4BEX8

early 9/17 ①

Exponential Growth

- arises from following (differential) equation:

$$y'(x) = ky(x)$$

some constant (+ or −)

"rate of growth/decay proportional to amount present" x is usually (not always) a time variable

$$\frac{dy}{dx} = ky \Rightarrow \frac{dy}{y} = k\,dx$$

$$\int \frac{dy}{y} = \int k\,dx$$

$$\boxed{\ln|y| = kx + C}$$

$$\ln y = kx + C$$

$$e^{\ln y} = e^{kx+c}$$

$$y = e^{kx} \cdot e^{c} \to C$$

$$\boxed{y = Ce^{kx}}$$

exponential growth if $k > 0$
decay if $k < 0$

$$F = Pe^{.02t}$$ PERT

P E R T

^{14}C

$$Q(t) = Q_0 e^{-.00012t}$$

$$\frac{Q(t)}{Q_0} = e^{-.00012t}$$

$$.92 = e^{-.00012t}$$

3

$\ln(.92) = -.00012t$

$$t = \frac{\ln(.92)}{-.00012} = 695$$

Lascaux $\sim 16,000$

$$\frac{Q(\text{now})}{Q(\text{then})} = e^{-.00012(16,000)}$$

$t_{1/2} \sim \underline{5776}$ yrs.

$$\frac{Q_{1/2}}{Q} = (.50) = e^{-.00012 t_{1/2}}$$

$$\ln(\tfrac{1}{2}) = -.69315 = -.00012 t_{1/2}$$

Separable Equations

$$\frac{dy}{dx} = f(x,y)$$

Solve: $\frac{dy}{dx} = (1+y)\cdot e^x \qquad y > -1$

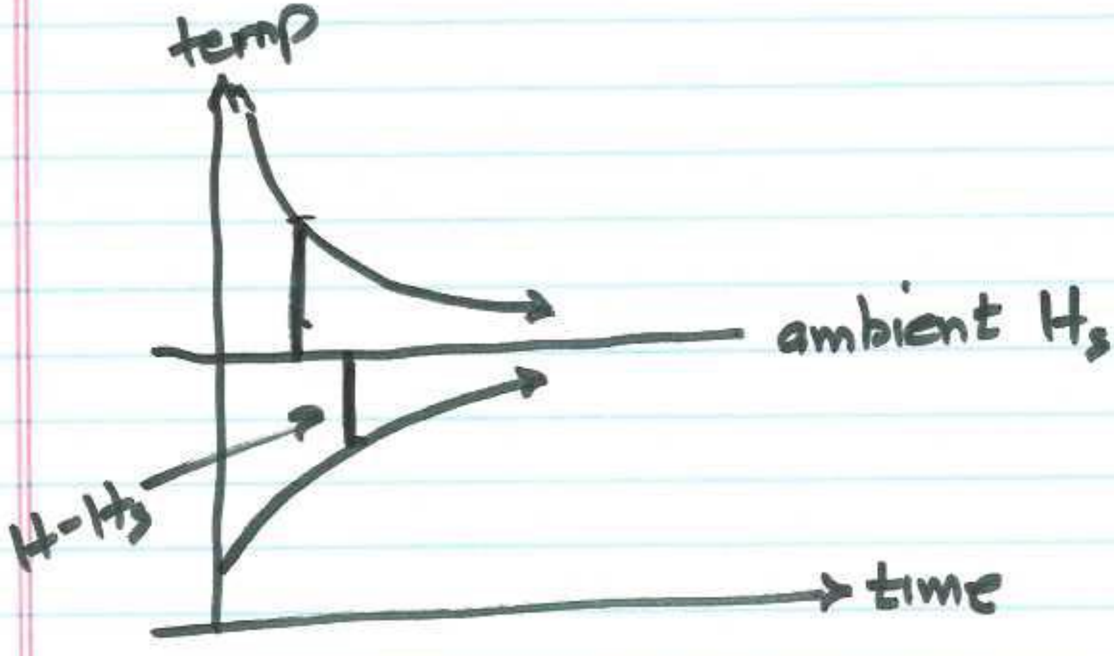
$$\int \frac{dy}{1+y} = \int e^x dx$$

"

$$e^{\ln|1+y|} = e^{e^x + C}$$

$$1+y = e^C \cdot e^{e^x}$$

$$y = \left[e^C \cdot e^{e^x} - 1\right]$$



Newton's Law :

