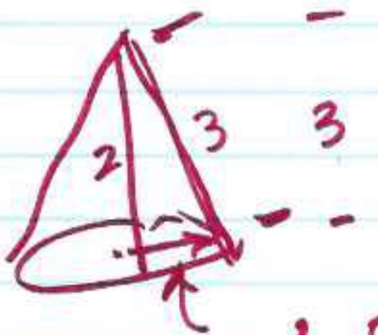
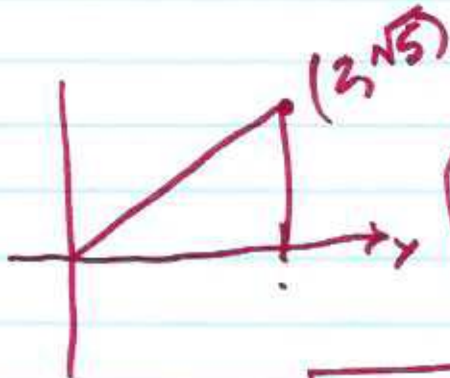
9SMXT

early
9/15
1



$3^2-2^2=5$ so rad $=\sqrt{5}$

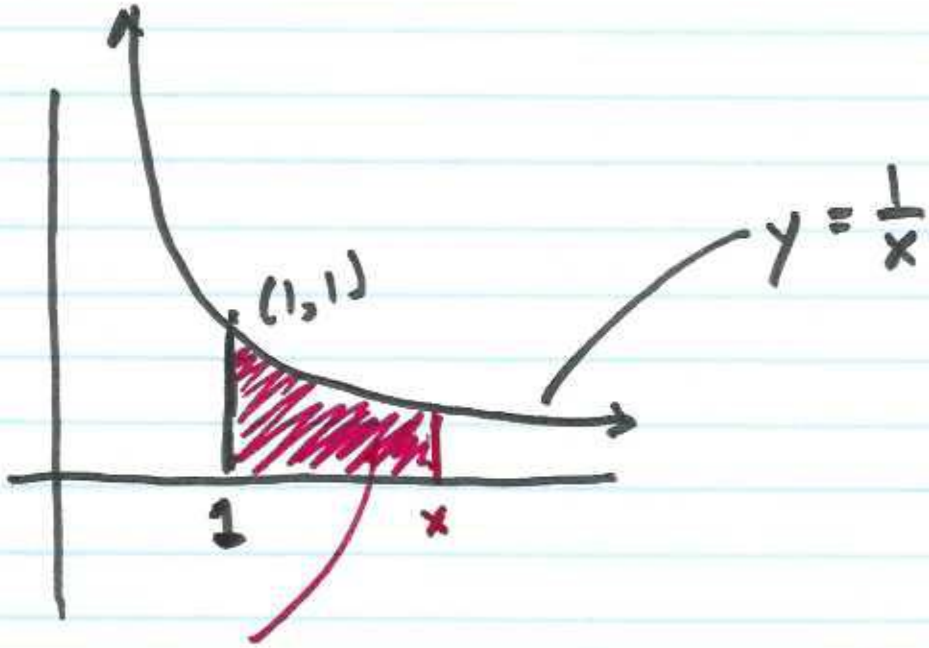


$$y=\frac{\sqrt{5}}{2}x$$

$$\frac{dy}{dx}=\frac{\sqrt{5}}{2}$$

$$\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$1+\frac{5}{4} \quad 9/4$$



This area is <u>defined</u> as $\ln x$

What is $\ln(1) = 0$

$$\ln x = \int_1^x \frac{dx}{x}$$

bad form to use same symbol

Cleaner $\ln x = \int_1^x \frac{dt}{t}$

<u>FTC</u> $(\ln x)' = \left.\frac{1}{t}\right|_{t=x} = \frac{1}{x}$

By def^{n} $x = \ln(e^x)$

$= e^{\ln x} \leftarrow$

$x = e^{\ln x}$ $\qquad (e^{g(x)})'$

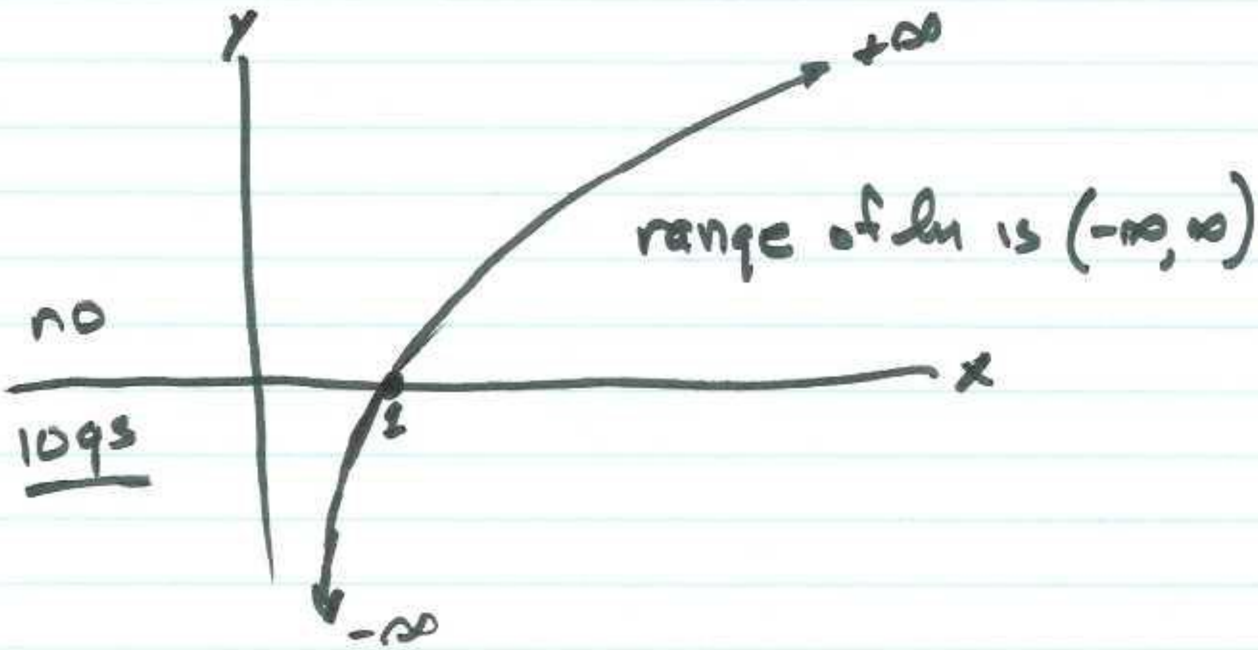
$1 = e^{\ln x} \cdot \underline{(\ln x)'}$

