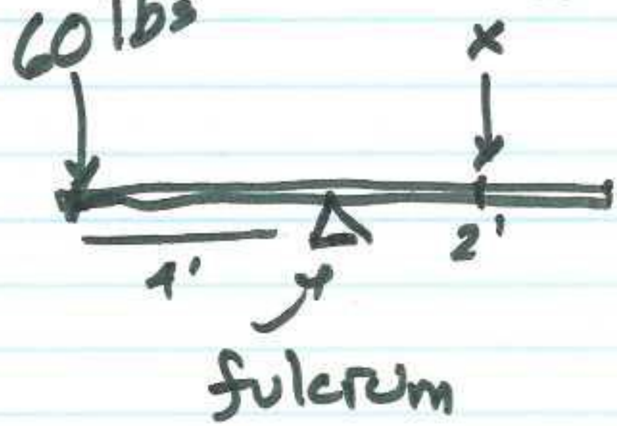
① 9/3 early



moment

<u>moment</u> is $F \cdot l$

torque = force × lever arm

τ for torque

CCW torque (or moment) is +

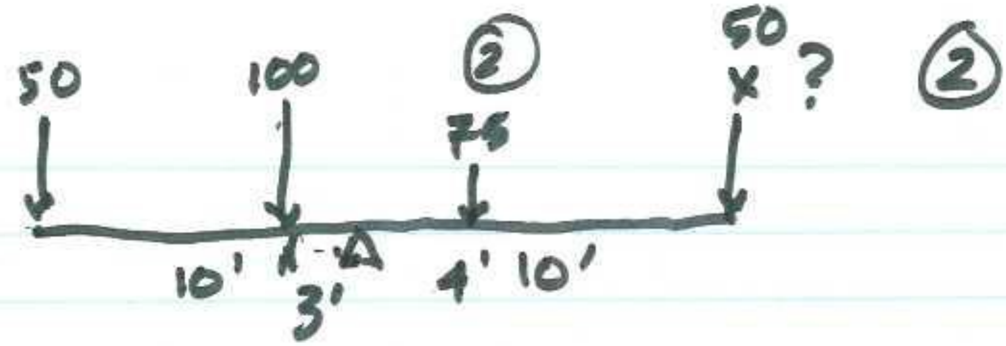
CW torque is −

τ_{ccw} = 240 ft-lbs

τ_{cw} = 2x ft-lbs

For stability

$2x = 240$ or $x = \underline{120 \text{ lbs}}$

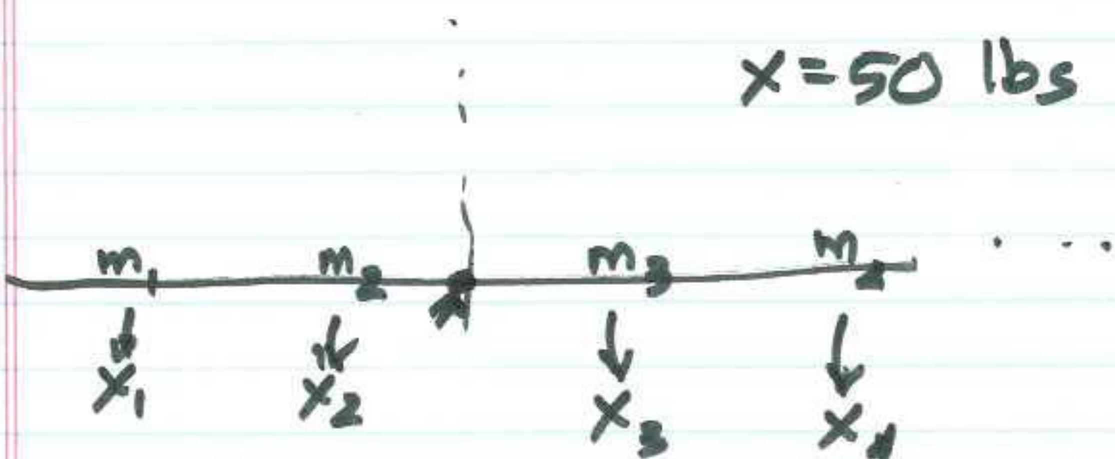


$+$ CCW $\quad 50 \cdot 10$, $\; 100 \cdot 3 \quad = 800$ ft-lbs CCW

