

XZP32

late
10/13
①

Study Day - Oct 27
Exam Day - Oct 29

Pythagorean Th^{ms}

BIG 1) $\cos^2 x + \sin^2 x = 1$
Still important 2) $1 + \tan^2 x = \sec^2 x$
ignore 3) $1 + \cot^2 x = \csc^2 x$

Ex 1 $\int \tan^2 x dx \stackrel{?}{=} \int (\sec^2 x - 1) dx =$

$$\int \sec^2 x dx - \int dx = \boxed{\tan x - x + C}$$

Ex 2 $\int \tan^4 x dx \stackrel{?}{=} \int \tan^2 x (\sec^2 x - 1) dx \rightarrow$

$$\int \tan^2 x \underbrace{\sec^2 x dx}_{d(\tan x)} - \int \tan^2 x dx$$

$\downarrow$ $\downarrow$

$\frac{\tan^3 x}{3}$ $\tan x - x$

$\left. \begin{array}{l} \frac{\tan^3 x}{3} \\ \tan x - x \end{array} \right\}$

(2)

$$\text{Ex 3} \quad \int \sec^3 x dx \stackrel{?}{=} \int \underbrace{\sec x}_u \cdot \underbrace{\sec^2 x dx}_{dv} \quad \rightarrow$$

$$u = \sec x \quad du = \sec x \tan x$$

$$dv = \sec^2 x dx \quad v = \tan x$$

$$= \sec x \tan x - \int \tan x (\sec x \tan x) dx$$

$$= \sec x \tan x - \int \sec x \tan^2 x dx$$

$\downarrow$
 $\sec^2 x - 1$

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1) dx$$

$$= \sec x \tan x - \int \sec^3 x - \sec x dx$$

$$\int \sec^3 x dx = \sec x \tan x - \int \sec^3 x dx - \int \sec x dx$$

(3)

$$2 \int \sec^3 x dx = \sec x \tan x - \ln |\sec x + \tan x| + C$$

Aside $\int \sec x \left(\frac{\sec x + \tan x}{\sec x + \tan x} \right) dx \rightarrow$

$$\int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx = \ln |\sec x + \tan x|$$

Products of Sines & Cosines

$$\int \sin mx \cos nx dx = ?$$

Need product to sum trig formulas

$$\sin(\overset{\Theta_1}{mx} + \overset{\Theta_2}{nx}) = \sin mx \cos nx + \cos mx \sin nx$$

$$\sin(mx - nx) = \underline{\sin mx \cos nx - \cos mx \sin nx}$$

$$\sin(m+n)x - \sin(m-n)x = \underline{2 \sin mx \cos nx}$$

④

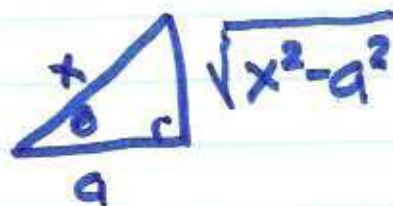
$$\int \sin^{\textcircled{m}} 3x \cos^{\textcircled{n}} 5x dx =$$

$$\frac{1}{2} \int [\sin 8x + \sin (12x)] dx$$

$$\frac{1}{2} \left[-\frac{\cos 8x}{8} + \frac{-\cos 12x}{12} \right]$$

$$\int \frac{x^2 dx}{\sqrt{a^2 + x^2}}$$

Magic Triangles



$$\frac{x}{a} = \tan \theta$$

$$x = a \tan \theta$$

$$dx = a \sec^2 \theta d\theta$$

$$\sqrt{a^2 + x^2} = a \sec \theta$$

$$\frac{x}{a} = \sin \theta$$

$$x = a \sin \theta$$

$$dx = a \cos \theta d\theta$$

$$\sqrt{a^2 - x^2} = a \cos \theta$$

$$\frac{x}{a} = \sec \theta$$

$$x = a \sec \theta$$

$$dx = a \sec \theta \tan \theta d\theta$$

$$\sqrt{x^2 - a^2} = a \tan \theta$$

⑤

Ex. $\int \frac{dx}{\sqrt{4+x^2}} = ?$ $a=2$
 $x = 2 \tan \theta$
 $dx = 2 \sec^2 \theta d\theta$
 $\sqrt{4+x^2} = 2 \sec \theta$

$$\int \frac{2 \sec^2 \theta d\theta}{2 \sec \theta} = \int \sec \theta d\theta$$

$$\boxed{\ln |\sec \theta + \tan \theta|}$$

$$\tan(\arctan \theta) = \theta$$

in terms of x : $\ln \left| \sec\left(\arctan \frac{x}{2}\right) + \frac{x}{2} \right|$