$1 = x \cdot (\ln x)'$

$\frac{1}{x} = (\ln x)'$

Range / Domain ??

domain of ln is $(\underline{0}, \infty)$



③

Let A, B be in domain of $\ln$ i.e. >0

also $x = \ln A$, $y = \ln B$

① $\ln(A \cdot B) = ?$ $A = e^x$, $B = e^y$

so $A \cdot B = e^x \cdot e^y = e^{x+y}$

$$\boxed{\ln(A \cdot B) = x + y = \ln A + \ln B}$$

② $\ln\left(\frac{A}{B}\right) = ?$ $\frac{A}{B} = \frac{e^x}{e^y} = e^{x-y}$

$$\boxed{\ln(A/B) = x - y = \ln A - \ln B}$$

③ $\ln(A^m) = ?$ $A^m = (e^x)^M = e^{Mx}$

$$\boxed{\ln(A^m) = Mx = M(\ln A)}$$

④ $\ln(\sqrt[m]{A}) = ?$ $\sqrt[m]{A} = A^{1/m}$ use (3)

$$= \frac{1}{m} \ln A$$

④

$$\ln\left(\sqrt[5]{7^2}\right) = ? = \frac{2}{5}\ln 7$$

$$\ln\left(\sqrt[8]{32}\right) = ? \quad \text{given } \ln 2 = .69315+$$

$$= (.625)(.69315) = \underline{\quad}$$

$$\frac{d}{dx}\left(2^x\right) = ? \qquad 2 = e^{\ln 2},\ 2^x = e^{x\ln 2}$$

$$\to \underbrace{e^{x\ln 2}}_{2^x} \cdot (\ln 2) = \ln 2 \cdot 2^x$$

$$\ln\left(\sqrt[5]{\frac{1}{100}}\right) \quad \ln\left(10^{-2/5}\right)$$

$$10 = e^{\ln 10} \qquad \ln 10 = 2.30258$$

$$\ln\left(e^{\ln 10 \cdot (-2/5)}\right) = \ln\left(e^{-\frac{2\ln 10}{5}}\right)$$

$\log_a A = x \Rightarrow a^x = A$

$\Leftarrow$

logarithmus naturalis

ln	means base is e	natural
log	means base is 10	common
$\log_b$	" " " b	arbitrary
lg	" " " 2	binary

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1. $\int_{-3}^{-2} \frac{dx}{x} = [\ln x]_{-3}^{-2} \rightarrow [\ln x]_{3}^{2}$ $\ln 2 - \ln 3 =$ $\ln 2/3$

$$\boxed{\int \frac{dx}{x} = \ln|x|}$$

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$$\int \sec x \, dx = ?$$

$$\int \sec x \left(\frac{\sec x + \tan x}{\sec x + \tan x} \right) dx$$

$$\int \frac{\sec^2 x + \sec \tan x}{\sec x + \tan x} \, dx \rightsquigarrow \int \frac{du}{u}$$

$$(\sec x + \tan x)'$$

Let $u = \sec x + \tan x$

$du = \sec^2 x + \sec \tan x \, dx$

$$\ln|\sec x + \tan x| + C$$

6 $\displaystyle\int \frac{\sec y \tan y}{2+\sec y}\,dy$

Let $u = 2+\sec y$

$\frac{du}{dy} = \sec y \tan y$

$du = (\sec y \tan y)\,dy$

$\Rightarrow \displaystyle\int \frac{du}{u} = \ln|u| = \ln|2+\sec y| + C$

17 $\displaystyle\int 2te^{-t^2}\,dt = ?$

Let $u = t^2$ so $du = 2t\,dt$

$\displaystyle\int e^{-u}\,du = -e^{-u} + C = -e^{-t^2} + C$

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$$\int_0^{\sqrt{\ln \pi}} 2x e^{x^2} \cos(e^{x^2})\, dx$$

$$u = e^{x^2} \Rightarrow du = e^{x^2}(2x)\, dx$$

$$\int \cos u \, du = \sin u$$

$$\left[\sin(e^{x^2})\right]_0^{\sqrt{\ln \pi}} = \sin \pi - \sin 1$$

$$\downarrow$$

$$0 = -\sin 1$$