$-$ CW $\quad 75 \cdot 4$, $\; x \cdot 10 \quad = 300 + 10x$ CW

500 ccw $= 10x$ cw

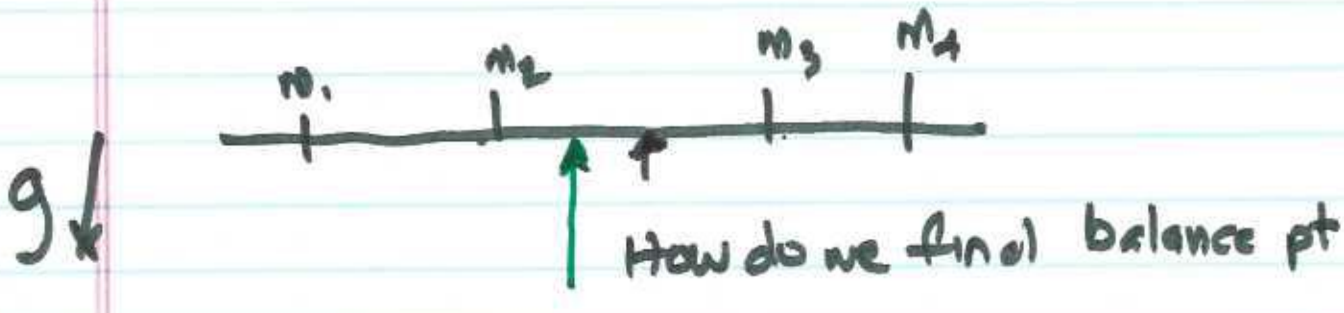
$x = 50$ lbs

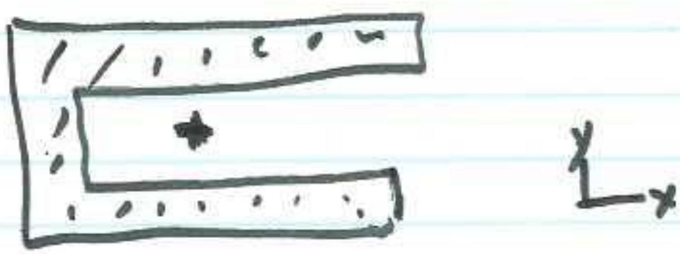


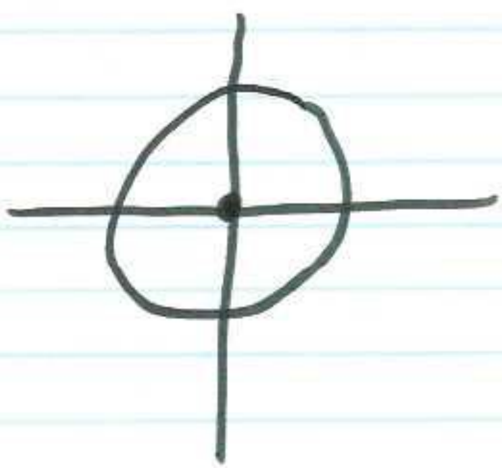
$$m_1 \cancel{g} x_1 + m_2 \cancel{g} x_2 = m_3 \cancel{g} x_3 + m_4 \cancel{g} x_4$$

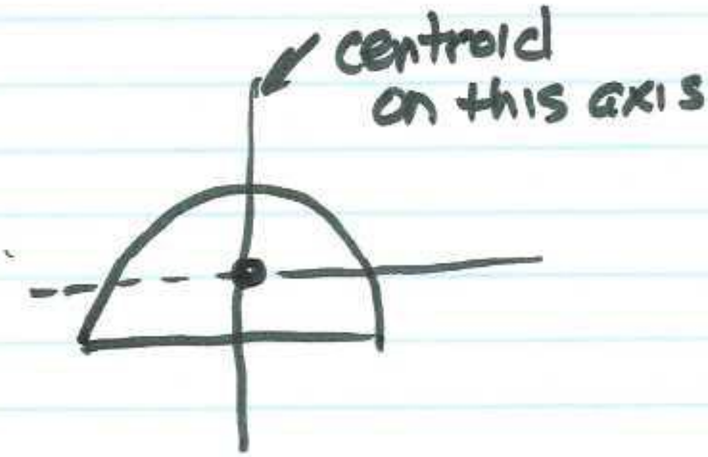
mass moments $\quad m_1 x_1 + m_2 x_2 = m_3 x_3 + m_4 x_4$