Ex. $\int \frac{x^2 dx}{\sqrt{9-x^2}} = ?$ $a=3$
 $x = 3 \sin \theta$
 $dx = 3 \cos \theta d\theta$
 $\sqrt{9-x^2} = 3 \cos \theta$

$$\int \frac{(9 \sin^2 \theta)(3 \cos \theta) d\theta}{3 \cos \theta} = 9 \int \sin^2 \theta d\theta$$

⑥

$$\sin^2 \frac{\theta}{2} = \frac{1 - \cos \theta}{2}$$

$$\cos^2 \frac{\theta}{2} = \frac{1 + \cos \theta}{2}$$

$$\tan^2 \frac{\theta}{2} = \frac{1 - \cos \theta}{1 + \cos \theta}$$

$$9 \int \frac{1 - \cos 2\theta}{2} d\theta = 9 \left[\int \frac{d\theta}{2} - \int \frac{\cos 2\theta}{2} d\theta \right]$$

$$= \frac{9}{2} \left[\theta - \frac{\sin 2\theta}{2} \right] -$$

$$= \frac{9}{2} \left[\theta - \frac{\sin(2 \arcsin \frac{x}{3})}{2} \right]$$

⑦

Ex. $\int \frac{dx}{\sqrt{25x^2 - 4 \cdot \frac{25}{25}}} = \int \frac{dx}{5\sqrt{x^2 - (\frac{2}{5})^2}}$

$\frac{1}{5} \int \frac{dx}{\sqrt{x^2 - (\frac{2}{5})^2}} \quad x = \frac{2}{5} \sec \theta$
 $dx = \frac{2}{5} \sec \theta \tan \theta d\theta$
 $\sqrt{x^2 - (\frac{2}{5})^2} = \frac{2}{5} \tan \theta$

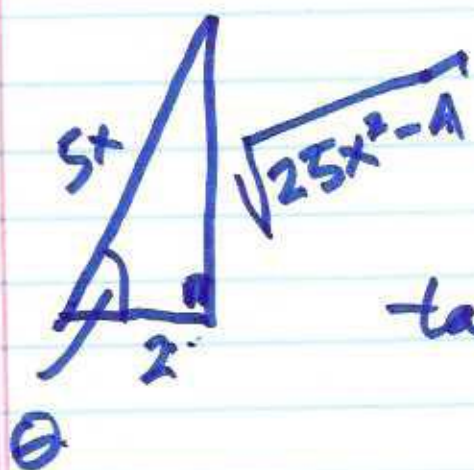
$\frac{1}{5} \int \frac{\frac{2}{5} \sec \theta \tan \theta d\theta}{\frac{2}{5} \tan \theta} = \frac{1}{5} \int \sec \theta d\theta$

$\frac{1}{5} \ln |\sec \theta + \tan \theta|$

Note: $\theta = \operatorname{arcsec} \frac{5x}{2}$ $\tan x = \sqrt{\sec^2 x - 1}$

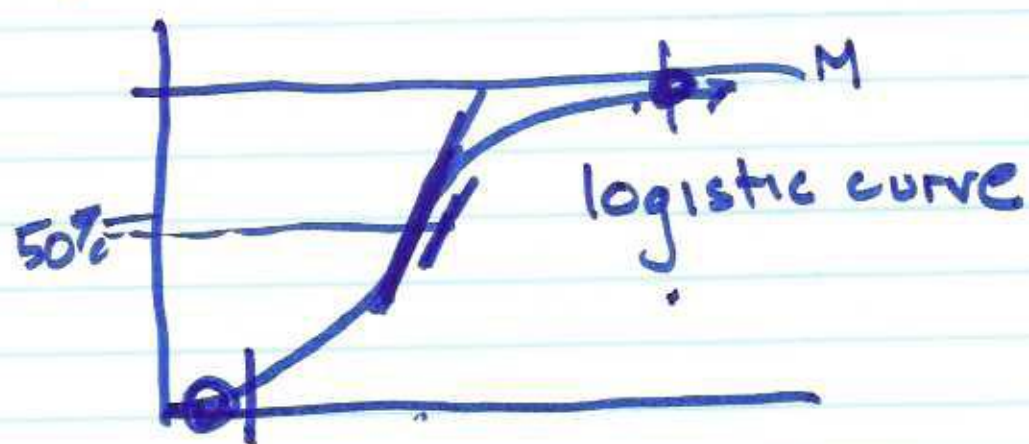
$\frac{1}{5} \ln \left| \frac{5x}{2} + \tan(\operatorname{arcsec} \frac{5x}{2}) \right|$

⑧



$$\tan \theta = \frac{1}{2} \sqrt{25x^2 - 1}$$

$y'(t) = k y(t)(M - y(t))$ von Holst c 1840



$$\frac{dy}{dt} = k y \cdot (M - y)$$

$$\frac{dy}{y \cdot (M - y)} = k dt \quad \frac{1}{y(M - y)} = \frac{\textcircled{A}}{y} + \frac{\textcircled{B}}{M - y}$$