H = temp of object (°C)

H_s = ambient temp (°C)

t = time (min)

k = cooling /heating parameter

$$\frac{dH}{dt} = -k\left(H - H_s\right)$$

$$\frac{dH}{H - H_s} = -k\,dt$$

Another shot: let $y = H - H_s$

$$\frac{dy}{dt} = \frac{dH}{dt} - \underbrace{\cancel{\frac{dH_s}{dt}}}_{0} = \frac{dH}{dt} = -ky$$

$$\frac{dH}{dt} = -ky \qquad \frac{dy}{dt} = -ky$$

$$\frac{dy}{y} = -k\,dt$$

$$\ln|y| = -kt + c$$

★ $$y = e^{-kt} \cdot \underbrace{e^{c}}_{y_0} = y_0 e^{-kt}$$

7

Egg @ 98°C is put in 18°C water.

After 5 min., egg is @ 38°C

Q: When will egg be @ 20°C.

$$y = H - H_s = \underbrace{(H_0 - H_s)}_{y_0} e^{-kt}$$

$H_s = 18°C \quad H_0 = 98°C$

$$H = 18 + (98-18)e^{-kt} = \boxed{18 + 80e^{-kt}}$$

Note: $38 = 18 + 80e^{-5k}$

$$0.25 = e^{-5k}$$

$$-1.3862 = -5k$$

$$k = 0.277$$

$$H = 18 + 80e^{-.277t}$$

$$20 = 18 + 80e^{-.277t_{20}}$$

$$\frac{1}{40} = \frac{2}{80} = e^{-.277 t_{20}}$$

$$\ln\left(\frac{1}{40}\right) = -.277\, t_{20}$$

$t_{20} = 13$ min.

time from now ($38°C$) to cool ($20°C$) is

$= 13 - 5 = $ 8 minutes

Hyperbolic functions:

$$\sinh x := \frac{1}{2}(e^x - e^{-x})$$

$$\cosh x := \frac{1}{2}(e^x + e^{-x})$$

$$\tanh x := \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\operatorname{csch} x := \frac{1}{\sinh x}$$

catenary

$$\operatorname{sech} x := \frac{1}{\cosh x}$$

$$\coth := \frac{1}{\tanh x}$$

$\int \rightarrow$ $\cosh^2 x = 1 + \sinh^2 x$

$y = \sinh x$ want $x = f(y)$?

$$y = \frac{e^x - e^{-x}}{2}$$

$$2y = e^x - e^{-x}$$

$$e^x - e^{-x} - 2y = 0$$

e^x: $e^{2x} - 1 - 2ye^x = 0$

$$(e^x)^2 - 2y(e^x) - 1 = 0$$

$$e^x = \frac{2y \pm \sqrt{4y^2+4}}{2} = y \pm \sqrt{y^2+1}$$

$$e^x = y + \sqrt{y^2+1}$$

$$x = \ln\left(y + \sqrt{y^2+1}\right)$$

$$\frac{f'(x)}{f(x)} = \frac{d}{dx}\left(\ln f(x)\right)$$

$$\lim_{x \to \infty}\left(\frac{x^n}{e^x}\right) = 0$$

Given: $f(x)$, $g(x)$ and $\lim\limits_{x \to \infty}\left(\frac{f(x)}{g(x)}\right) = 0$

we say g grows faster than f

but if $\lim\limits_{x \to \infty}\left(\frac{f(x)}{g(x)}\right) = L > 0$