$$0 = \sum_{i=1}^{n} m_i x_i$$ ← can be negative





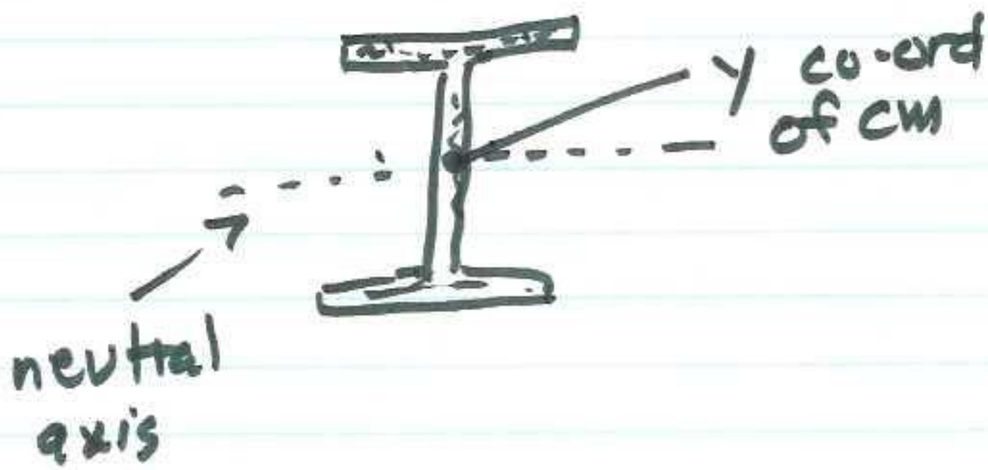




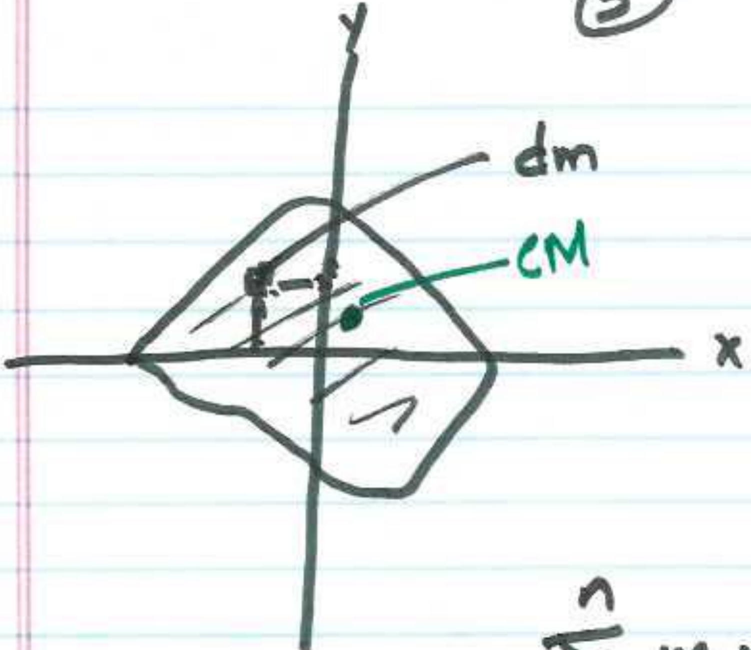
To find balance pt

$$\{m_1, m_2, \cdots, m_n\}$$
$$\{x_1, x_2 \cdots x_n\}$$

$$\frac{\sum_{n=1}^{4} m_i x_i}{\sum_{n=2}^{1} m_i} \qquad \sum m_i \longrightarrow M$$



