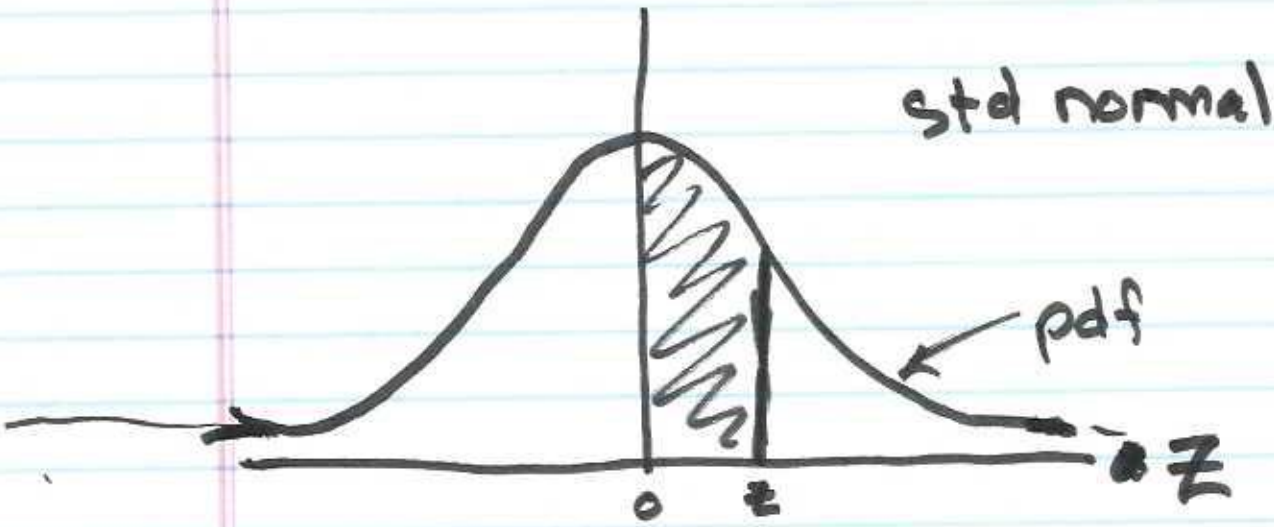
$\left[\operatorname{arcsec}(x+1)\right]'_{x=0} = \operatorname{arcsec}' 2 - \operatorname{arcsec}' 1$

$\operatorname{arcsec}' 1 \to 0$

$\tan \operatorname{arcsec} 2 \cdot 0$



$= \sqrt{3}$



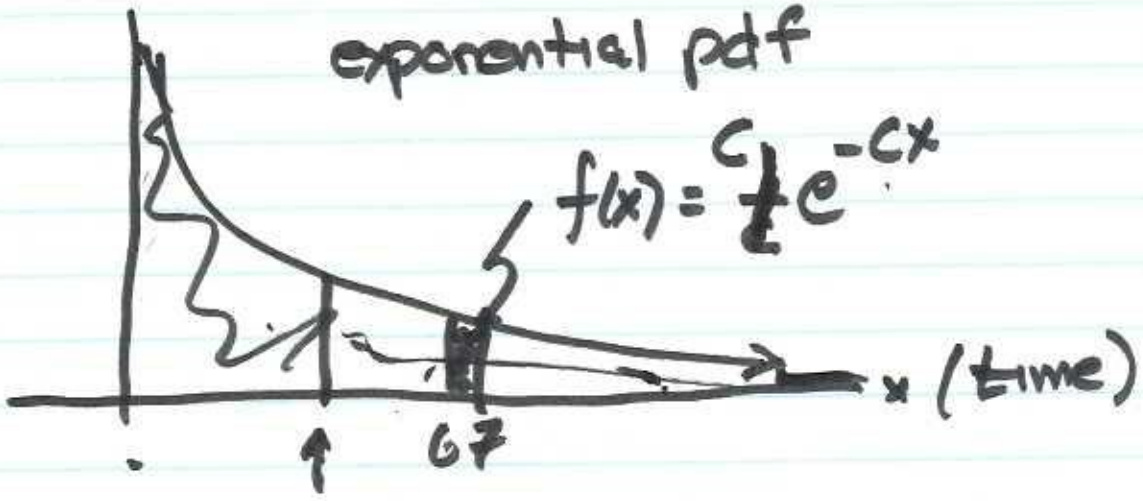
pdf ≥ 0

$\int_R pdf = 1$

$\lim_{x \to \pm\infty} f(x) = 0$

$$z(0;1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$\lim_{x\to\infty} f(x) = 0 \quad \text{and} \quad \lim_{x\to-\infty} f(x) = 0$$



$$c\int_0^\infty e^{-cx}\,dx = \left[\frac{1}{c}\frac{e^{-cx}}{-c}\right]_0^\infty = 2$$

$$-\frac{1}{c}e^{-cx}\Big|_0^\infty = 0 - (-1) = \boxed{1}$$

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