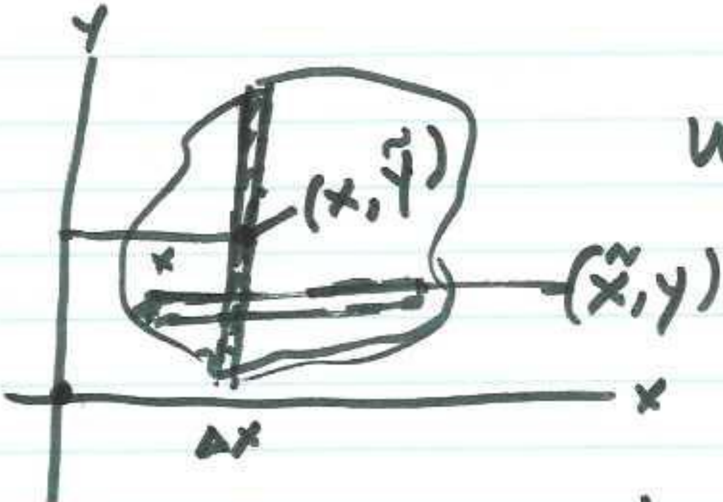
⑤



$(\bar{x}, \bar{y})$

$$\bar{x} = \frac{\sum_{i=1}^{n} m_i x_i}{\sum_{i=1}^{n} m_i} \rightarrow M$$

$$\bar{y} = \frac{\sum_{i=1}^{n} m_i y_i}{\sum_{i=1}^{n} m_i}$$



What are $\bar{x}$ & $\bar{y}$?

$$M_y = \frac{\sum \tilde{x}\,\Delta m}{\sum \Delta m} \qquad M_x = \frac{\sum \tilde{y}\,\Delta m}{\sum \Delta M}$$

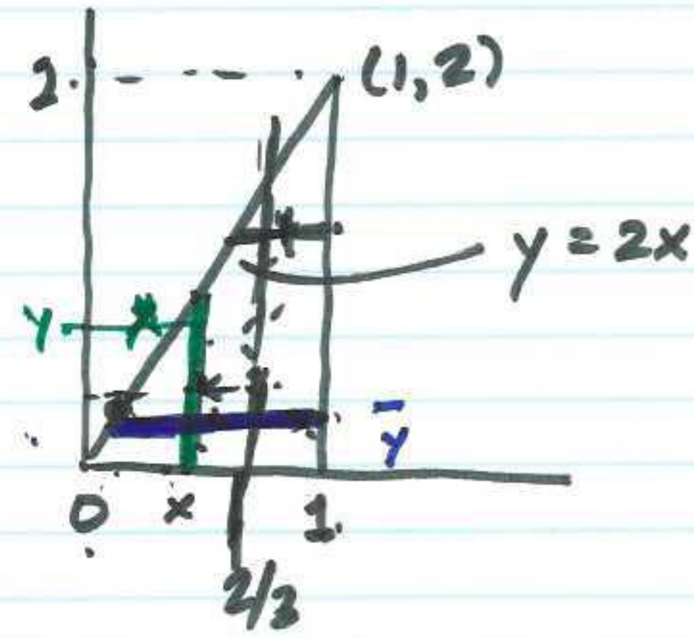
$$\bar{x} = \frac{\int \tilde{x}\,dm}{\int dm} \qquad \bar{y} = \frac{\int \tilde{y}\,dm}{\int dm}$$

total mass

linear:
areal δ
volumetric ρ

ρ or δ

$\delta = 3 g/cm^2$



tilde ~

① Find M_y

length of vert strip @ x is $2x$
width " " " " " is dx
area " " " " " is $2x\,dx$
dm mass " " " " " is $\underbrace{2 \cdot 3 \cdot x\,dx}_{6x\,dx}$

$\tilde{x} = x$

$\tilde{x}\,dm = 6x^2\,dx$

$$M_y = \int_0^1 \tilde{x}\,dm = \int_0^1 6x^2\,dx = \left[2x^3\right]_0^1 = 2$$

$$M = \int_0^1 dm = \int_0^1 6x\,dx = 3x^2\Big|_0^1 = 3$$

so... $\bar{x} = \dfrac{M_y}{M} = \dfrac{2}{3}$

Want M_x (because $\bar{y} = \dfrac{M_x}{M}$)

"blue" strip goes from $\dfrac{y}{2}$ to 1

$\dfrac{1-\frac{y}{2}}{2} + \dfrac{y}{2}$ is location of

$$\frac{2-y}{4} + \frac{y}{2} = \frac{2-y+2y}{4} = \frac{y+2}{4} = \tilde{x}$$

length of strip $= 1 - \dfrac{y}{2} = \dfrac{2-y}{2}$

width " " dy

area $= \dfrac{2-y}{2}\,dy$

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mass of strip $\frac{2-y}{2} \cdot 3\,dy$

$\tilde{x} = \frac{y+2}{4}$

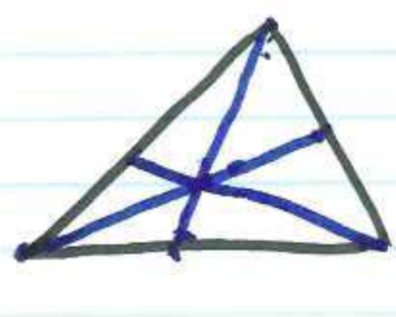
moment about y axis $\tilde{x}\,dm = \left(\frac{y+2}{4}\right)\left(\frac{2-y}{2}\right) \cdot 3\,dy$

$\tilde{x}\,dm = 3 \cdot \frac{4-y^2}{8} = \frac{3}{8}(4-y^2)\,dy$

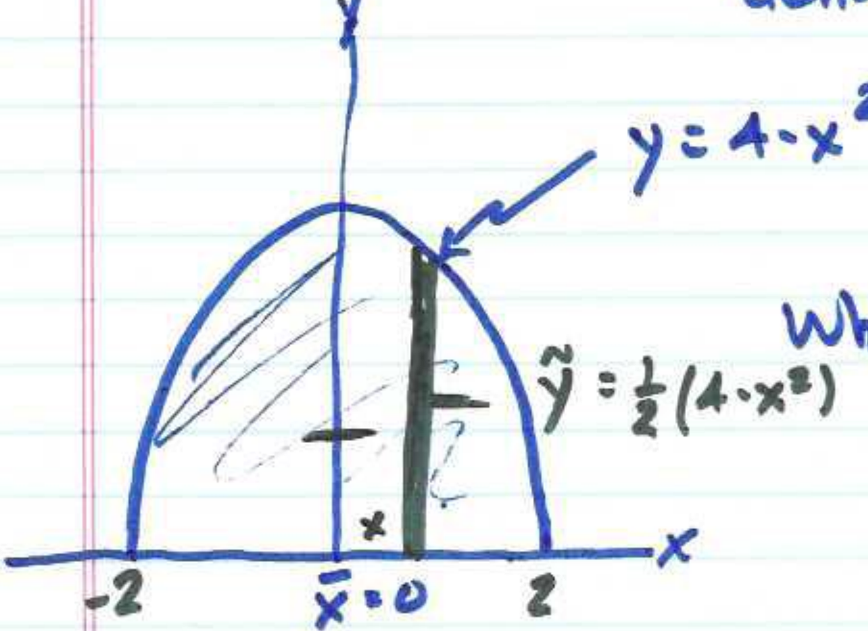
$M_y = \frac{3}{8}\int_0^2 (4-y^2)\,dy = \frac{3}{8}\left[4y - \frac{y^3}{3}\right]_0^2 = 2$

$\frac{3}{\cancel{8}}\left[\frac{\cancel{16}}{3}\right]^2 = 2$

So $\bar{x} = \frac{M_y}{M} = \frac{2}{3}$



density = δ



height of strip $= 4 - x^2$
location of $\tilde{y} = \frac{1}{2}(4 - x^2)$

length of strip $4 - x^2$
width " " dx
area " " $(4 - x^2)\,dx$
dm " " $\delta(4 - x^2)\,dx$

$$\tilde{y}\,dm = \frac{1}{2}(4 - x^2) \cdot \delta(4 - x^2)\,dx$$

$$M_x = \frac{\delta}{2}\int_{-2}^{2} (4 - x^2)^2\,dx$$

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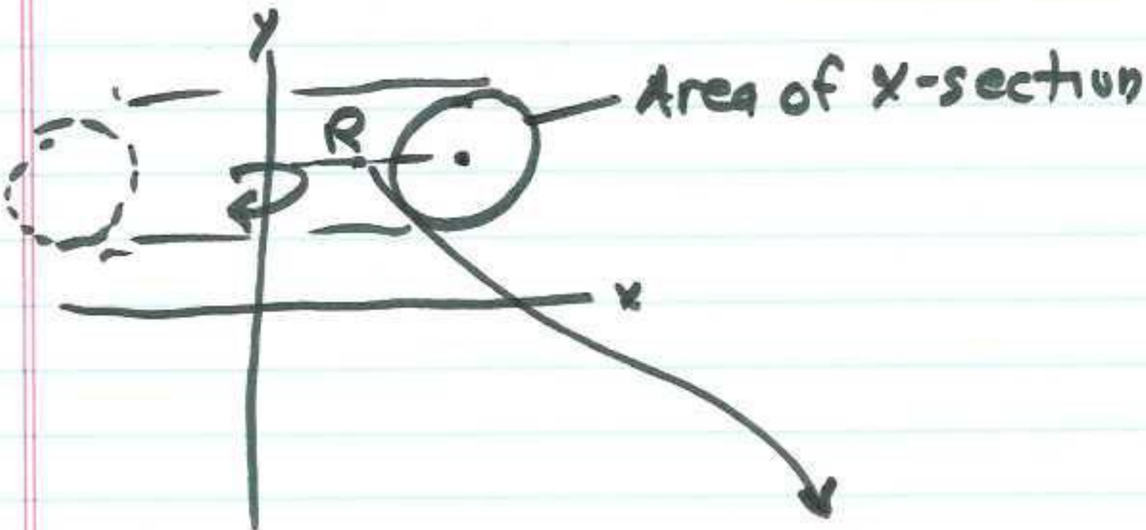
$$M_x = \frac{\delta}{2}\int_{-2}^{2}(16-8x^2+x^4)\,dx$$

$$M_x = \frac{\delta}{2}\left[16x-\frac{8x^3}{3}+\frac{x^5}{5}\right]_{-2}^{2}$$

$$M = \int_{-2}^{2} dm = \int_{-2}^{2}\delta(4-x^2)\,dx = \delta\left[4x-\frac{x^3}{3}\right]_{-2}^{2}$$

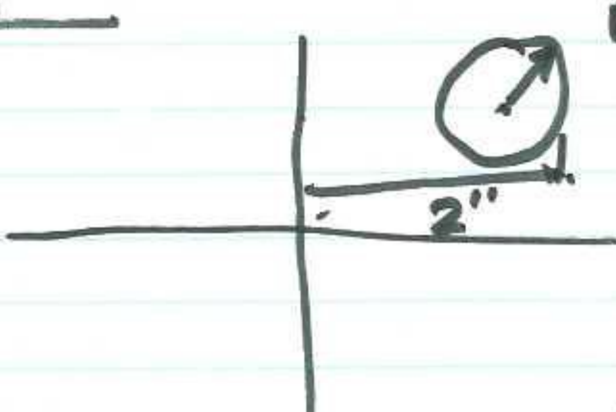
$$\frac{M_x}{M} = \bar{y}$$

Theorem of Pappus:

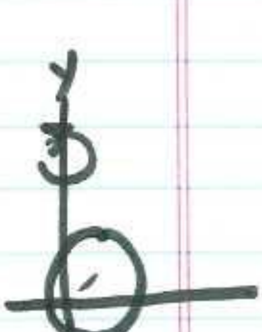


Area of section × $2\pi R$ = Volume of figure

radius of tube = 1"



R = 1.5"



$2\pi R(1) = 2\pi(1.5) = 3\pi = \underline{9.4 \text{ cu in}}$